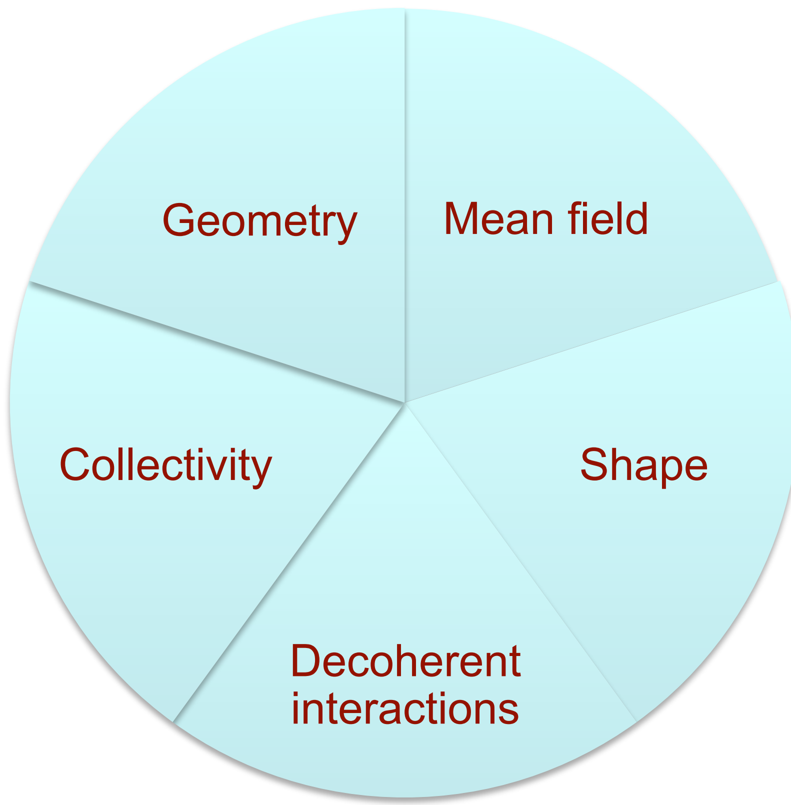
Collectivities in many-body ensembles with random interactions.

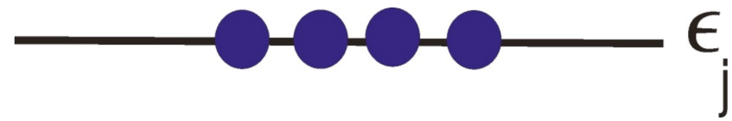
Alexander Volya

Florida State University

Mesoscopic system



The simple model



- Single-j level
- Ω=2j+1 single-particle orbitals: m=-j, j-1, … j
- Number of nucleons N: $0 \leq N \leq \Omega$
- Number of many-body states: $\Omega!/((N!(\Omega-N)!)$
- Many-body states classified by rotational symmetry: (J,M)

Dynamics

- Rotational invariance and two-body interactions
  particle-particle pair operator $P_{L\,M}=(a\ a)_{L\,M}$
  particle-hole pair operator $M_{K\,\kappa}=(a\ a^{\dagger})_{K\,\kappa}$
- Hamiltonian $H = \sum_L V_L \sum_M P^{\dagger}_{LM} P_{LM}$
- Dynamics is fully determined by j+1/2 parameters V_L

Ground state statistics[1]

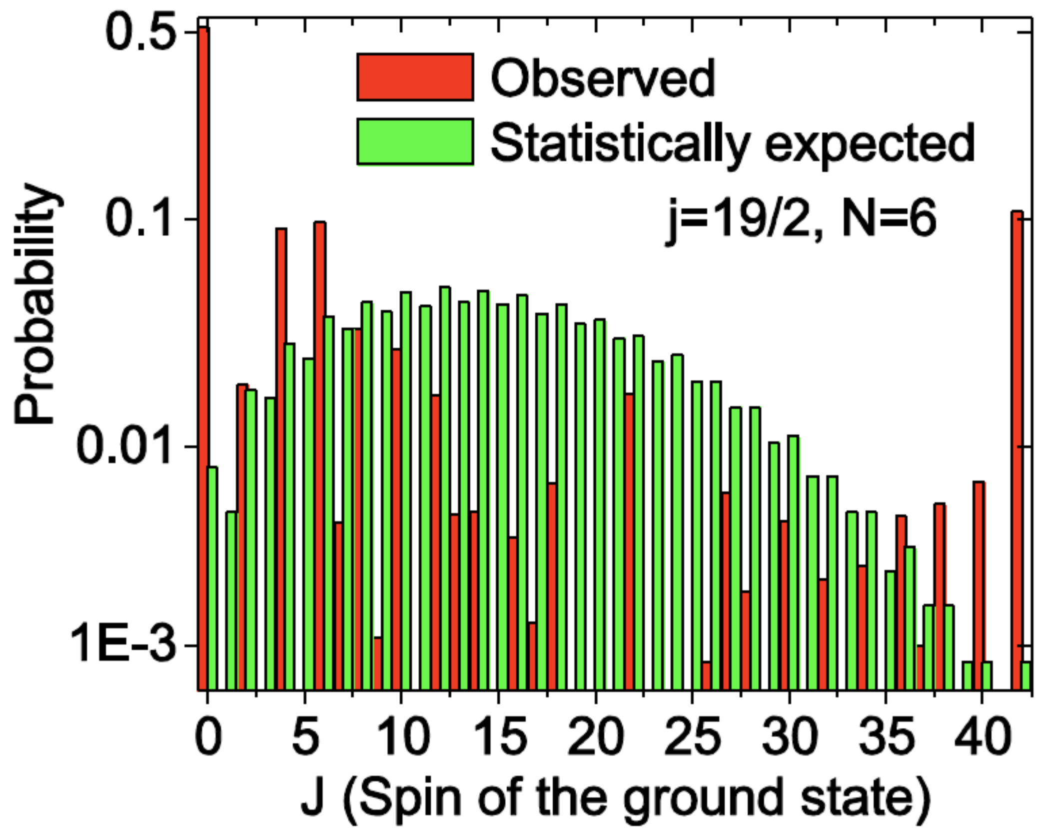
Dynamics versus symmetry

Take V_L at random

(Gaussian distribution centered at 0, width 1)

What is the probability for the ground state to have spin J?

- J=0 is enhanced
- $J=J_{max}$ is enhanced



[1] C. W. Johnson, G. F. Bertsch, and D. J. Dean, Phys. Rev. Lett. **80**, 2749 (1998).

Observables

Reduced transition probability $$B(E2, J_i \to J_f) = \sum_{\mu, M_f} |\langle J_f M_f | \mathcal{M}_{2\mu} | J_i M_i \rangle|^2$$

Total transition strength $$S(J_i) = \sum_{J_f} B(E2, J_i \to J_f)$$

Fractional collectivity $$b(E2, J_i \to J_f) = B(E2, J_i \to J_f)/S(J),$$

Quadrupole moment Q $$\mathcal{Q}(J) = \langle JJ | \mathcal{M}_{20} | JJ \rangle$$

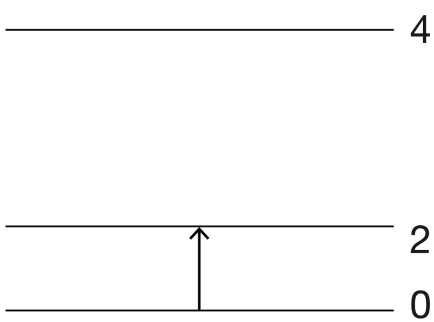
Normalized body-fixed moment $$q_\lambda(J) = \frac{Q_\lambda(J)}{\sqrt{S_\lambda(0_{gs})}} \qquad \mathcal{Q}(2_1) = -2/7\, Q(2_1)$$

Ratio of de-excitation rates $$B_{42} = \frac{B(E2, 4_1 \to 2_1)}{B(E2, 2_1 \to 0_{gs})}$$

