# A New Microscopic Multiphonon Approach to Nuclear Spectroscopy

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| Semiclassical | Microscopic |
| --- | --- |
| $\{\alpha_\lambda, \pi_\lambda\} \rightarrow \{O_\lambda, O_\lambda^\dagger\}$ | **TDA mapping** $O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h$ |
| **EofM** $[H, O_\lambda^\dagger] = \hbar\omega_\lambda\, O_\lambda^\dagger$ | **EoM** $[H, O_\lambda^\dagger] \mid > = \hbar\omega_\lambda\, O_\lambda^\dagger \mid >$ |
| **Collective modes** | **RPA mapping** $O_\lambda^\dagger = \Sigma_{ph}\, [X_{ph}(\lambda)\, a^\dagger_p\, a_h - Y_{ph}(\lambda)\, a^\dagger_h\, a_p]$ **EoM** $[H, O_\lambda^\dagger]\, \mid 0> = \hbar\omega_\lambda\, O_\lambda^\dagger \mid 0>$ $\mid 0> \equiv$ correlated g.s |

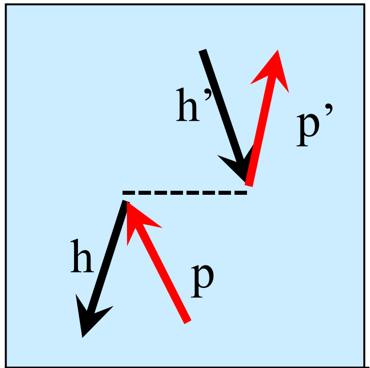
# TDA: Eigenvalue Equations

$$\langle\lambda|H\,|ph^{-1}\rangle = \langle\lambda|[H, a^\dagger_p\, a_h]\,|\,\rangle \quad = \quad \hbar\omega_\lambda\, c^\lambda_{ph}$$

$$\Downarrow$$

$$A\, c^\lambda = \hbar\omega_\lambda\, c^\lambda$$

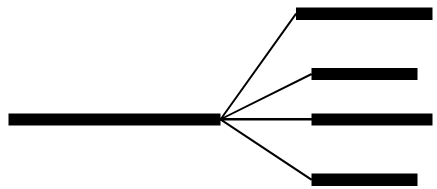
$$A_{(ph)(p'h')} = (\varepsilon_p - \varepsilon_h)\,\delta_{pp'}\,\delta_{hh'} + V_{ph'hp'}$$




$V_{ph'hp'}$

$$|\lambda\rangle = O^\dagger_\lambda\,|\rangle$$

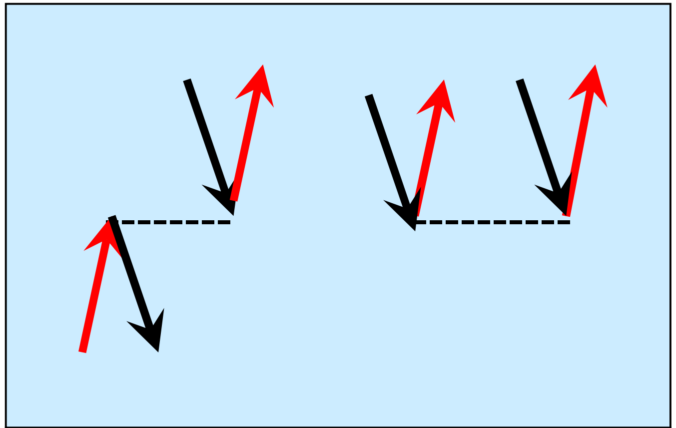
$$O^\dagger_\lambda = \Sigma_{ph}\, c^\lambda_{ph}\, a^\dagger_p\, a_h\,|\,\rangle$$



**Landau damping**

# RPA: Eigenvalue Equations

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X(\lambda) \\ Y(\lambda)\end{pmatrix} = \hbar\omega_\lambda \begin{pmatrix} X(\lambda) \\ -Y(\lambda)\end{pmatrix}$$



A B

$$A_{php'h'} = (\varepsilon_p - \varepsilon_h)\, \delta_{pp'}\, \delta_{hh'} + V_{ph'hp'}$$

