Statistics of Nuclear Observables: From Level Density to Neutron Width Distribution

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NSCL/ Michigan State University

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Workshop on Level Density and Gamma Strength

Oslo, May 25, 2011

OUTLINE

1. **Shell model and level density**
2. **Statistics of unstable quantum states**
3. **Porter-Thomas distribution or what ?**
4. **Quantum signal transmission**

THANKS

- Mihai Horoi (Central Michigan University)
- Roman Sen’kov (Central Michigan University)
- Alexander Volya (Florida State University)
- Naftali Auerbach (Tel Aviv University)
- Felix Izrailev (University of Puebla)
- Suren Sorathia (University of Puebla)
- Luca Celardo (University of Breschia)
- Lev Kaplan (Tulane University)

Shell Model and Nuclear Level Density

M. Horoi, J. Kaiser, and V. Zelevinsky, Phys. Rev. C 67, 054309 (2003).
M. Horoi, M. Ghita, and V. Zelevinsky, Phys. Rev. C 69, 041307(R) (2004).
M. Horoi, M. Ghita, and V. Zelevinsky, Nucl. Phys. A785, 142c (2005).
M. Scott and M. Horoi, EPL 91, 52001 (2010).
R.A. Sen'kov and M. Horoi, Phys. Rev. C 82, 024304 (2010).
R.A. Sen'kov, M. Horoi, and V. Zelevinsky, Phys. Lett. B (to be published)

Statistical Spectroscopy:

S. S. M. Wong, *Nuclear Statistical Spectroscopy (Oxford,* University Press, 1986).

V.K.B. Kota and R.U. Haq, eds., *Spectral Distributions in Nuclei and Statistical Spectroscopy (World Scientific,* Singapore, 2010).

$$\rho(E,\alpha) = \sum_{\kappa} D_{\alpha\kappa} \cdot G_{\alpha\kappa}(E)$$

$$\alpha = \{n, J, T_z, \pi\}$$

Quantum numbers

$$\kappa = \{n_1, n_2, \dots, n_g\}$$

Partitions

$$G_{\alpha\kappa}(E) = G(E + E_{g.s.} - E_{\alpha\kappa}, \sigma_{\alpha\kappa})$$

$$G(x,\sigma) = C \cdot \begin{cases} \exp\left(-x^2/2\sigma^2\right) & , \ |x| \le \eta \cdot \sigma \\ 0 & , \ |x| > \eta \cdot \sigma \end{cases}$$

Finite range Gaussian

$$D_{\alpha\kappa}$$

Many-body dimension

$$E_{\alpha\kappa} = \langle H \rangle_{\alpha\kappa};$$

$$\sigma_{\alpha\kappa} = \sqrt{\langle H^2 \rangle_{\alpha\kappa} - \langle H \rangle^2_{\alpha\kappa}}$$

$$\mathrm{Tr}^{(J)}[\dots] = \mathrm{Tr}^{(J_z)}[\dots]_{J_z=J} - \mathrm{Tr}^{(J_z)}[\dots]_{J_z=J+1}$$

$$\langle H \rangle_{\alpha\kappa} = \mathrm{Tr}^{(\alpha\kappa)}[H]/D_{\alpha\kappa},$$

Centroids

$$\langle H^2 \rangle_{\alpha\kappa} = \mathrm{Tr}^{(\alpha\kappa)}[H^2]/D_{\alpha\kappa}$$

Widths

$$\rho^{(0)}(E,J,0)=\rho(E,J,0)$$

$N\hbar\omega$ **classification**

Pure **Total** **(N=0)**

$$\rho^{(0)}(E,J,1)=\rho(E,J,1)-\sum_{J'=|J-1|}^{J+1}\rho(E,J',0)$$

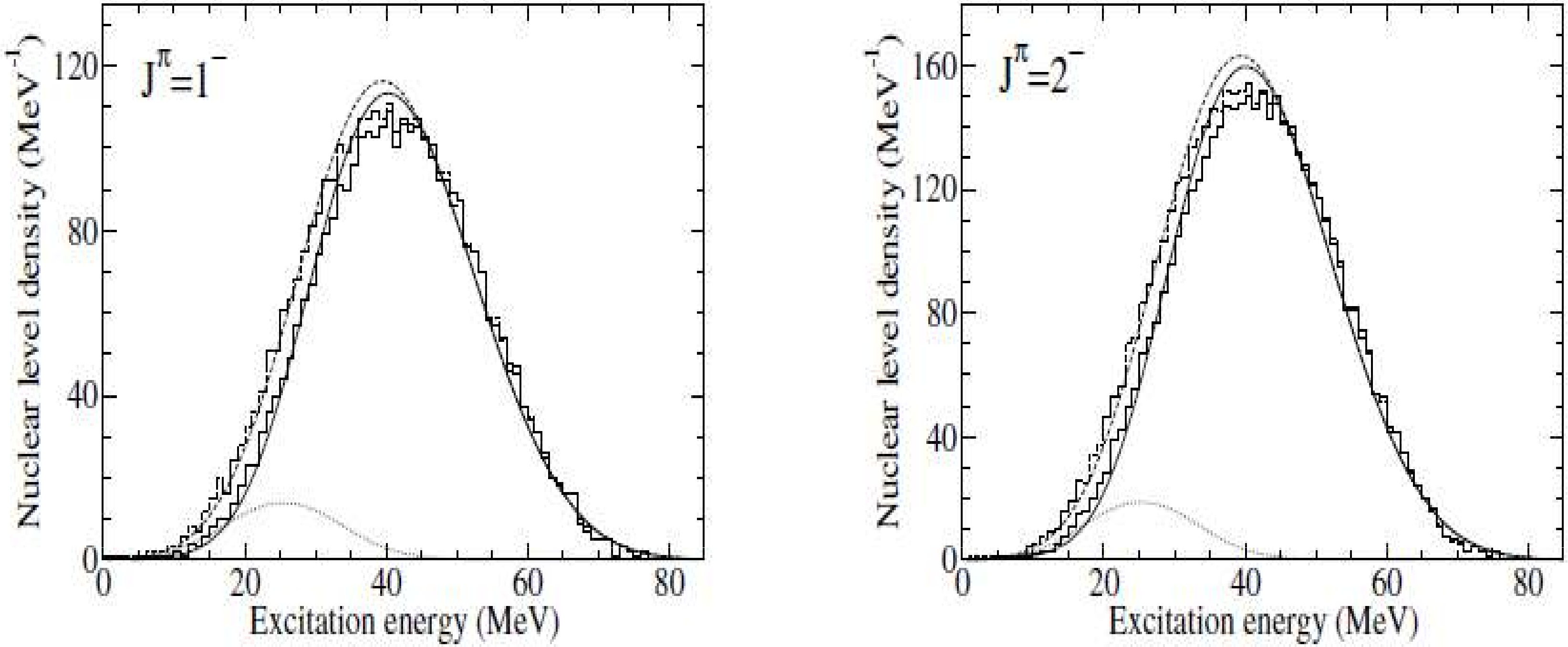
(N=1)

$$\rho^{(0)}(E,J,N)=\rho(E,J,N)-$$

$$-\sum_{K=1}^{N}\sum_{J_K=J_{\min}}^{N,\text{step }2}\sum_{J'=|J-J_K|}^{J+J_K}\rho^{(0)}(E,J',(N-K))$$

Recursive relation

20 Ne

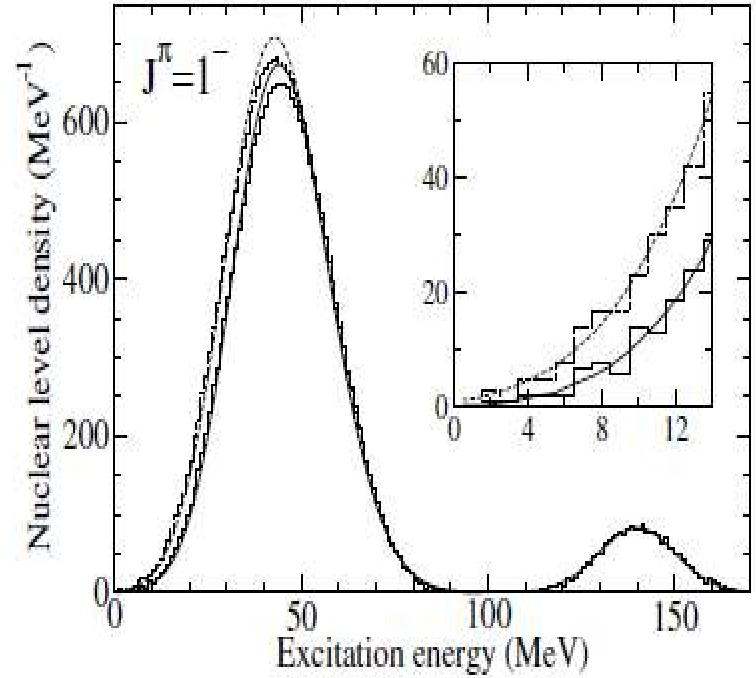


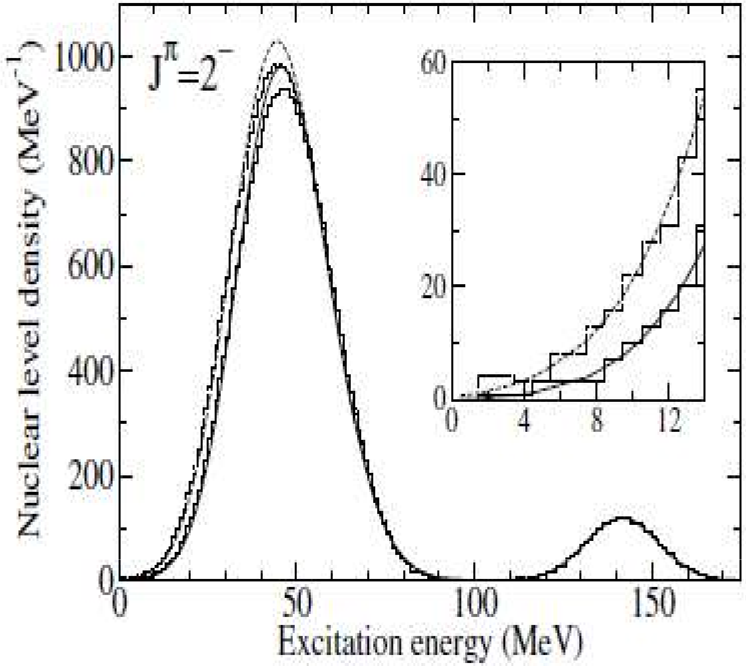
s + p + sd + pf shell space
WBT interaction,
negative parity