Ratio of excitation energies $$R_{42} = \frac{E(4_1)}{E(2_1)}$$

Collective modes

Rotations



$$b = 1$$

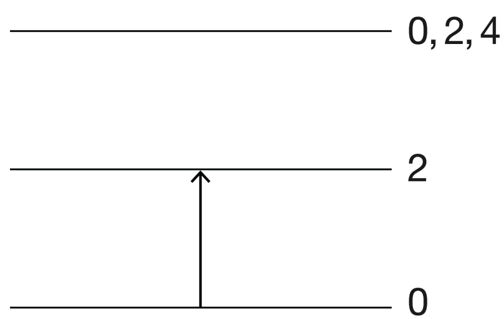
$$q = +1 \ (\text{prolate})$$

$$q = -1 \ (\text{oblate})$$

$$R_{42} = 10/3 \approx 3.33$$

$$B_{42} = 10/7 \approx 1.41$$

Vibrations



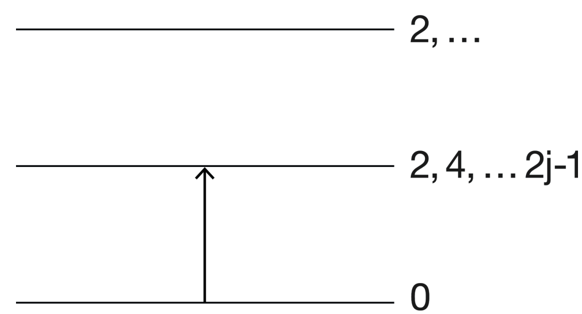
$$b = 1$$

$$q = 0$$

$$R_{42} = 2$$

$$B_{42} = 2$$

Pairing



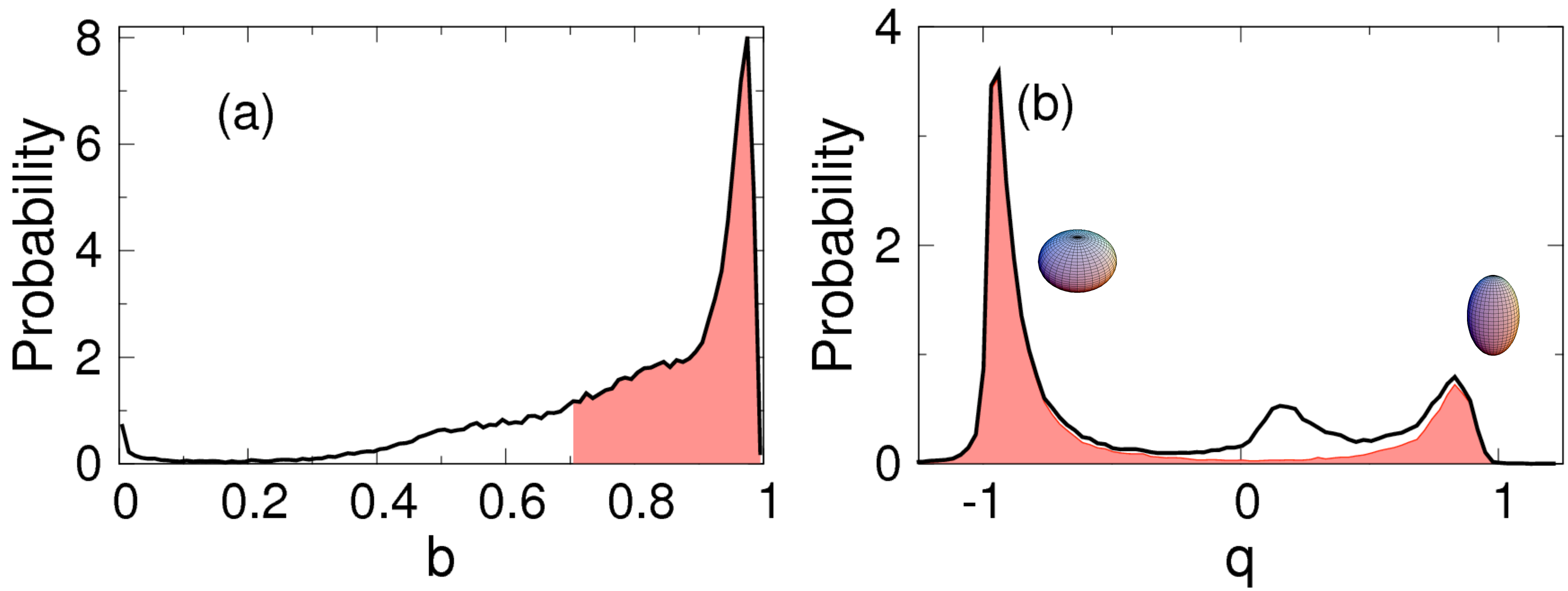
$$b = 1$$

$$q \approx 0$$

$$R_{42} = 1$$

$$B_{42} \approx 0$$

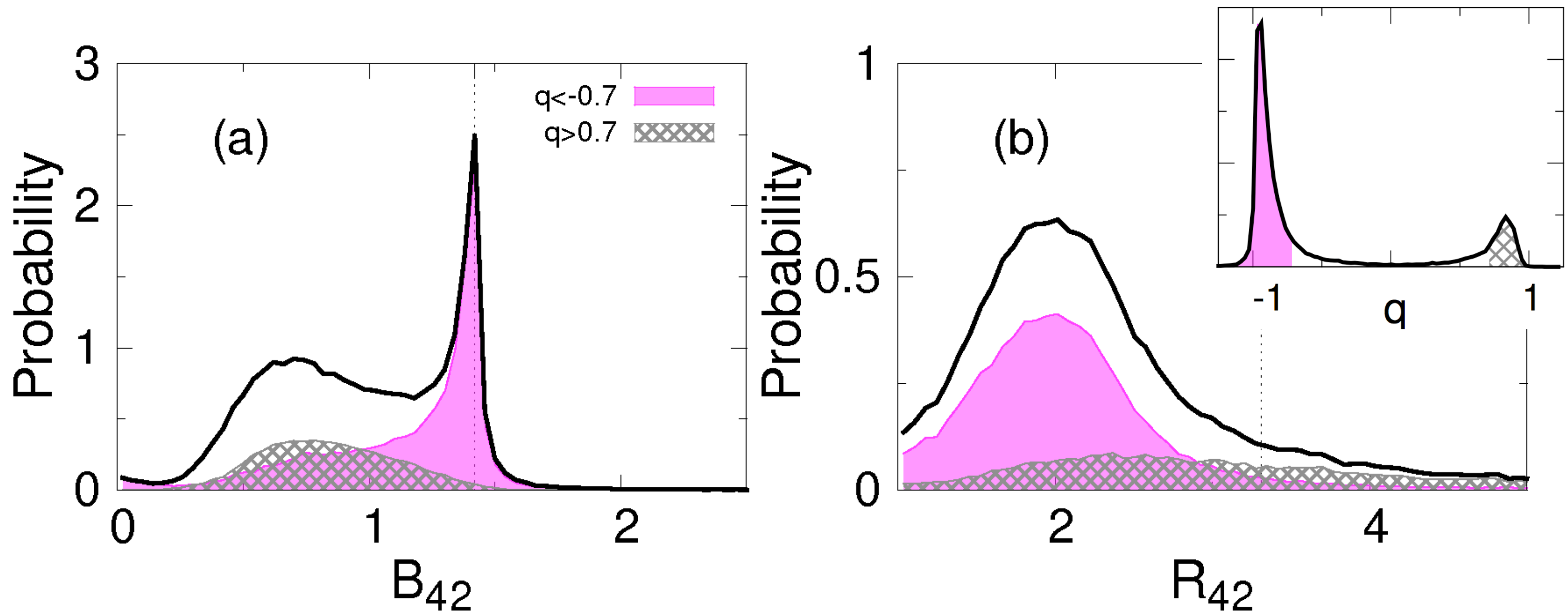
Collectivity in TBRE $(19/2)^6$



- Large fraction of realizations with $0_{g\,s}$, 2_1 (10%)
- Collectivity in E2 transition $b \approx 1$
- For *collective* (b>0.7) realizations the quadrupole moment is close to that of a symmetric top (|q|~1).
- Both prolate (q>0.7) and oblate (q<-0.7) shapes are observed.