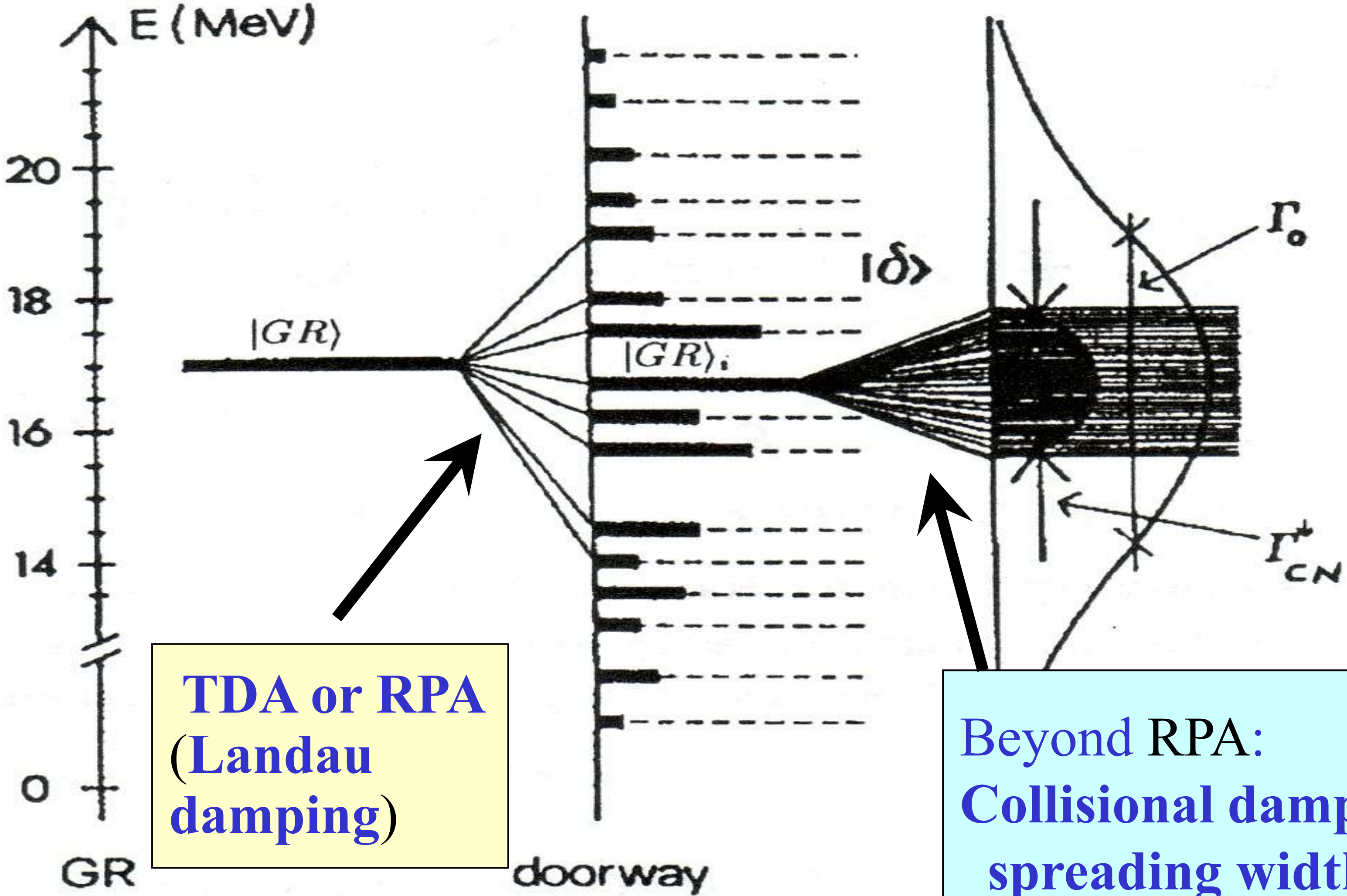
$$B_{hh'pp'} = V_{hh'pp'}$$

$$|\lambda\rangle = O_\lambda^\dagger |0\rangle$$

$$O_\lambda^\dagger = \Sigma_{ph} \left[X_{ph}(\lambda)\, a^\dagger_p\, a_h - Y_{ph}(\lambda)\, a^\dagger_h\, a_p\right] |0\rangle$$

Quasi-Boson Approximation: $|0\rangle \rightarrow \quad |\rangle$

# Collective modes: anharmonic features




**TDA or RPA (Landau damping)**

Beyond RPA: **Collisional damping spreading width**

From P.F. Bortignon, A. Bracco, R.A. Broglia, Giant Resonances (hap, 1998)