$1\hbar\omega$ subspace $H \rightarrow H' = H + \beta\left[\left(H_{CM} - \frac{3}{2}\hbar\omega\right)\frac{A}{\hbar\omega}\right]$

Exact shell model: stair-dashed (with CM) and stair-solid (no CM)
Method of moments: straight-dashed (with CM) and straight-solid (no CM)
Dotted line: spurious states

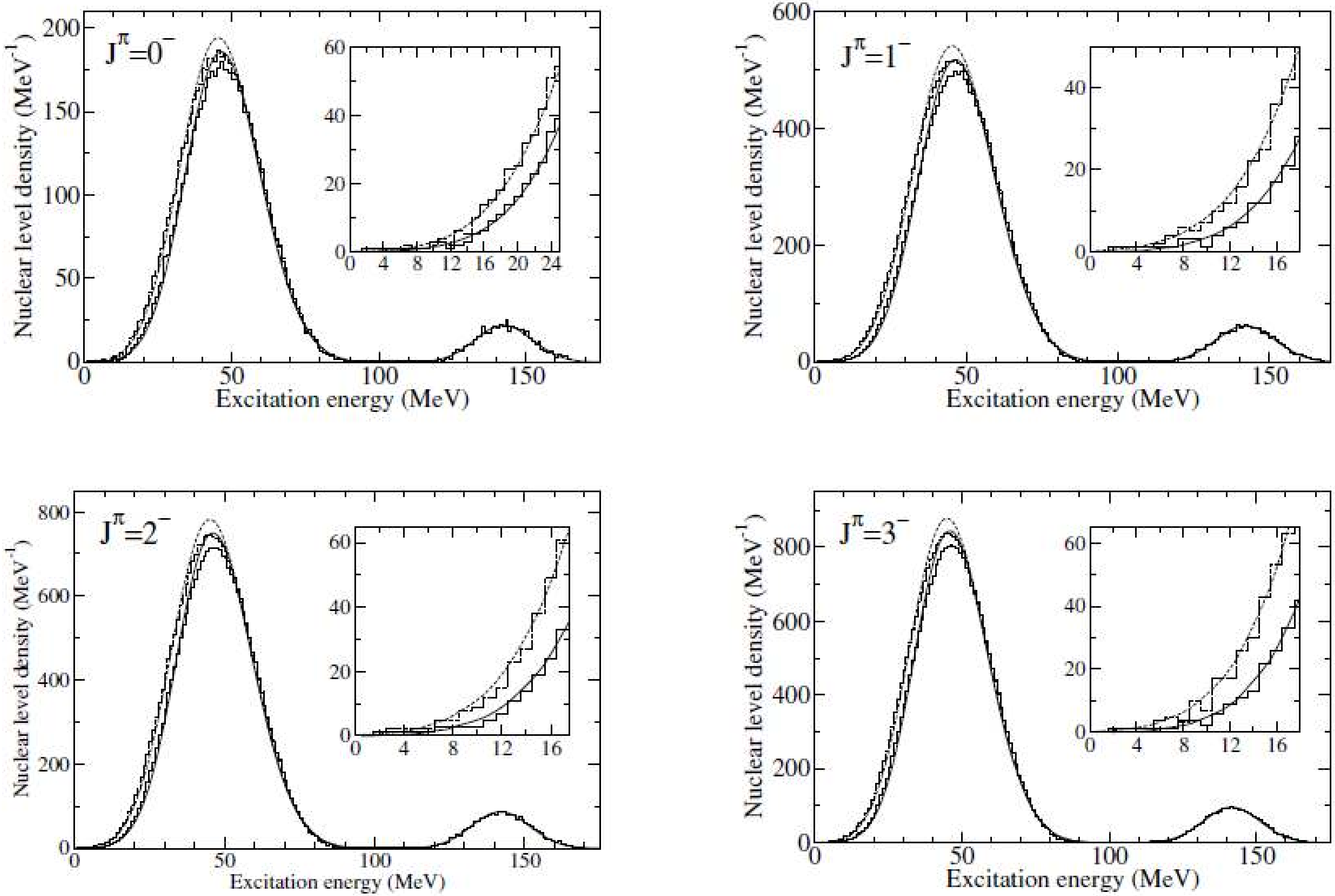
22 Na





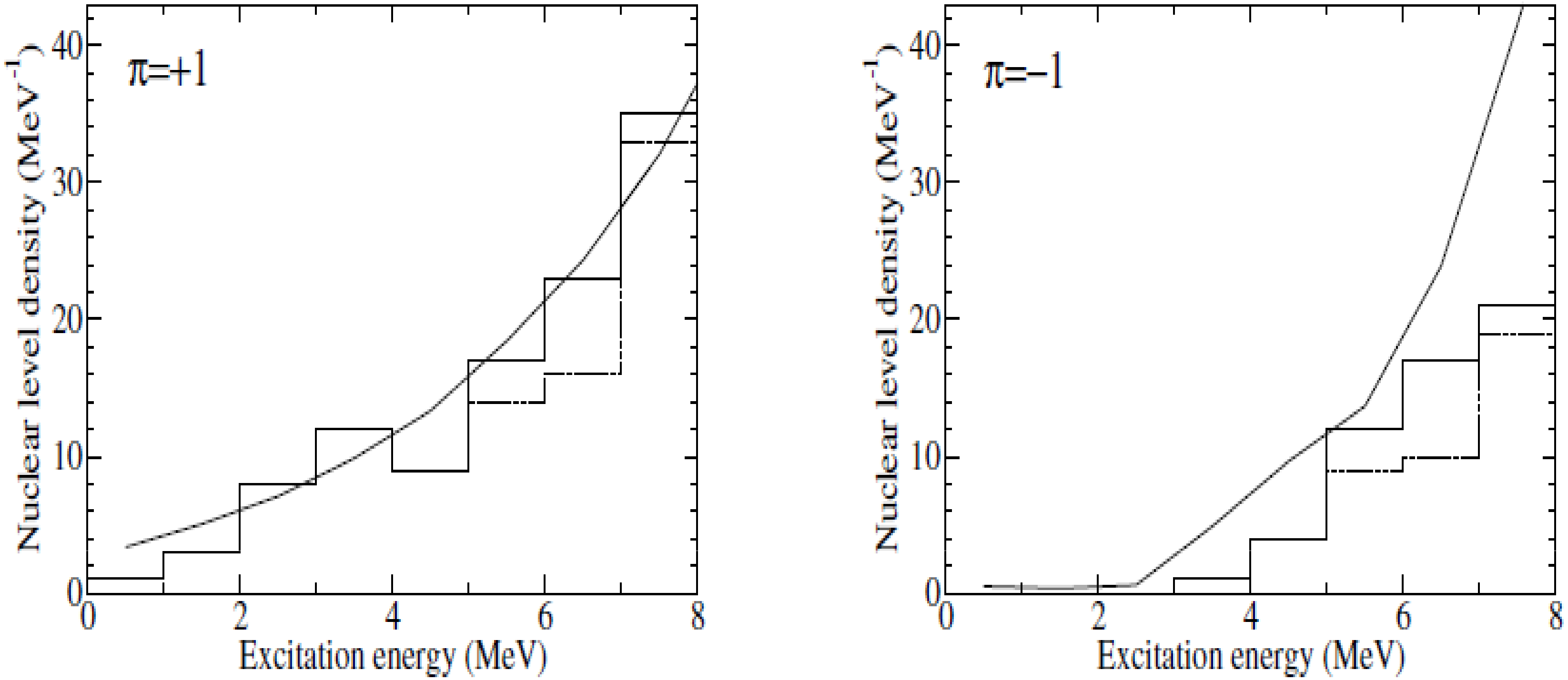
Density of spurious states is shifted to compare with the result of the shell model with the shift

22 Mg



$J^\pi=0^-$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
$J^\pi=1^-$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
$J^\pi=2^-$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
$J^\pi=3^-$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)

26Al



Staircase – experimental counting of levels ("pessimistic" and "optimistic")

PROBLEMS:

Oscillator classification
Parameter of the finite range Gaussians
Ground state energy
Inter-shell effective interactions
Excited states are in the continuum

STATISTICAL MECHANICS of CLOSED MESOSCOPIC SYSTEMS

* **SPECIAL ROLE OF MEAN FIELD BASIS**
  (separation of regular and chaotic motion;
  mean field out of chaos)

* **CHAOTIC INTERACTION as HEAT BATH**

* **SELF – CONSISTENCY** OF
  mean field, interaction and thermometer

* **SIMILARITY OF CHAOTIC WAVE FUNCTIONS**

* **SMEARED PHASE TRANSITIONS**

* **CONTINUUM EFFECTS** (IRREVERSIBLE DECAY)
  new effects when widths are of the order of spacings –
  restoration of symmetries
  super-radiant and trapped states
  conductance fluctuations ...