Rotations ➔ Deformed mean filed

Collectivity in TBRE $(19/2)^6$



- The distribution of B_{42} has a peak at the rotational value for the oblate realizations.
- R_{42} deviates from rotational

Size of deformation

Intrinsic moment and total transition strength $S(0_{gs}) = Q^2 \sim \beta^2$

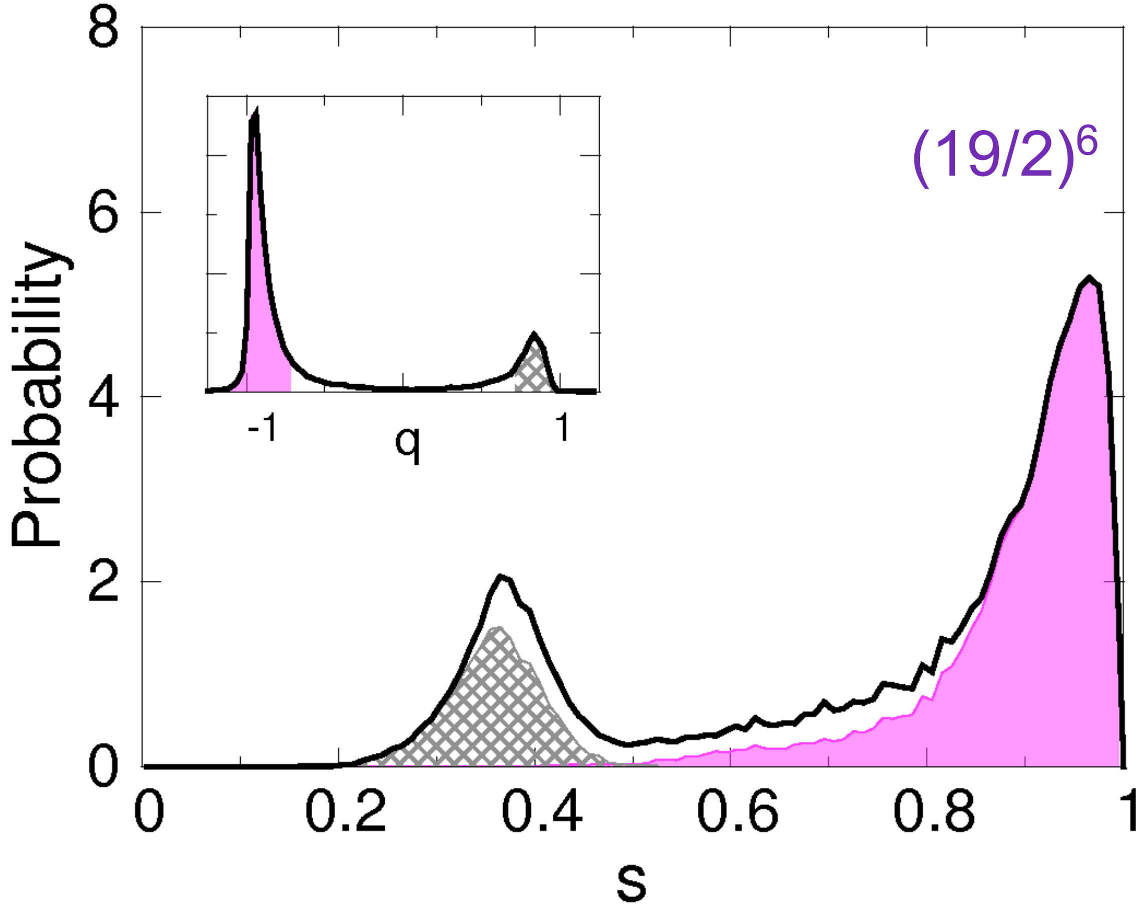
Sum rule for the transition strength

$$S = \sum_i B(E2, 0_{g.s.} \to 2_i) = \langle 0_{gs} \underbrace{\left| \sum_\mu \mathcal{M}^\dagger_{2\mu} \mathcal{M}_{2\mu} \right|}_{H_{QQ}} 0_{gs} \rangle$$

Use Quadrupole-Quadrupole (QQ) Hamiltonian $H_{\rm QQ} = -\sum_\mu \mathcal{M}^\dagger_{2\mu} \mathcal{M}_{2\mu}$

- Eigenvalue coincides with S
- Ground state has largest possible S $\quad s(J) = \dfrac{S(J)}{|E_{\rm QQ}(0_{gs})|},$
- For SU(3), S is Casimir operator, identifies representation

Distribution of s



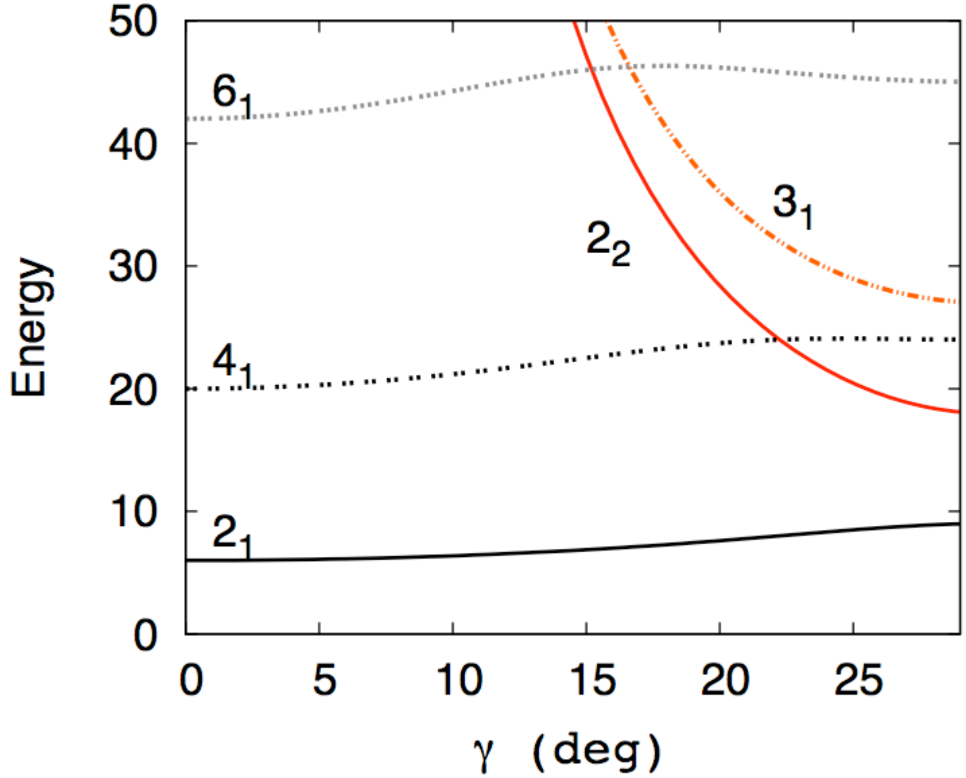
- Prolate shape, less likely, small deformation s=0.35.
- Oblate shape, most likely, high deformation, s=1, state is nearly that of QQ

Triaxial rotor Hamiltonian

Collective Rotor Hamiltonian $H_{rot} = \sum_{i=123} A_i I_i^2$

Three parameters $A_1,\ A_2,\ A_3$

Spectral relations
$$E(2_1) + E(2_2) = E(3_1)$$
$$4E(2_1) + E(2_2) = E(5_1)$$



QQ-Hamiltonian is triaxial rotor $(19/2)^6$

$$E(2_1) + E(2_2) = 1.005E(3_1)$$
$$4E(2_1) + E(2_2) = 1.026E(5_1)$$

Triaxiality Parameters

H_{rot} parameters $A_1,\ A_2,\ A_3$ instead we use:

1.) Overall energy scale

2.) K mixing angle for 2_1 and 2_2 states Γ $\qquad \tan 2\Gamma = \frac{\sqrt{3}(A_1 - A_2)}{A_1 + A_2 - 2A_3}$

3.) Energy ratio of $E(2_1)$ and $E(2_2)$ $\gamma^2_{\mathrm{DF}} \approx \frac{E(2_1)}{2E(2_2)}$