# Multiphonon excitations: Exp. evidence

## * High-energy

(N. Frascaria, NP A482, 245c(1988); T. Auman, P.F. Bortignon, H. Hemling, Ann. Rev. Nucl. Part. Sc. 48, 351 (1998))

Double

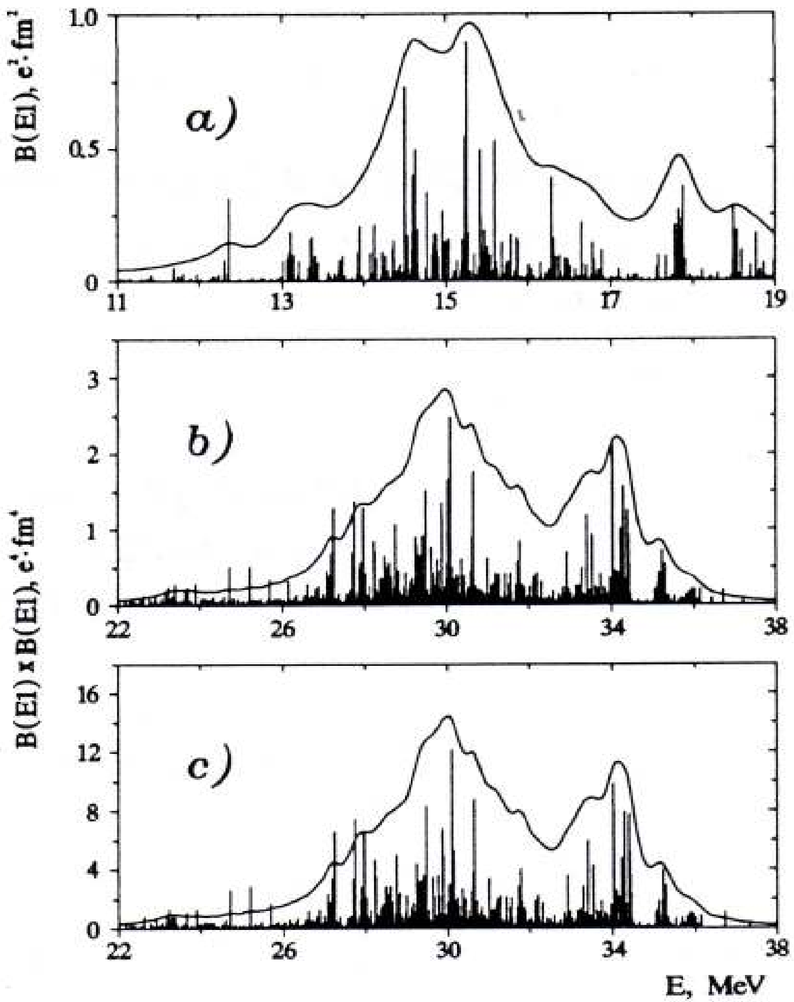
$$D \times D|0>$$

and

triple

$$D \times D \times D|0>$$

dipole giant resonances



# Multiphonon excitations: Exp. evidence

## ** Low-energy

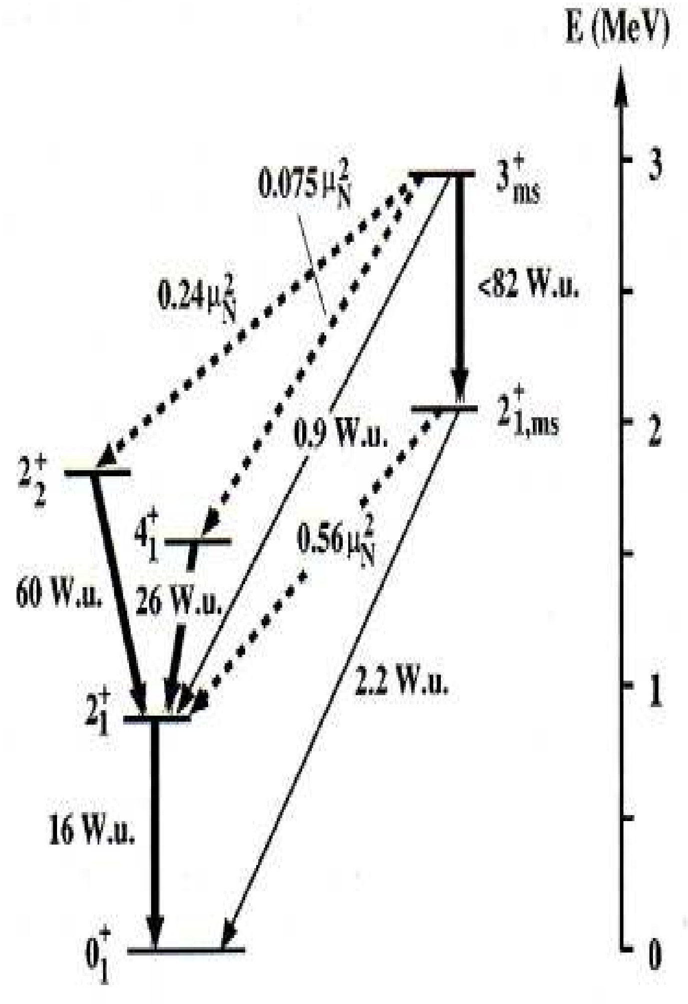
M. Kneissl. H.H. Pitz, and A. Zilges, Prog. Part. Nucl. Phys. 37, 439 (1996); M. Kneissl. N. Pietralla, and A. Zilges, J.Phys. G, 32, R217 (2006) :

- Two- and three-phonon multiplets

$Q_2 \times Q_3|0>$, $Q_2 \times Q_2 \times Q_3|0>$

- Proton-neutron (F-spin) mixed-symmetry states
  (N. Pietralla et al. PRL 83, 1303 (1999))

$[Q_2^{(p)} - Q_2^{(n)}] \, (Q_2^{(p)} + Q_2^{(n)})^N|0>$,



# From RPA to multiphonon approaches

Basic idea inspired by the **Boson expansion techniques (BET)**
(S. T. Belyaev and V. G. Zelevinsky, Nuc. Phys. 39, 582 (1962) )

$$O^\dagger \quad \Rightarrow \quad O^\dagger + O^\dagger O^\dagger + O^\dagger O^\dagger O^\dagger \; \ldots\ldots$$

$$O^\dagger = \Sigma_{ij} [X(ij)\, \alpha^\dagger_i \alpha^\dagger_j - Y(ij)\, \alpha_i \alpha_j]$$

**Recent calculations**

a. **Density functional approaches** (up to two-phonons)