COMPLEXITY of QUANTUM STATES

RELATIVE!

Typical eigenstate:

$$|\alpha\rangle = \Sigma_k C_k^\alpha |k\rangle; \quad |C_k^\alpha|^2 \approx \frac{1}{N}$$

GOE: $$\mathrm{Prob}(C_1, ..., C_N) \propto \delta(1 - \sum_k C_k^2)$$

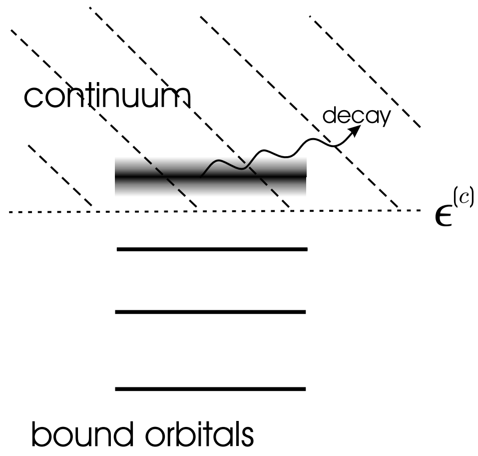
$$\text{at } N \gg 1, \ \mathrm{Prob}(C) \Rightarrow \sqrt{N/2\pi}\, e^{-NC^2/2}$$

Porter-Thomas distribution for weights: $$W_k^\alpha = (C_k^\alpha)^2$$

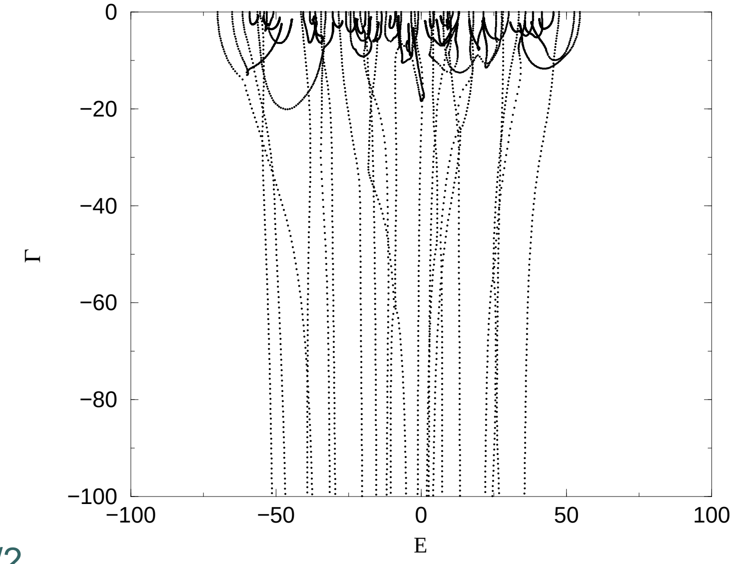
$$P_{PT}(W) = \frac{1}{\sqrt{2\pi\langle W\rangle}} \frac{1}{\sqrt{W}} e^{-W/2\langle W\rangle}$$ **(1 channel)**

Neutron width of neutron resonances as an analyzer

Single-particle decay in a many-body system



Evolution of complex energies ☜=E-i $\Gamma/2$
as a function of γ



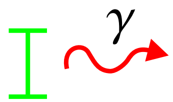
•System 8 s.p. levels, 3 particles
•One s.p. level in continuum $e=\varepsilon - i\gamma/2$

Total states 8!/(3! 5!)=56; **states that decay fast 7!/(2! 5!)=21**
Quasistationary states are determined by continuum

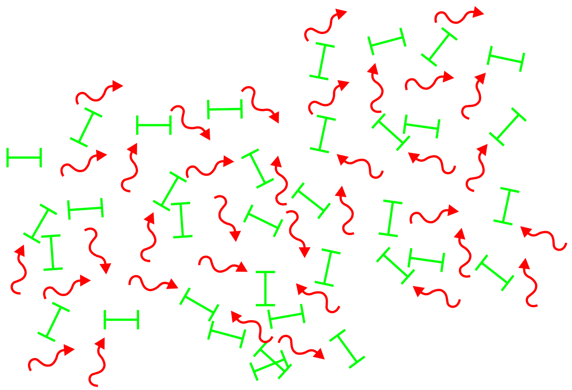
Doorway states

Superradiance, collectivization by decay

Dicke coherent state
N identical two-level atoms
coupled via common radiation

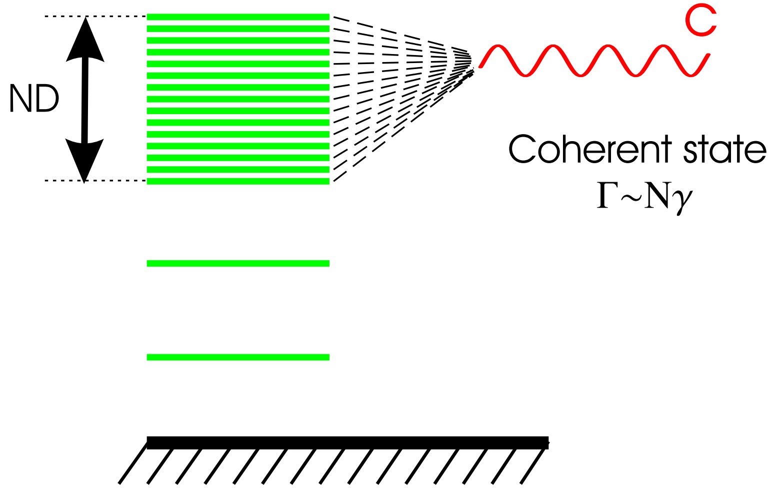
Single atom γ 

Coherent state $\Gamma \sim N\gamma$



Volume ¿ λ^3

Analog in a complex system
Interaction via continuum
Trapped states) self-organization



♑ ~ D and few channels

- Nuclei far from stability
- High level density (states of same symmetry)
- Channel thresholds

COUPLING THROUGH CONTINUUM

$$\Sigma_{12}(E) \sim \sum_{c(\mathrm{all})} \int d\tau_c \frac{(c \to 1)(2 \to c)}{E - E_c + i0}$$

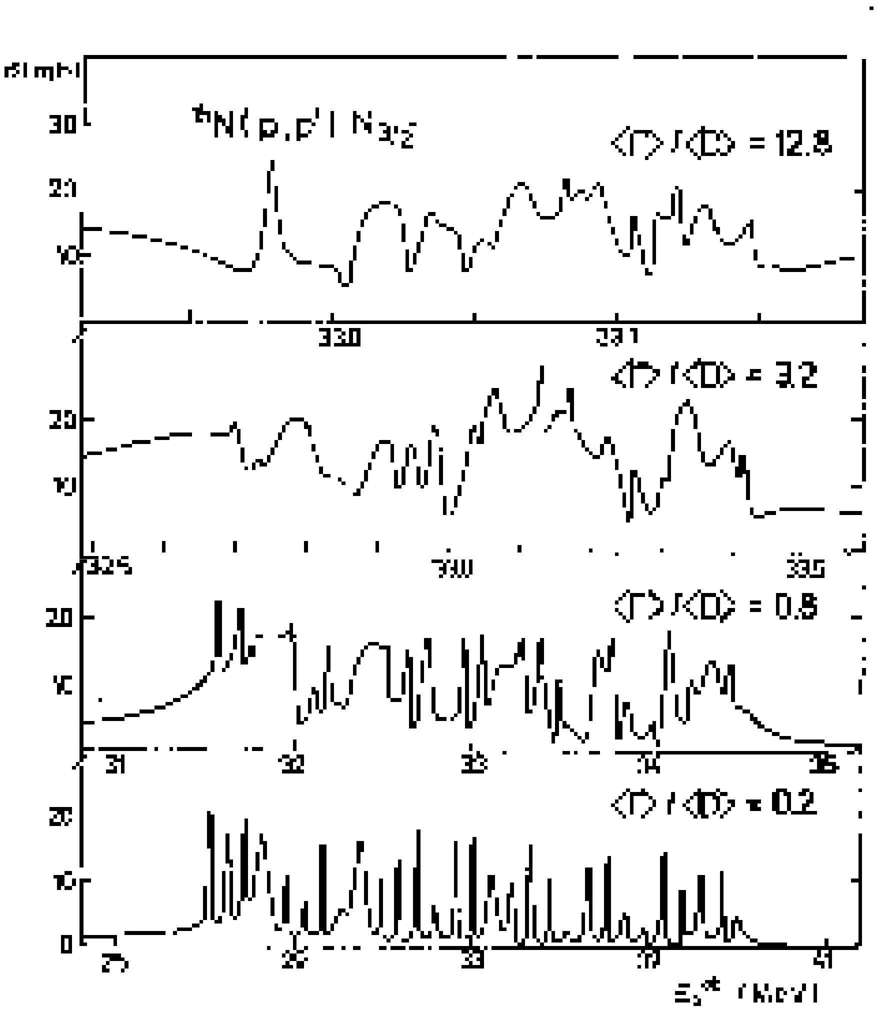
Real (dispersive) part: principal value (virtual, off-shell processes), *closed* **and** *open* **channels ⇒ renormalization of Hermitian Hamiltonian**