Shape parameters: β γ define $\mathcal{M}_{\lambda\mu}$

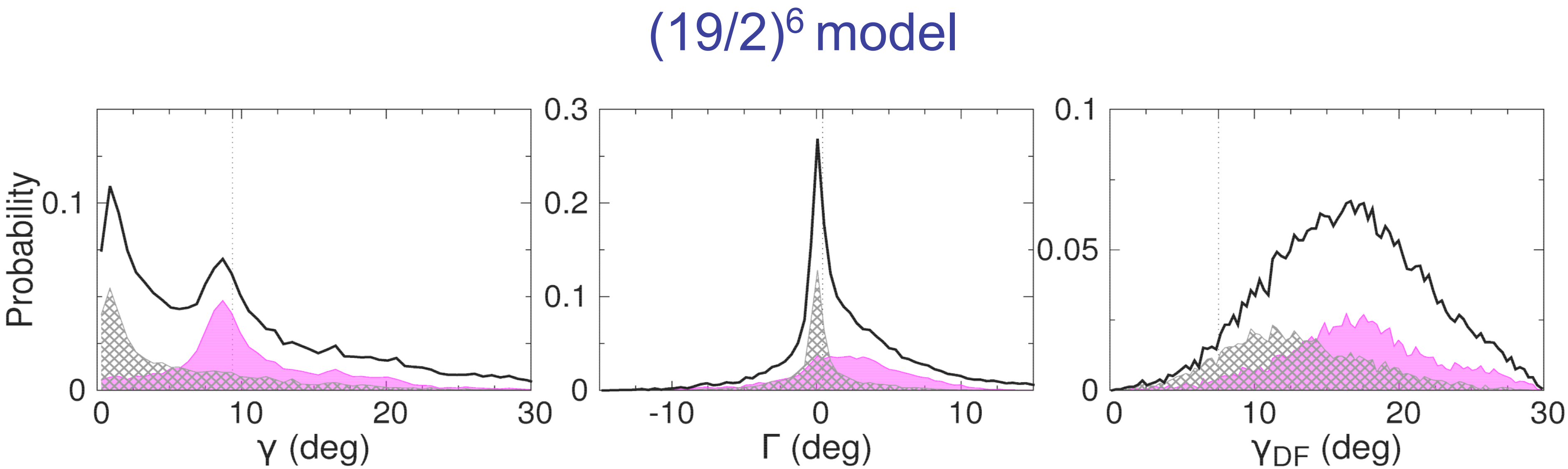
Relationship between H_{rot} and β γ is model-dependent.
For irrotational flow model $\gamma_{DF} = \gamma \quad \Gamma \ll \gamma$

how to measure?

$$\tan^2(\gamma - \Gamma) = \frac{B(E2, 0 \to 2_2)}{B(E2, 0 \to 2_1)} \qquad \tan^2(\gamma + 2\Gamma) = \frac{2B(E2, 2_1 \to 2_2)}{7Q^2(2_1)}$$

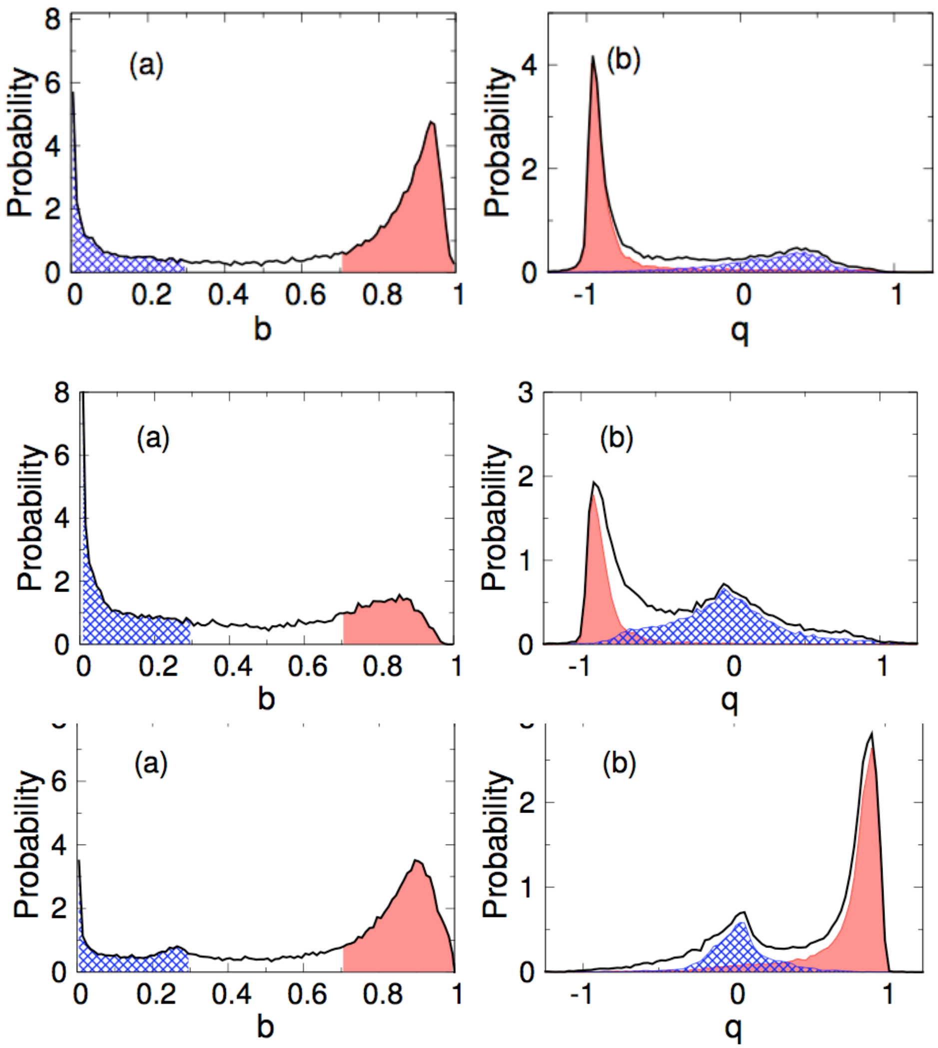
See also: J. M. Allmond, et.al. Phys. Rev. C 78, 014302 (2008).

Distribution of triaxiality parameters



- Prolate realization are axially symmetric.
- Oblate realizations are triaxial. The values of the triaxiality parameters are consistent with those of the QQ Hamiltonian $\gamma = 9.79 \;\; \Gamma = 0.73$
- Energy ratios are too sensitive to non-collective features.

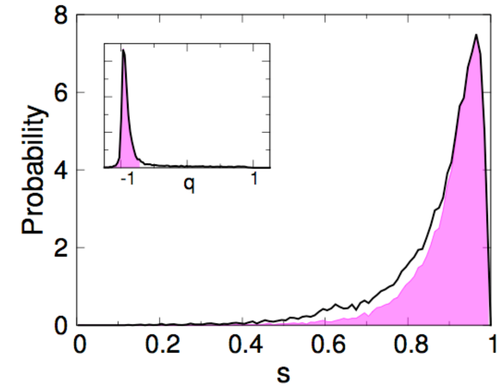
Collectivity in other models



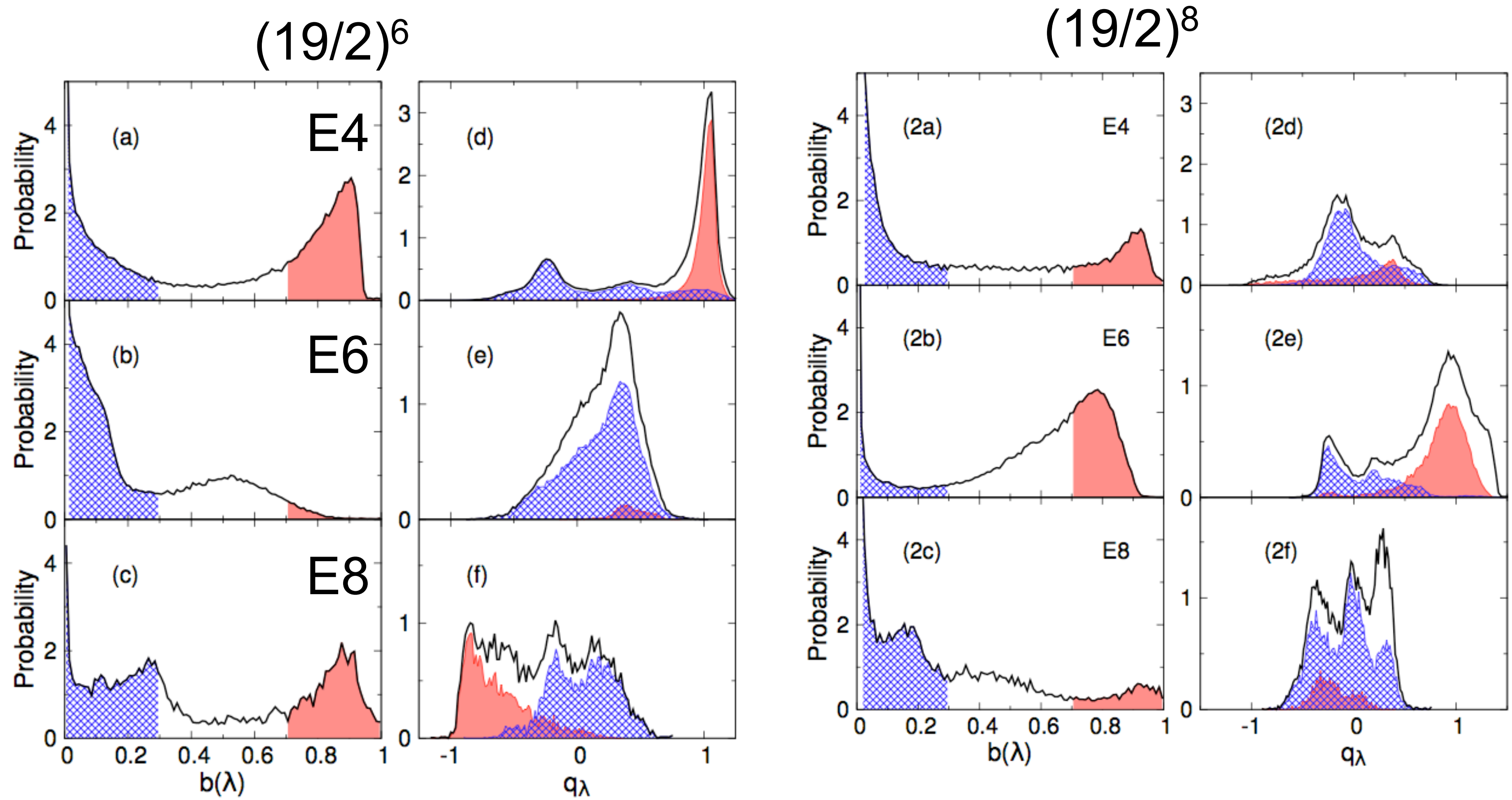
$(19/2)^8$

$(13/2,13/2)^6$

$(13/2,13/2)^{22}$

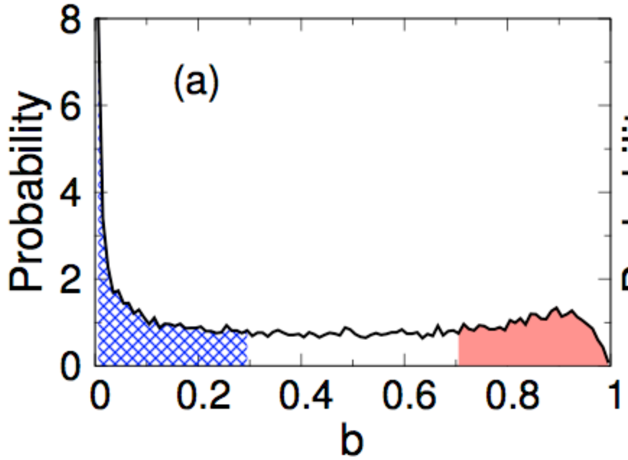


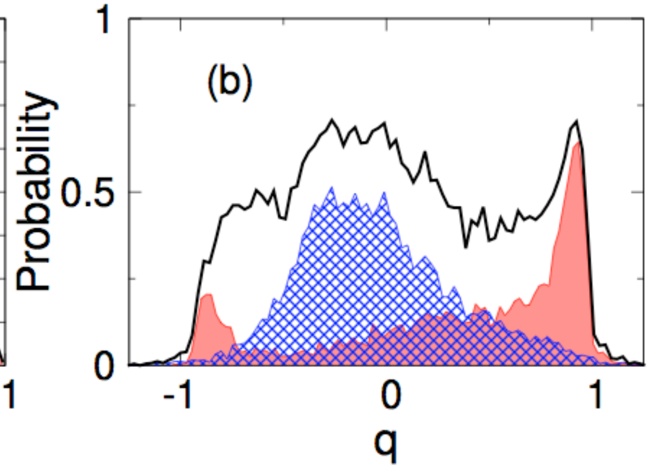
Collectivity in higher multipole moments

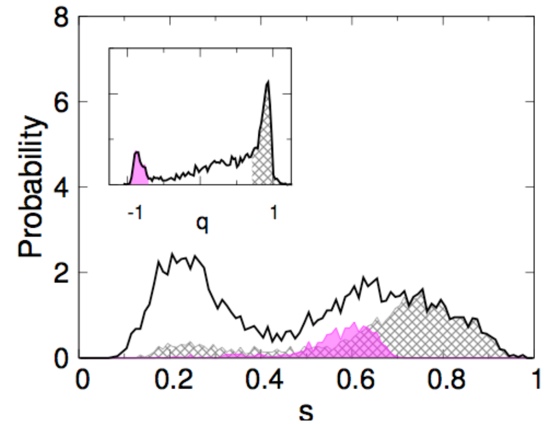


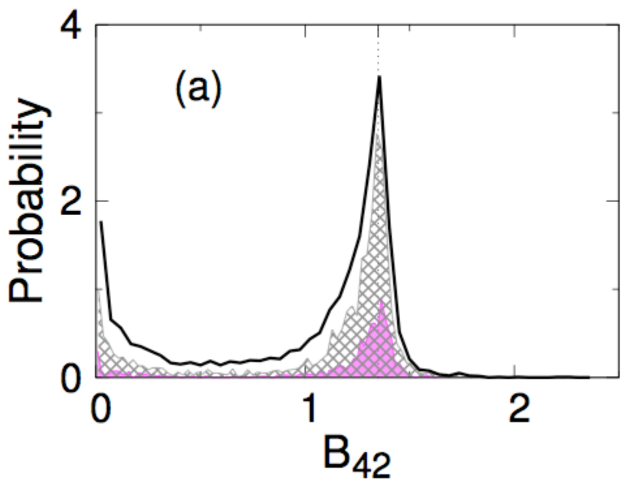
Collectivities are seen for higher multipolairities.