Imaginary (absorptive) part: δ-function (real, on-shell processes), only *open* **channels ⇒ non-Hermitian energy-dependent Hamiltonian**

$$\mathcal{H}(E) = H_0(E) + \Delta(E) - \frac{i}{2} W(E)$$

$$W_{12}(E) = \sum_{c(\mathrm{open})} A_1^c(E)\, A_2^{c*}(E)$$

$$Factorization \Leftrightarrow Unitarity$$



Kleinwaechter & Rotter, 1985

Dipole resonance in 16O
From 15N(p,p’) reaction

Shell model calculation
changing as a function
of level density <D>

Overlapping resonances

Narrow isolated resonances

Ingredients

- Intrinsic states: Q-space
  - States of fixed symmetry
  - Unperturbed energies ε_1; some $\varepsilon_1 > 0$
  - Hermitian interaction V
- Continuum states: P-space
  - Channels and their thresholds E^c_{th}
  - Scattering matrix $S^{ab}(E)$
- Coupling with continuum
  - Decay amplitudes $A^c_1(E)$
  - Typical partial width $\gamma = |A|^2$
  - Resonance overlaps: level spacing vs. width

EFFECTIVE HAMILTONIAN

$$\mathcal{H}(E) = H - \frac{i}{2}W(E)$$ - non-Hermitian

$$W_{12} = \sum_{c;\mathrm{open}(E)} A_1^c A_2^c$$

One open channel

Internal representation: $H \to \epsilon_n$,

$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)A_1^2 & -(i/2)A_1A_2 & -(i/2)A_1A_3 \\ -(i/2)A_1A_2 & \epsilon_2 - (i/2)A_2^2 & -(i/2)A_2A_3 \\ -(i/2)A_1A_3 & -(i/2)A_2A_3 & \epsilon_3 - (i/2)A_3^2 \end{pmatrix}$$

Weak coupling, $\kappa \ll 1$ – isolated resonances

$$\mathcal{E}_n = E_n - (i/2)\Gamma_n \approx \epsilon_n - (i/2)A_n^2$$

Strong coupling, $\kappa > 1$

k open channels $\Rightarrow k$ nonzero eigenvalues of W.

Doorway representation:

$$k = 1 \Rightarrow \Gamma_d = \text{Trace}\, W$$

$$\mathcal{H} = \begin{pmatrix} \tilde{\epsilon}_1 - (i/2)\Gamma_d & h_2 & h_3 \\ h_2 & \tilde{\epsilon}_2 & 0 \\ h_3 & 0 & \tilde{\epsilon}_3 \end{pmatrix}$$

Width collectivization:

broad *super-radiant* state $\Gamma_1 \approx \Gamma_d[1 - O(\kappa^{-2})]$,

narrow (*trapped*) states $\Gamma_{2,3} \sim \Gamma_d/[(N-1)\kappa^2]$

Dynamics is determined by alignment
to open decay channels

TIME HIERARCHY

ONE-CHANNEL CASE

Weak coupling: $\kappa = \gamma / D \ll 1$

Separated narrow resonances:

$\Delta E = ND \gg \Gamma = N\gamma$

Strong coupling: $\kappa > 1$,

overlap

Direct process $\tau_{\rm dir} \sim \hbar/\Gamma_{SR}$
Fragmentation $\tau_f \sim \hbar/\Delta E \sim \kappa\tau_{\rm dir}$
Recurrence $\tau_{\rm rec} \sim \hbar/D \sim \kappa\tau_{\rm dir}N$ (Weisskopf time)
Trapped (compound) states $\tau_t \sim \hbar/\Gamma_t \sim \kappa\tau_{\rm rec}$

No room for Ericson fluctuations - only for k (many) open channels, $1 < \kappa < k$.

GAUSSIAN ENSEMBLES

Hermitian:

$$P_\beta(E) = C_{\beta N}\, \Pi_{m<n}\, |E_m - E_n|^\beta \, \exp\left[-\beta \, \frac{N}{a^2} \, \Sigma_n \, E_n^2\right]$$

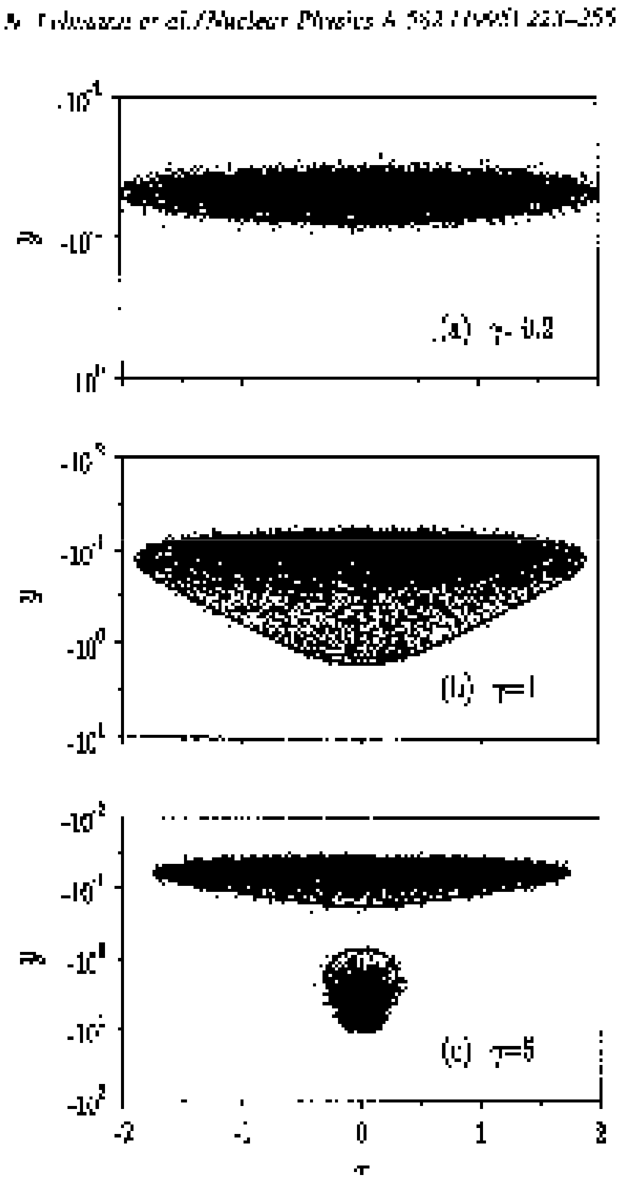
Complex:

$$P(z) = C_N \, \Pi_{m<n} \, |z_m - z_n|^2 \, \exp\left[-2 \, \frac{N}{a^2} \, \Sigma_n \, |z_n|^2\right]$$

Unstable: $\mathcal{E}_n = E_n - \frac{i}{2}\,\Gamma_n$

$$P(\mathcal{E}) = C_N \, \Pi_n \, \frac{1}{\sqrt{\Gamma_n}} \, \Pi_{m<n} \, \frac{|\mathcal{E}_m - \mathcal{E}_n|^2}{|\mathcal{E}_m - \mathcal{E}_n^*|^2} \times$$

$$\exp\left\{-N\left[\frac{1}{a^2}\,\Sigma_n \, E_n^2 + \frac{1}{\eta}\,\Sigma_n \, \Gamma_n + \frac{1}{2a^2}\,\Sigma_{m<n}\,\Gamma_m \Gamma_n\right]\right\}$$



Super-radiant transition

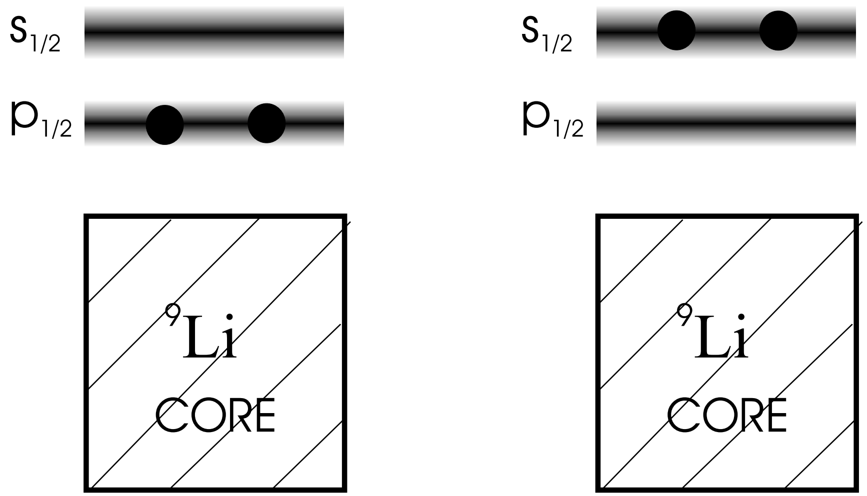
in Random Matrix Ensemble

N= 1000, m=M/N=0.25

^{11}Li model

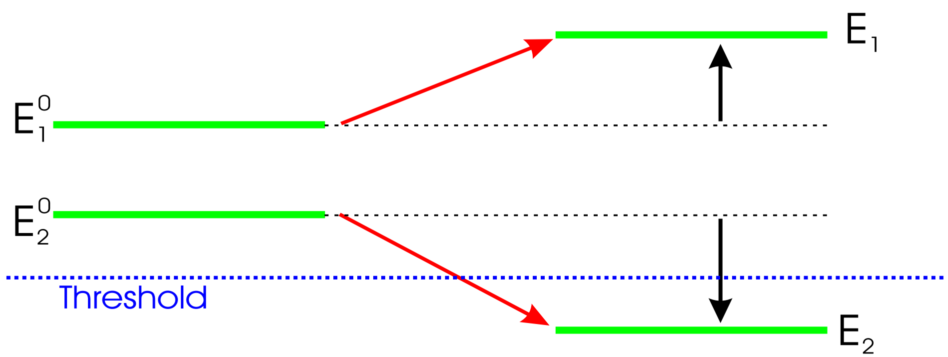
Dynamics of two states coupled to a common decay channel