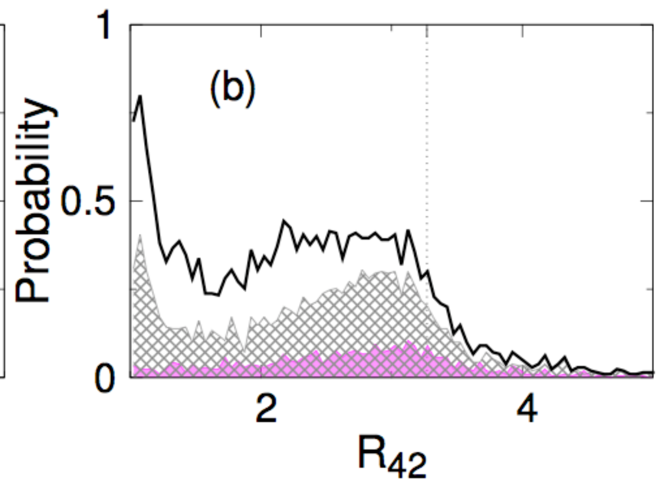
Realistic Model space $(f_{7/2}, p_{3/2})^8$









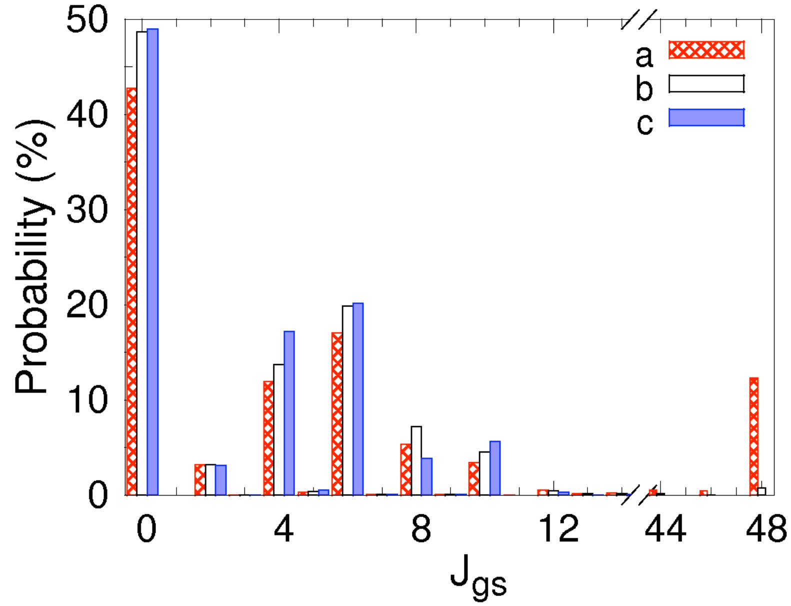


- Similar to other models
- Prolate shape
- Rotational energy spacing in R_{42}

Role of QQ component $(19/2)^8$

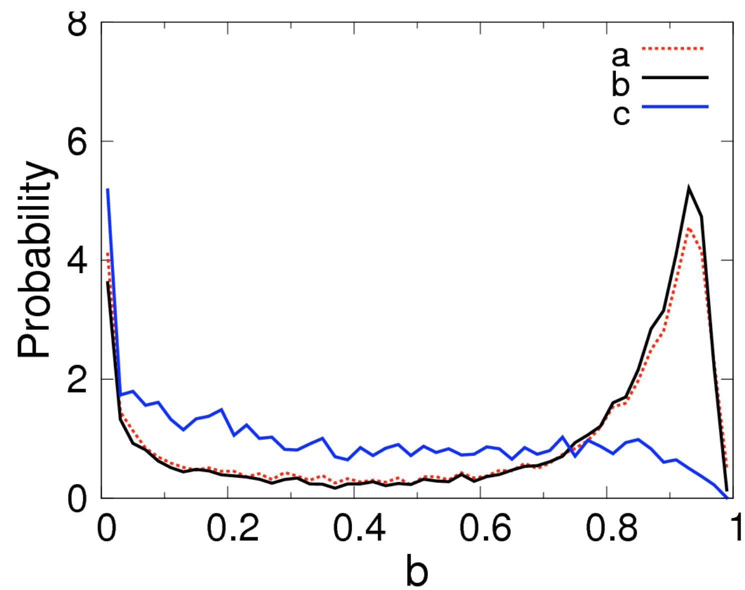
$$H = \sum_L V_L \sum_\Lambda P^\dagger_{L\Lambda} P_{L\Lambda} = \epsilon N - \sum_{\mathcal{K}} \tilde{V}_{\mathcal{K}} \sum_k M^\dagger_{\mathcal{K}k} \mathcal{M}_{\mathcal{K}k}$$

- The K=0 term is proportional to the number of particles.
- The dipole term, K=1, is related to the components of J.
- K=2 is a quadrupole term.



What happens if quadrupole componet is removed?

(a) two-body random ensemble
(b) J^2 is excluded
(c) J^2 and QQ are excluded

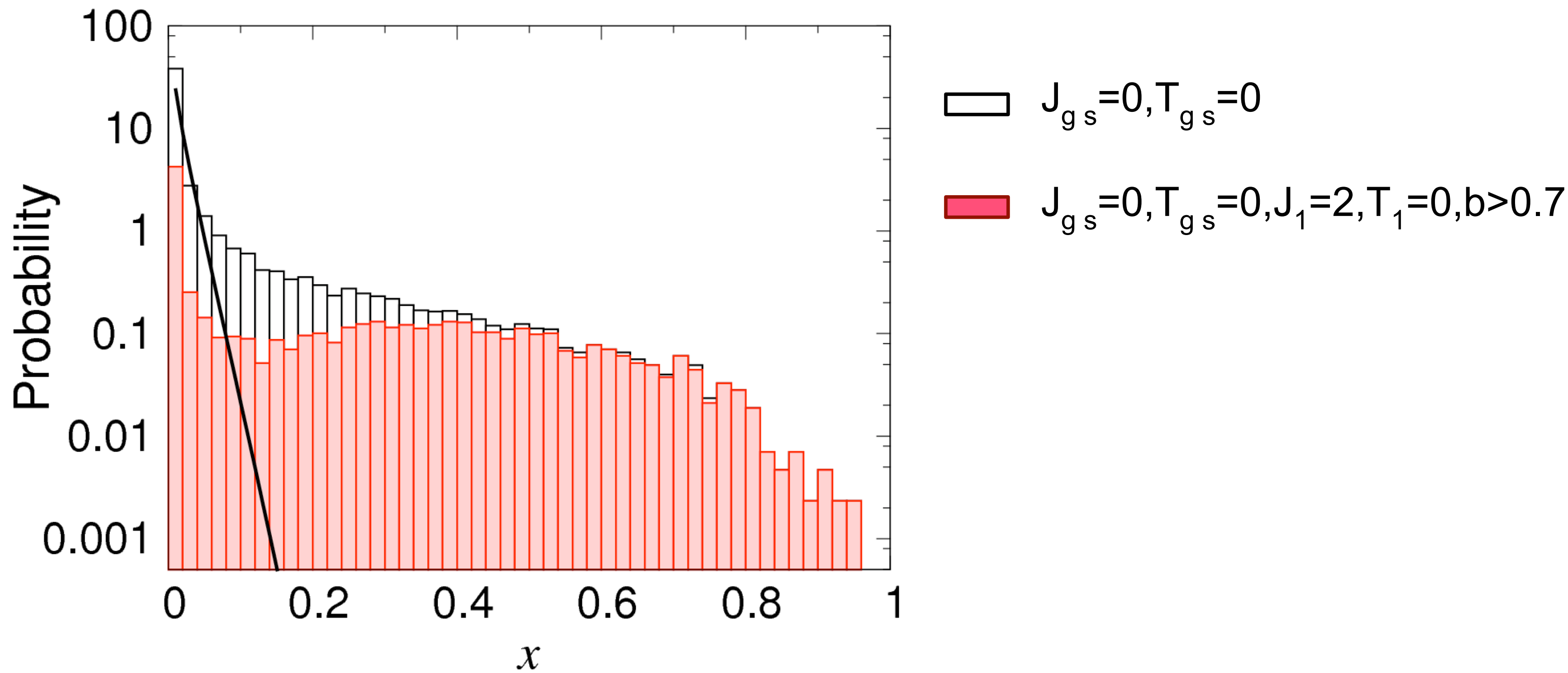


QQ interaction component is necessary for quadrupole collectivity in transition

Role of QQ component $(f_{7/2},p_{3/2})^8$

How big is the QQ component in the wave function?

Overlap between TBRE and QQ wave functions: $x = |\langle 0_{gs}(\mathrm{TBRE})|0_{gs}(\mathrm{QQ})\rangle|^2$



The states in TBRE exhibit QQ structure.