- Model $\mathcal{H}$



$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1A_2 \\ v - \frac{i}{2}A_1A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$

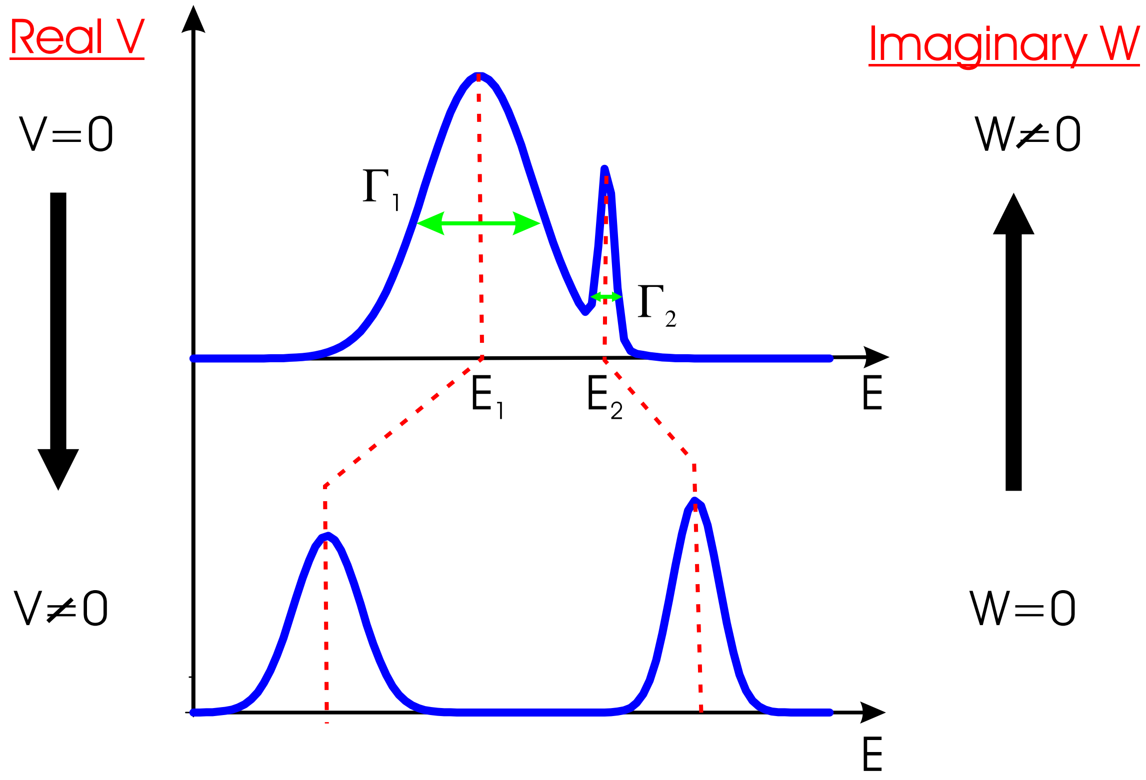
- Mechanism of binding by Hermitian interaction



Interaction between resonances

- Real V
  - Energy repulsion
  - Width attraction
- Imaginary W
  - Energy attraction
  - Width repulsion

$$\mathcal{H} = H^0 + V - iW/2$$



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V.V. Sokolov, V.G. Zelevinsky, Ann. Phys. (N.Y.) 216 (1992) 323.

S. Mizutori, V.G. Zelevinsky, Z. Phys. A 346 (1993) 1.

F.M. Izrailev, D. Sacher, V.V. Sokolov, Phys. Rev. E 49 (1994) 130.

Porter – Thomas distribution of neutron widths (**recent ongoing story**)

P. E. Koehler et al., Phys. Rev. Lett. 105 (2010) 072502.

E. S. Reich, Nature 466 (2010) 1034. <Nuclear theory nudged.>

H. A. Weidenmüller, Phys. Rev. Lett. 105 (2010) 232501.

J.L. Celardo et al. , Phys. Rev. Lett. 106 (2011) 042501.

A. Volya, Phys. Rev. C 83 (2011) 044312.

Loosely stated, the PTD is based on the assumptions that s-wave neutron scattering is a single-channel process, the widths are statistical, and time-reversal invariance holds; hence, an observed departure from the PTD implies that one or more of these assumptions is violated

P.E. Koehler et al.
PRL 105, 072502 (2010)

the combined probability that the PTD is valid is less than 3×10^{-5}.

Attempts to fit by the PTD: $\nu < 1$

(a) Time-reversal invariance holds

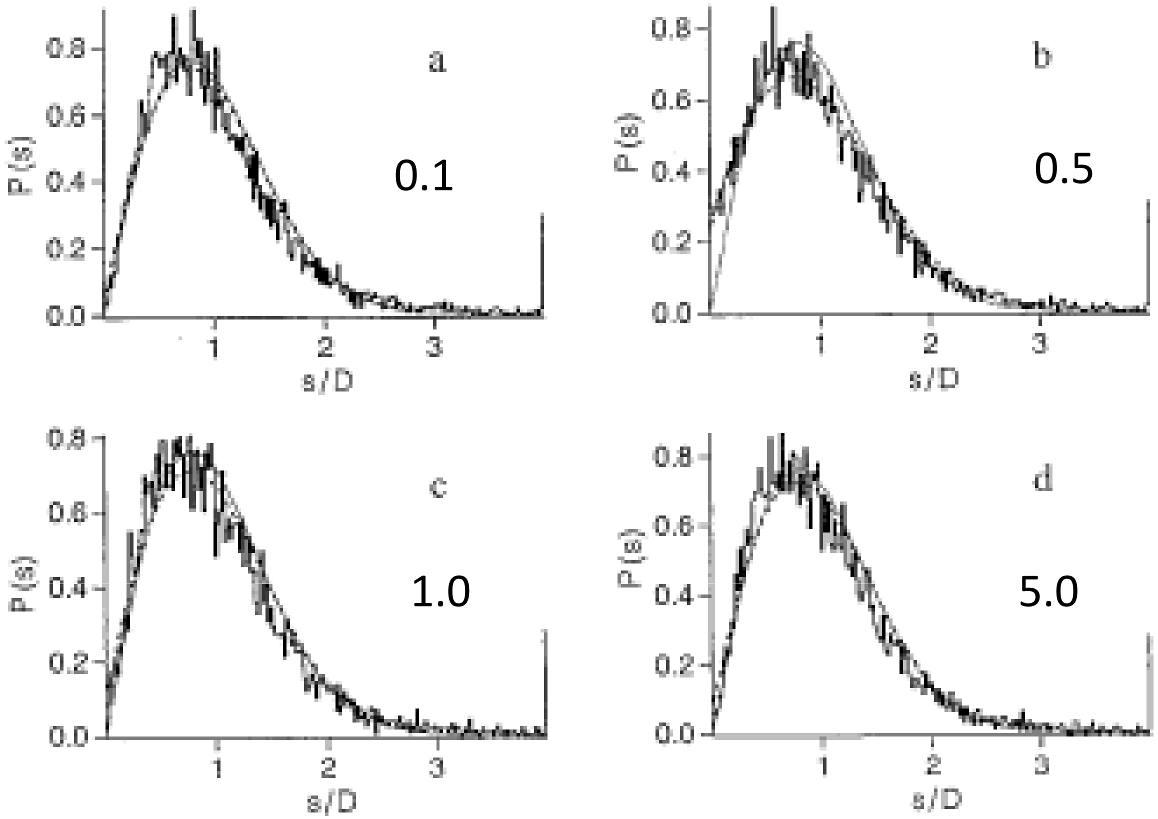
(b) Single-channel process

(c) Widths are statistical ? Whatever it means …

(d) Intrinsic “chaotic” states are correlated through common decay channel

(e) Single-particle resonance – doorway state?

(f) Combination of (c), (d) and (e)



No level repulsion at short distances!

(**Energy of an unstable state is not well defined**)

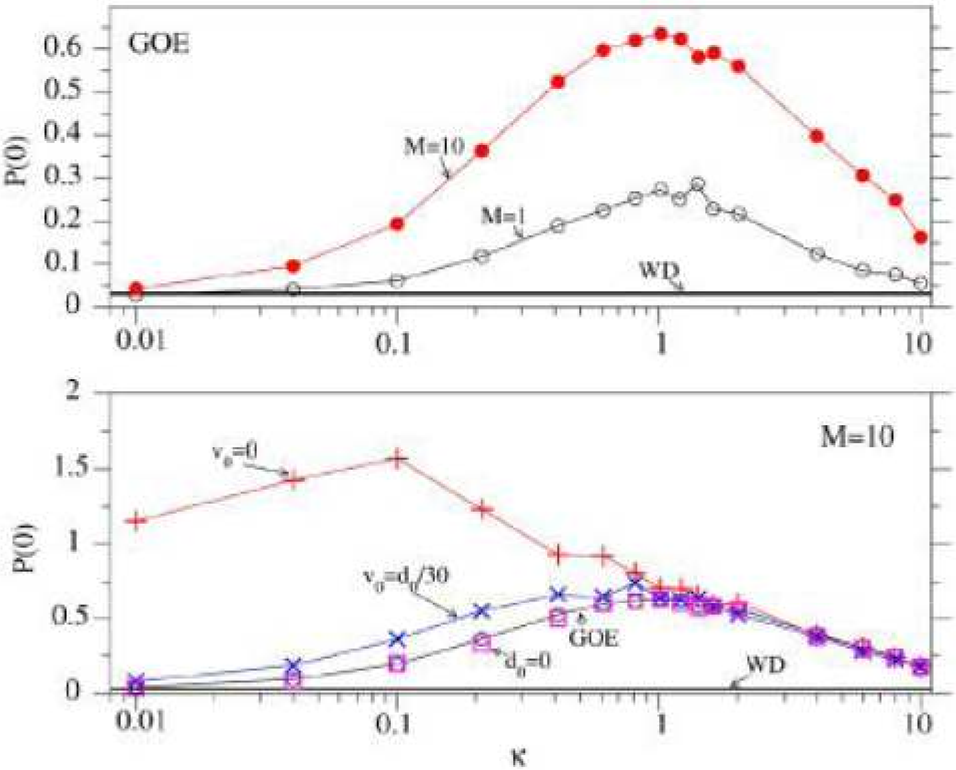


Figure 7: The probability $P(0)$ of finding the resonance energy spacing (in units of the mean spacing) $s < 0.04$ [47]. The intrinsic dynamics corresponds to the GOE, upper panel, with different numbers of open channels M, and to the TBRE, lower panel, for $M = 10$, and different strength of the two-body random interaction, from $v = 0$ through the critical value for onset of intrinsic chaos to the strong interaction for degenerate single-particle levels, when the results are identical to those for the GOE case. Note that for the TBRE the vertical scale is different; at weak interaction the deviations from $P(0) = 0$ start at a very small continuum coupling strength.

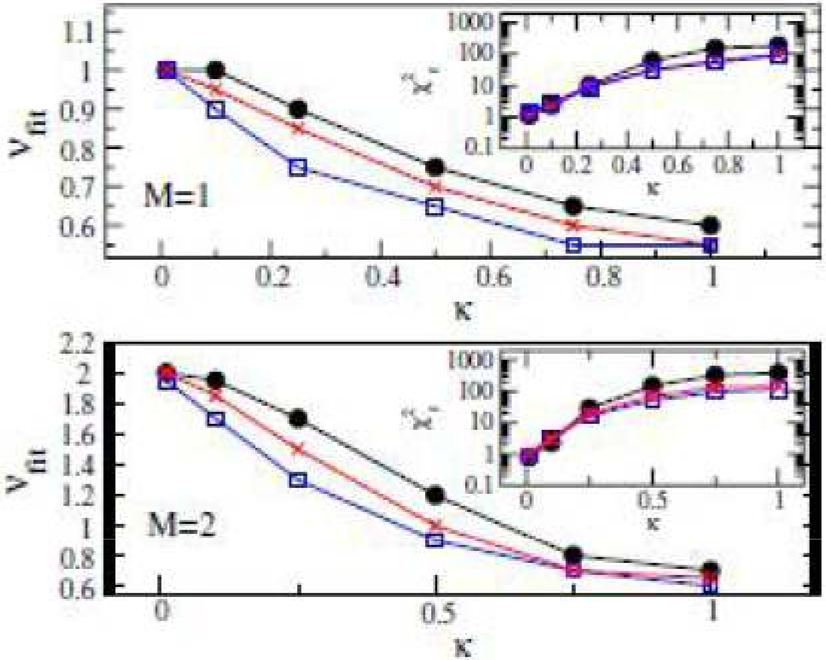


Figure 8: The best fitted parameter ν for the chi-square distribution with ν degrees of freedom used for the description of the width distribution found with averaging over 1000 random realizations of intrinsic interactions for one, upper panel, and two, lower panel, open channels [47]. Full circles refer to the GOE intrinsic Hamiltonian, crosses stand for the TBRE with the interaction below onset of chaos, and squares for TBRE with no intrinsic chaos. The inserts show the unsatisfactory growing chi-square criterion of the fit as a function of κ.

Theoretical framework

$$\mathcal{H} = H - \frac{i}{2}W, \quad W_{ij} = \sum_{c=1}^{M} A_i^c A_j^c.$$

$$\langle A_i^c A_j^{c'} \rangle = \delta_{ij}\delta^{cc'}\frac{\gamma^c}{N}$$

$$\sigma^{ba}(E) = |T^{ba}(E)|^2$$

$$T^{ba}(E) = \sum_{i,j}^{N} A_i^b \left(\frac{1}{E - \mathcal{H}}\right)_{ij} A_j^a$$

Cross section

Transition amplitude

$$S^{ba} = \delta^{ba} - iT^{ba}$$

$$S = \frac{1 - iK}{1 + iK}$$

Scattering matrix in space of channels (unitary)

$$K = \frac{1}{2}\mathrm{A}\frac{1}{E - H}\mathrm{A}^T$$

Analog of R−matrix $\langle K^{ab} \rangle = -i\pi\delta^{ab}\frac{\gamma^a}{2N}\rho(0) = -i\delta^{ab}\kappa^a$

Averaging over intrinsic states (GOE or TBRE)

$$\langle K^{ab}\rangle = -i\pi\delta^{ab}\frac{\gamma^a}{2N}\rho(0) = -i\delta^{ab}\kappa^a$$

transmission coefficient

Maximum at perfect coupling ("super-radiance")

$$T^a = 1 - |\langle S^{aa}\rangle|^2 = \frac{4\kappa^a}{(1+\kappa^a)^2}$$

$C(\epsilon) = \langle\sigma(E)\sigma(E+\epsilon)\rangle - \langle\sigma(E)\rangle^2$

Standard (Ericson) theory predicts small fluctuations

$$w(\Gamma)/\langle\Gamma\rangle^2 \ll 1$$

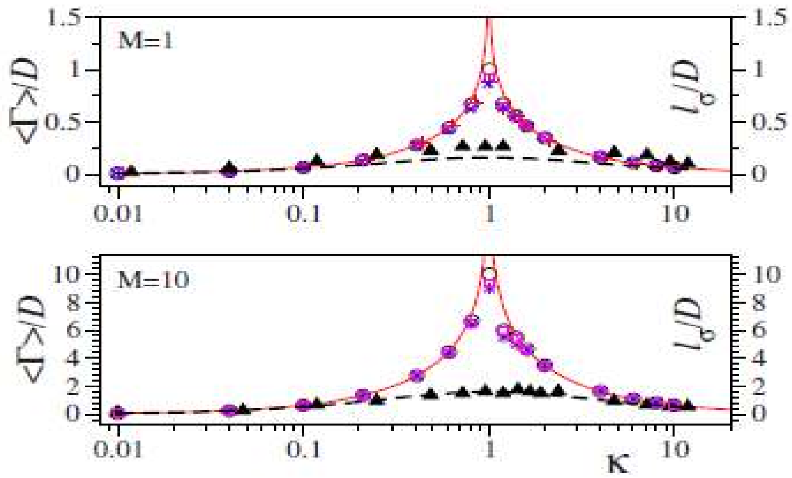
If the number of channels M >> 1

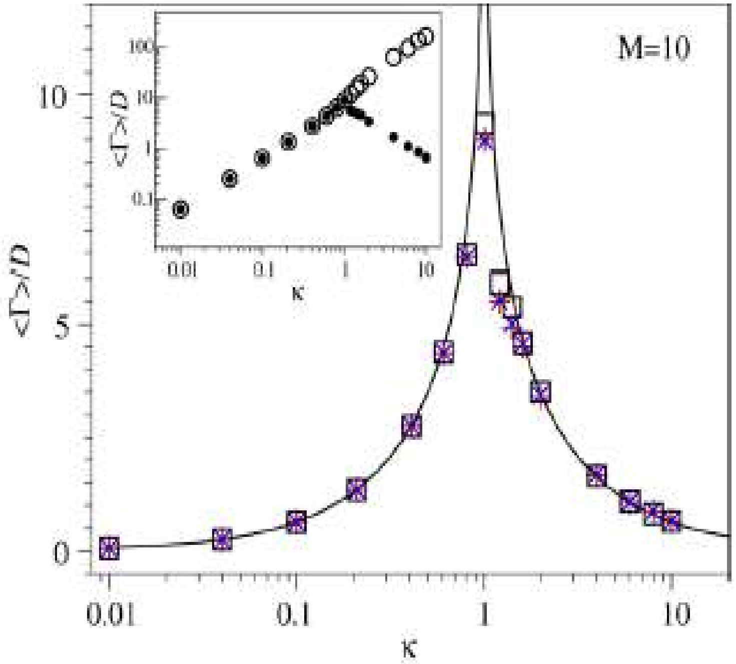
$$C(\epsilon) = \langle\sigma(E)\sigma(E+\epsilon)\rangle - \langle\sigma(E)\rangle^2$$