Characteristics of the QQ interaction

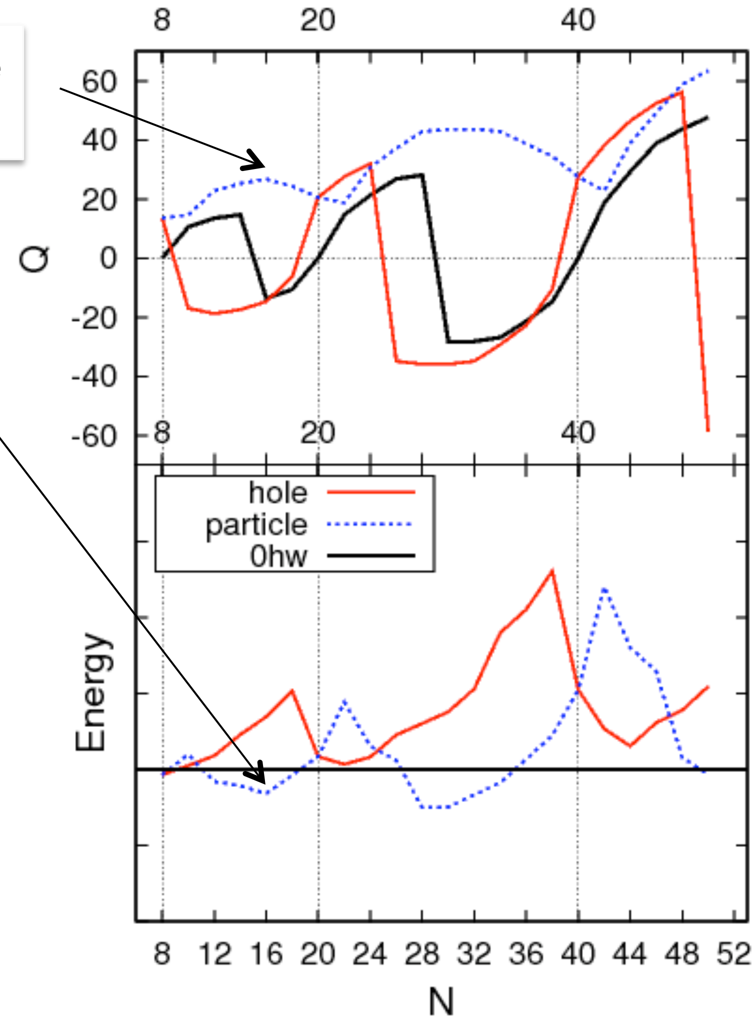
	b	q	B_{42}	R_{42}	γ(deg)	Γ(deg)	γ_{DF}(deg)
$(19/2)^6$	0.97	-0.979	1.42	3.31	9.8	0.43	7.5
$(0f_{7/2}, 1p_{3/2})^8$	0.97	0.996	1.35	3.27	4.7	-0.03	13.1

- Spectral characteristics of the dominant collective mode in the TBRE are consistent with those of the QQ Hamiltonian.

Remarks on prolate dominance

Figure: $H=H_{HO.}+H_{su(3)}$ perturbative approach.
$\hbar\omega=41/A^{1/3}$ $\kappa=14/A^{5/3}$

- States of the QQ-Hamiltonian determine shape
- QQ-Hamiltonian is defined by the model space
- Particle-hole symmetry
- Model predictions



Example: 8 nucleons on sd shell

configuration	irrep	Energy	shape
$(p)^2(sd)^4$	(0,6)	216	oblate
$(sd)^8$	(2,4)	184	oblate
$(sd)^4(pf)^2$	(12,0)	720	prolate

n-body Random Ensemble (n-BRE)

n-body Hamiltonian

$$H^{(n)} = \sum_{\alpha\beta} \sum_{L} V_L^{(n)}(\alpha\beta) \sum_{M=-L}^{L} T_{LM}^{(n)\dagger}(\alpha)\, T_{LM}^{(n)}(\beta)$$

Operator

$$T_{LM}^{(n)\dagger}(\alpha) = \sum\nolimits_{12\ldots n} C_{12\ldots n}^{LM}(\alpha)\, a_1^\dagger a_2^\dagger \ldots a_n^\dagger$$

n-body operator is eigenstate $T_{LM}^{(n)\dagger}(\alpha)|0\rangle$

of the reference 2-body Hamiltonian $H_0^{(2)}$

Random Gaussian ensemble of interactions

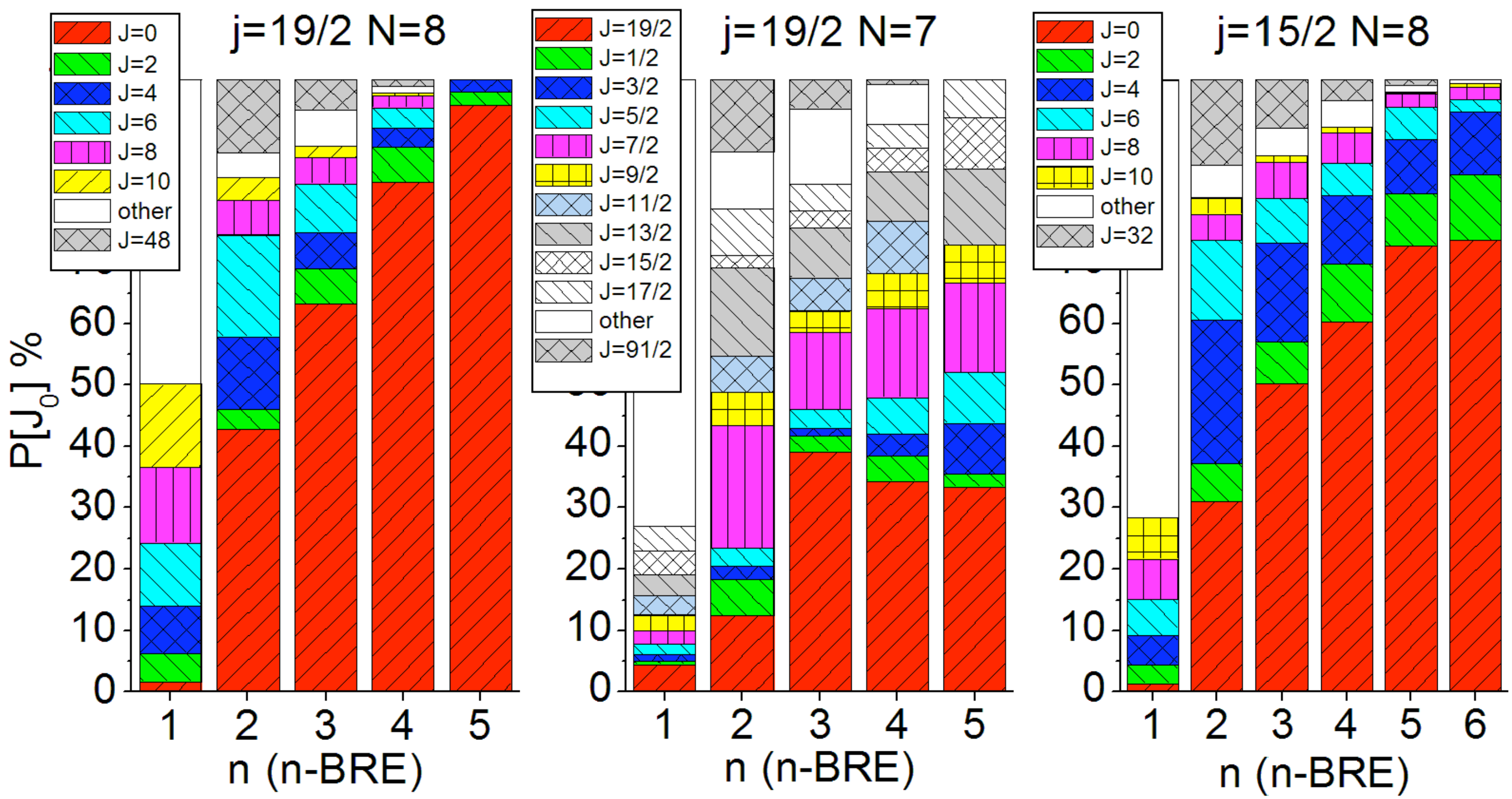
$$\langle V_L^{(n)}(\alpha,\beta)\rangle = 0 \qquad V_L^{(n)}(\alpha,\beta) = V_L^{(n)}(\beta,\alpha)$$

$$\langle V_L^{(n)}(\alpha,\beta)\, V_{L'}^{(n)}(\alpha',\beta')\rangle = \delta_{LL'}\delta_{\alpha\alpha'}\delta_{\beta\beta'}(1+\delta_{\alpha\beta})/2$$