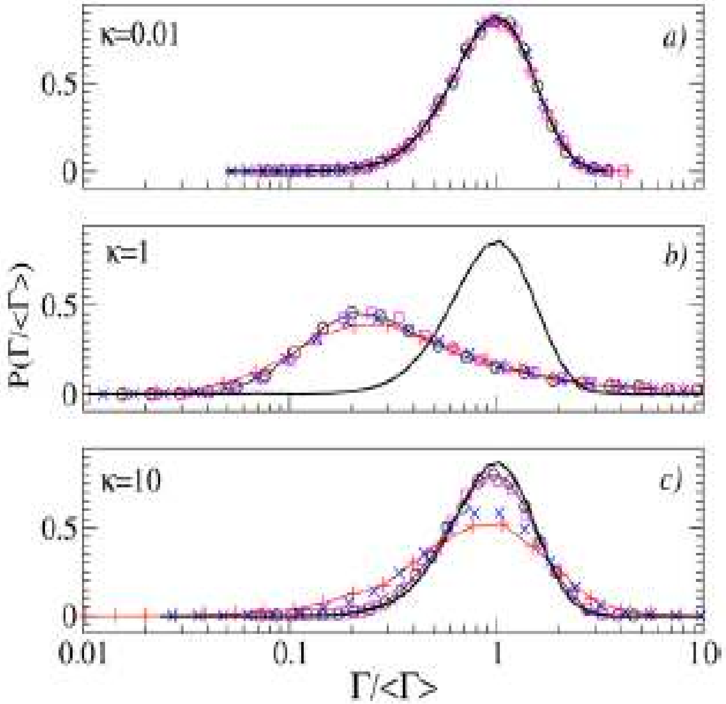
$$\frac{C(\epsilon)}{C(0)} = \frac{l^2}{l^2+\epsilon^2} \qquad l = \langle\Gamma\rangle$$

$$\frac{l}{D} = \frac{MT}{2\pi} = \frac{M}{2\pi}\frac{4\kappa}{(1+\kappa)^2}$$

Weisskopf relation







Average width and width distribution for M=10 equivalent channels

$$\frac{\langle \Gamma \rangle}{D} = \frac{M}{\pi} \ln \left| \frac{1+\kappa}{1-\kappa} \right|.$$

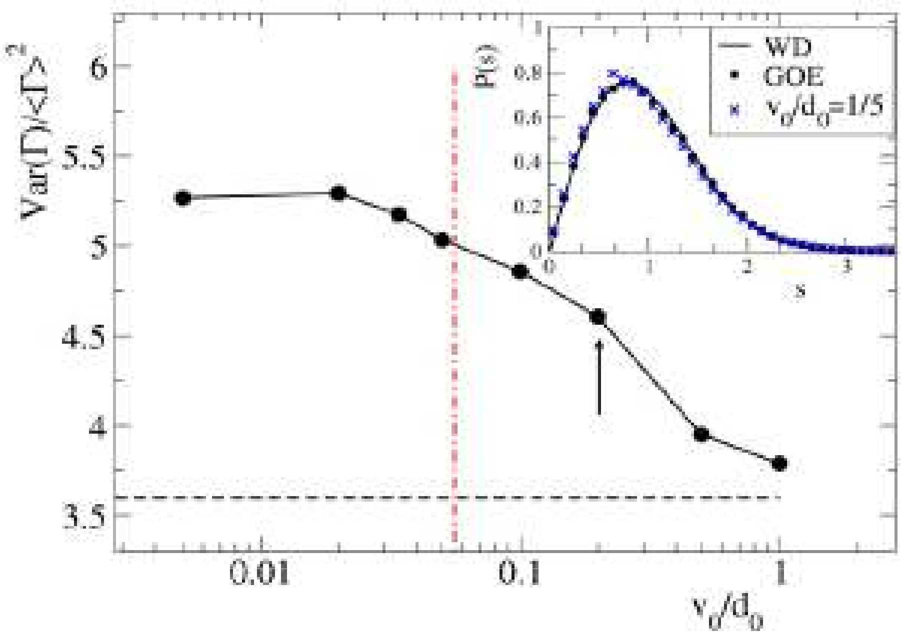
Moldauer – Simonius relation
(also follows from non-Hermitian Hamiltonian)

Average quantities depend only on transmission coefficients

Width fluctuations

in different models

of intrinsic interactions



GOE level

Fig. 3. (Color online.) Dimensionless variance of the widths versus v_0/d_0 for $M = 10$ (connected circles): the GOE value is shown by horizontal dashed line; the vertical dot-dashed line marks the value $v_0 = v_{cr}$ corresponding to the onset of chaos. In the inset the level spacing distribution is shown for $\kappa = 0$ and $v_0/d_0 = 0.2$ (crosses), see the arrow in the main part, and for the GOE (circles). The smooth curve is the WD-distribution.

At the arrow position:

P(s) is not sensitive

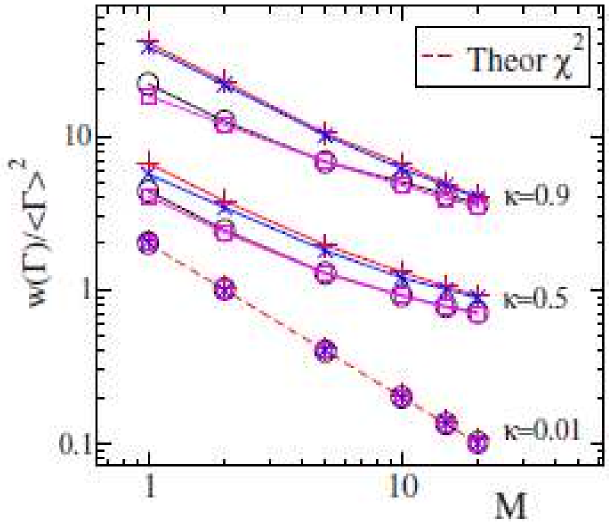


FIG. 3. (Color online) Normalized variance of the width as a function of the number of channels M, for different coupling strengths κ (symbols are the same as in Fig. 1). While for small coupling $\kappa=0.01$, the variance decreases with the number of channels very fast in accordance with the expected χ^2 distribution (dashed line), for large couplings $\kappa=0.5$ and 0.9 the behavior is different from the $1/M$ dependence. Pluses, crosses, etc. stand for the same situations as in Fig. 1.

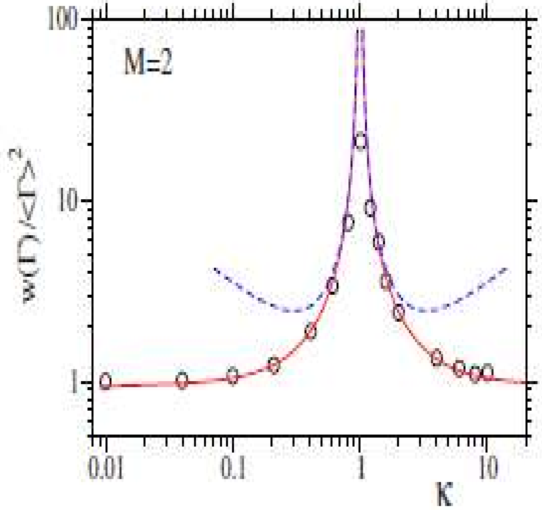


FIG. 4. (Color online) Numerical data for the normalized variance of the widths vs κ for GOE and $M=2$ (circles), in comparison with the result of numerical integration of Eq. (50) (solid curve), and with Eq. (51) (dashed curve) (see in the text).

$$\frac{w(\Gamma)}{\langle\Gamma\rangle^2} \propto \frac{1}{(1-\kappa)^2}[\ln(1-\kappa)]^2$$

Y. V. Fyodorov and H.-J. Sommers, J. Math. Phys. **38**, 1918 (1997); H.-J. Sommers, Y. V. Fyodorov, and M. Titov, J. Phys. A **32**, L77 (1999).

Weak intrinsic chaos =

Large width fluctuations

Divergence independently of M

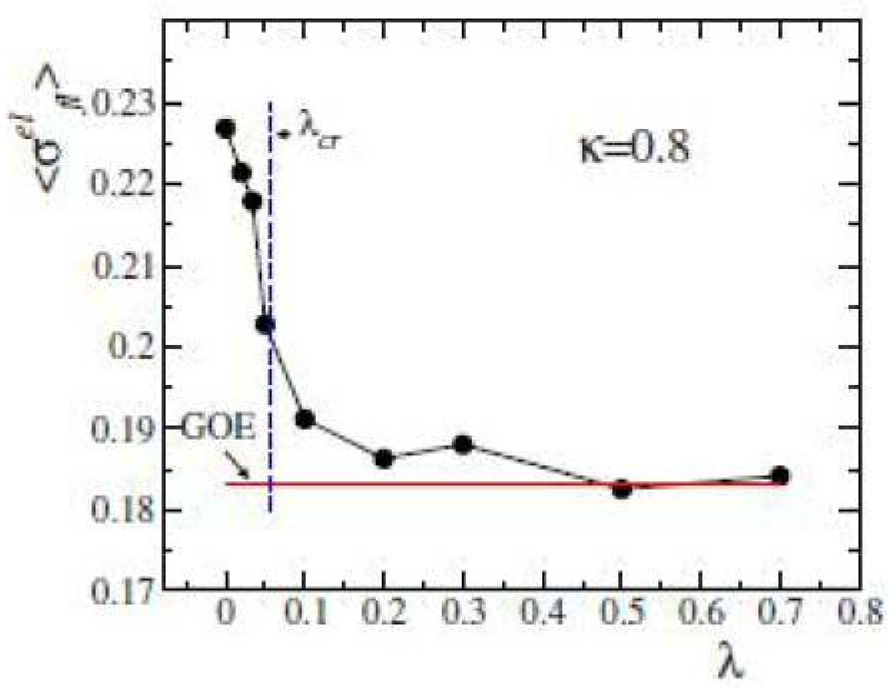


Figure 9: The fluctuating part of the elastic cross section for 10 open channels and $\kappa = 0.8$ as a function of the intrinsic interaction strength in the TBRE; the GOE value is shown by a horizontal line [46]. The dashed vertical line shows the critical value $\lambda_{\rm cr}$ for the transition to chaos in the TBRE.

Quantum signal transmission through a simple chain of wells

V.V. Sokolov, V.G. Zelevinsky, Ann. Phys. (N.Y.) 216 (1992) 323.

A. Volya and V. Zelevinsky, in *Nuclei and Mesoscopic Physics*, ed. V. Zelevinsky (AIP Conference Proceedings **777**, 2005) p. 229.

G.L. Celardo, F.M. Izrailev, V.G. Zelevinsky, and G.P. Berman, Phys. Rev. E **76**, 031119 (2007).

G.L. Celardo, F.M. Izrailev, V.G. Zelevinsky, and G.P. Berman, Phys. Lett. B **659**, 170 (2008).

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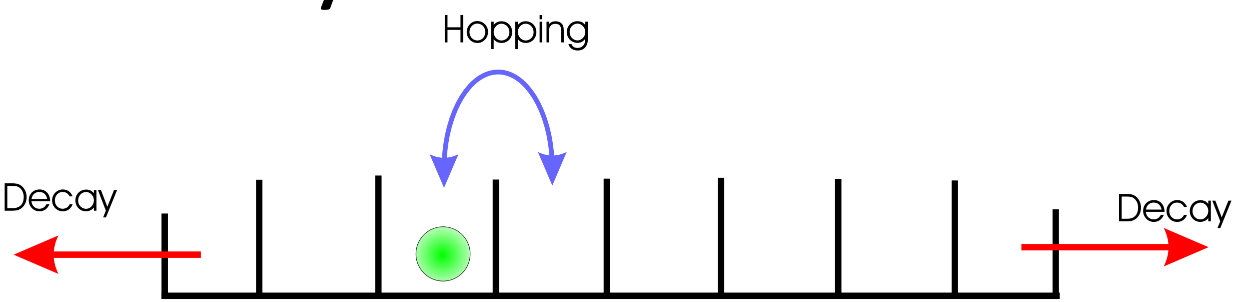
G.L. Celardo, A.M. Smith, S. Sorathia, V.G. Zelevinsky, R.A. Sen'kov, and L. Kaplan, Phys. Rev. B **82**, 165437 (2010).

Particle in Many-Well Potential



Hamiltonian Matrix:

$$H_{nm} = \epsilon\delta_{nm} + v(\delta_{mn+1} + \delta_{m,n-1}) - \frac{i}{2}\left(\gamma^L \delta_{n1}\delta_{m1} + \gamma^R \delta_{nN}\right)$$

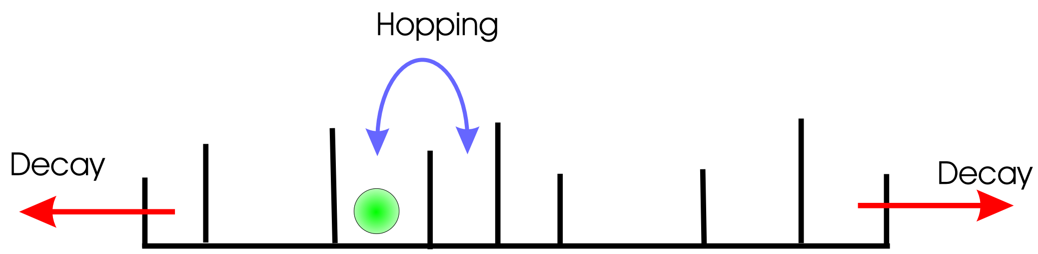
hopping

decay left

decay right

Solutions:
- No continuum coupling - analytic solution
- Weak decay - perturbative treatment of decay
- Strong decay – localization of decaying states at the edges

Disordered problem



$$H_{nm} = \epsilon_n \delta_{nm} + v(\delta_{mn+1} + \delta_{m,n-1}) - \frac{i}{2}\left(\gamma^L \delta_{n1}\delta_{m1} + \gamma^R \delta_{nN}\right)$$

random energy

hopping

decay left

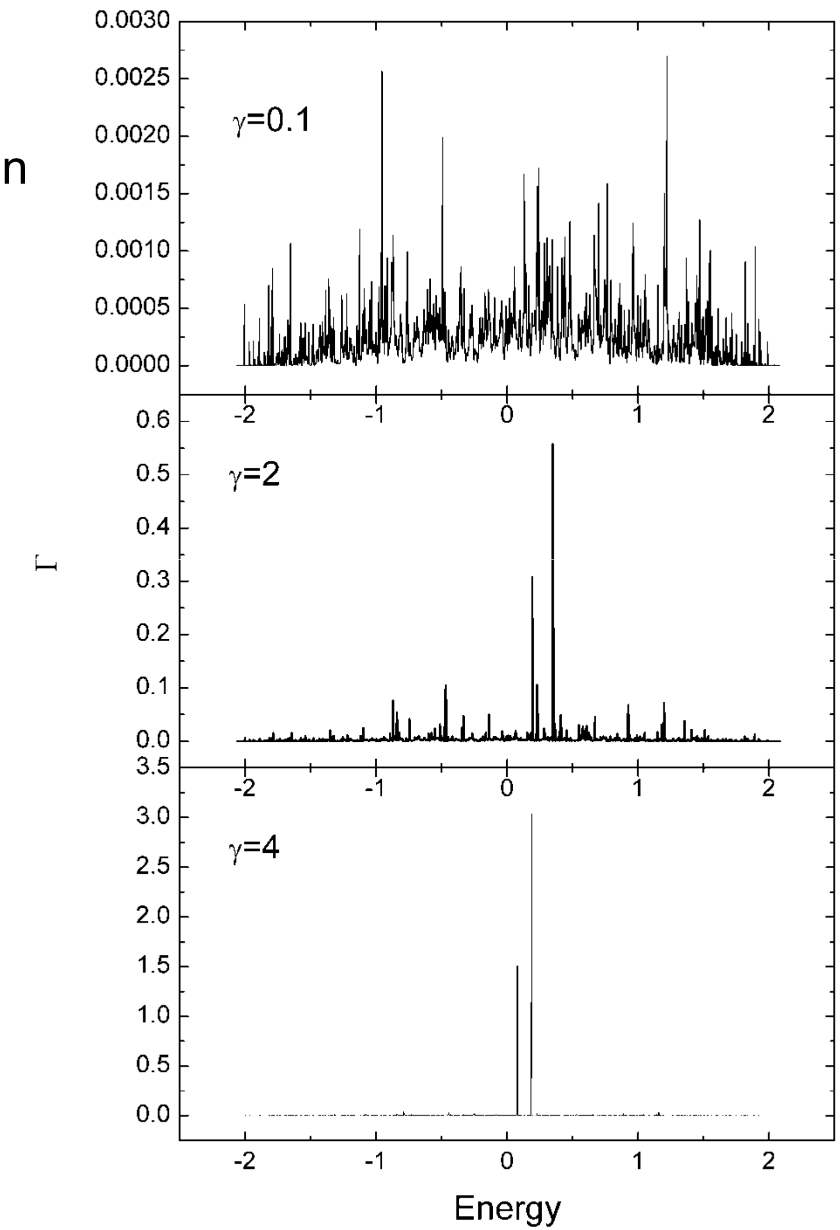
decay right

Distribution of widths as a function of decay strength

Weak decay: Random Distribution

Transitional region:
Formation of superradiant states

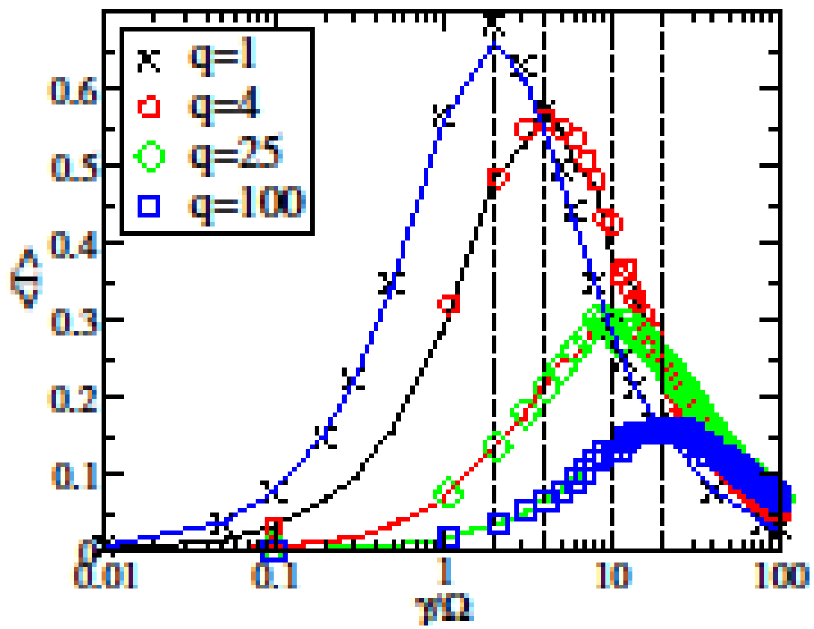
Strong decay: Superradiance



Transmission at the center of the energy band, disordered chain, N=100, asymmetric leads with ratio of width q.

Curves: Anderson disorder model

Points : sequence of potential barriers with random levels for each site



Full correspondence with universal conductance fluctuations
Various regimes of asymmetry and disorder
Various geometries: 1-d, quasi 1-d, 2-d, 3-d, Y, crossings, stars …
Attach reservoirs at the ends
Noise and decoherence

SUMMARY

1. **General method for open and marginally stable many-body quantum systems**

2. **Instrument for studying the intrinsic chaos by cross sections and their fluctuations and correlations**

3. **Broad range of applications: exotic nuclei, particle resonances, chemical reactions, micro- and nano-devices, engineering for quantum information and signal transmission**

4. **Many unsolved problems:**
   ^ theoretical description beyond canonical Gaussian ensembles
   ^ interaction with collective excitations and doorway structyre
   ^ interaction with external noise
   ^ distribution function in the complex plane – problem for experiment
   ....