The ensemble does not depend on the choice of reference Hamiltonian

For n = N the ensemble is GOE in each symmetry class

Statistics of g.s. spins



Low-lying quadrupole collectivity and 3-body force

$(19/2)^6$ model

Phonon-phonon interaction

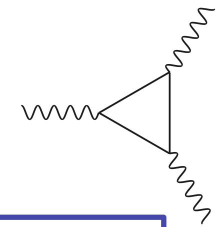
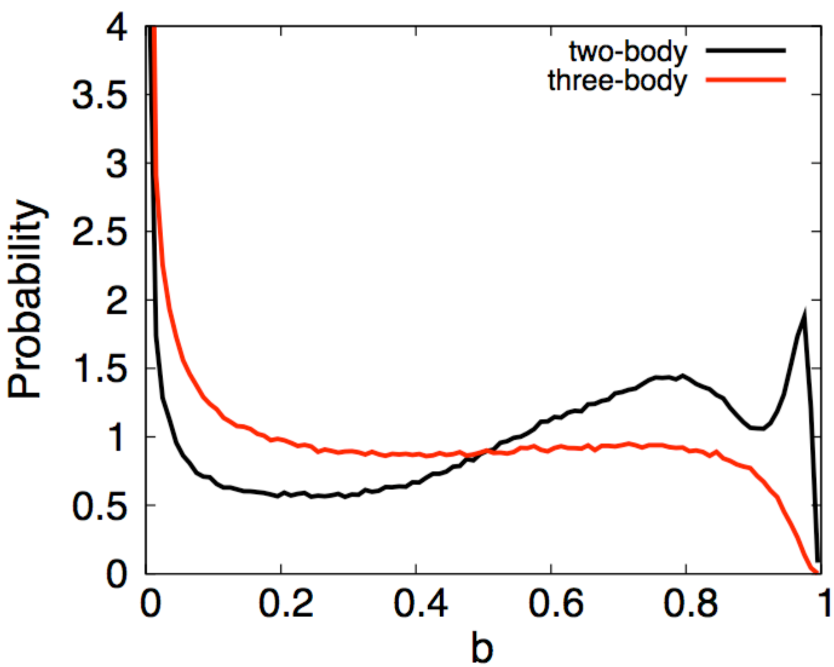
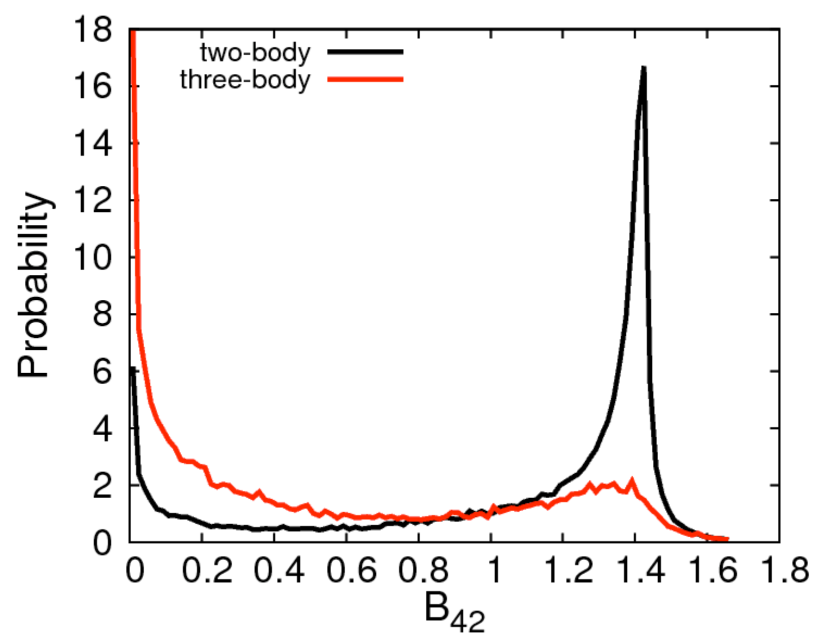


Figure: Fractional collectivity of the E2 between 0^+ g.s. and 2^+ j=19/2 system with 6 particles

Figure: Ratio of the E2 reduced transition rates between 0^+ g.s., 2^+ and 4^+ states j=19/2 system with 6 particles





Distribution of spectroscopic factors

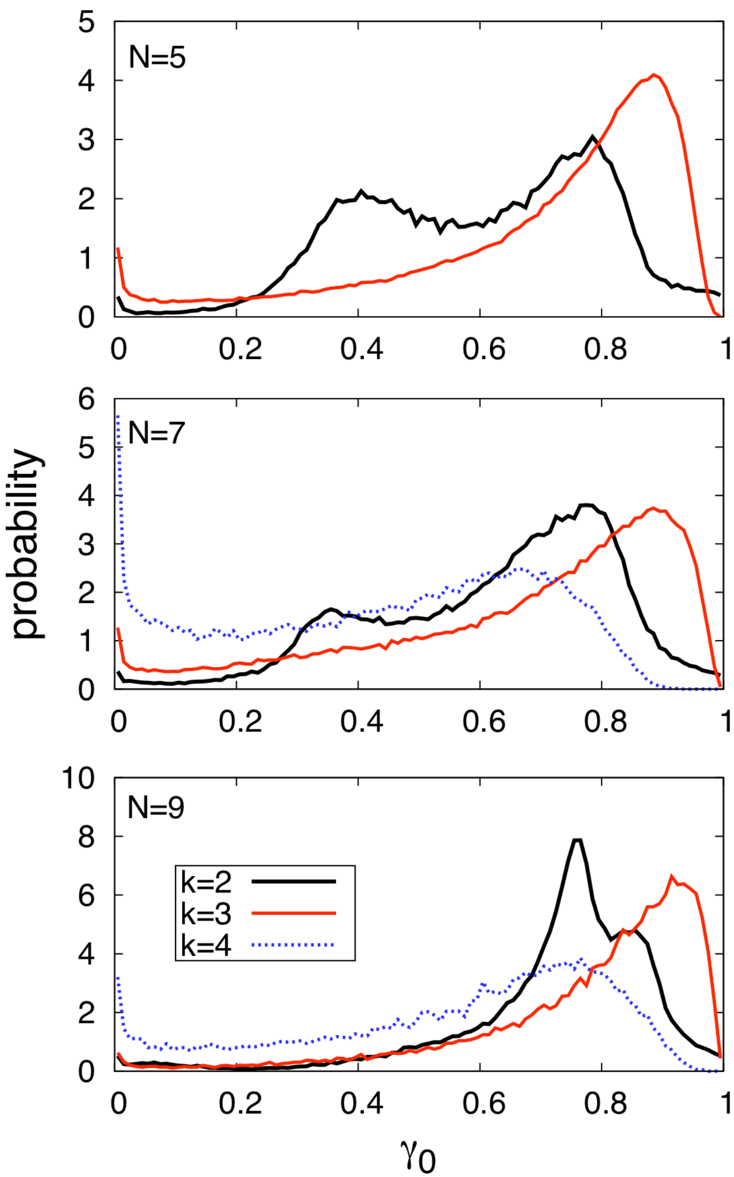


Figure: Distribution of the spectroscopic factors for decay of g.s. in N-particle system to final N-1 0^+ g.s. j=19/2

Single j=19/2 level

$|I\rangle$ -parent ground state $19/2^+$

$|F\rangle$ -daughter ground state 0^+

$$|c\rangle = \{a^\dagger |F\rangle\}$$

γ_0 -ground state to ground state reduced transition width

The spectroscopic factor of low-lying states is large

Conclusions

- Collective rotational features are common in TBRE (10%)
- Surprising coherent role of QQ-component
- Deformation type, triaxiality are consistent with those of QQ Hamiltonian
- Prolate versus oblate
- Many-body forces are destructive for collectivities and eventually lead to GOE

Acknowledgements:

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Further reading:

V. Abramkina and A. Volya, **arXiv:1105.1669v1** [nucl-th]