1. **SRPA** (Gambacurta et al. PRC**8 1, 054312 (2010)) based on Skyrme**

2. **RRPA-phonon** model (Litvinova et al. PRL 105, 022502 (2010)

b. A popular approach: **QPM** (Soloviev) (up to 3-phonons)

$$\mathbf{H}\ (\text{separable}) \quad \Rightarrow \quad H(O^\dagger O) = \Sigma \omega_\lambda O_\lambda^\dagger O_\lambda + H_{vq}(O^\dagger O\, O^\dagger O)$$

Common features:

1. **QBA** is the underline approximation
2. **No Correlations** **explicitly included** in the gs

# Eigenvalue problem in multiphonon space

$$H \,|\, \Psi_\nu > = E_\nu \,|\, \Psi\nu >$$

$| \Psi_\nu > \ni \mathcal{H} = \Sigma_n \oplus \mathcal{H}_n$ $\mathcal{H}_n \in |n; \beta> \equiv$ n-phonon **basis states**

An obvious (**prohibitive!!**) choice

$$|n; \beta> \;=\; O^\dagger_{\nu_1} \ldots O^\dagger_{\nu_i} \ldots . O^\dagger_{\nu_n} |0>$$

$$O^\dagger_\nu = \Sigma_{ph}\, c_{\alpha ph}\; a^\dagger_p\, a_h \,|\, 0>$$

**A viable route**

$$|n; \beta> \;=\; \Sigma_{\nu\alpha}\, C^\beta_{\nu\alpha}\, O^\dagger_\nu \,|\, n\text{-}1; \alpha >$$

# Construction of $|n;\beta>$ : EofM

$$< n;\beta\,|[H, O^{\dagger}_{\nu}]|\,n-1;\alpha> = (E_{\beta}^{(n)} - E_{\alpha}^{(n-1)}) < n;\beta\,|\,O^{\dagger}_{\nu}\,|\,n-1;\alpha>$$

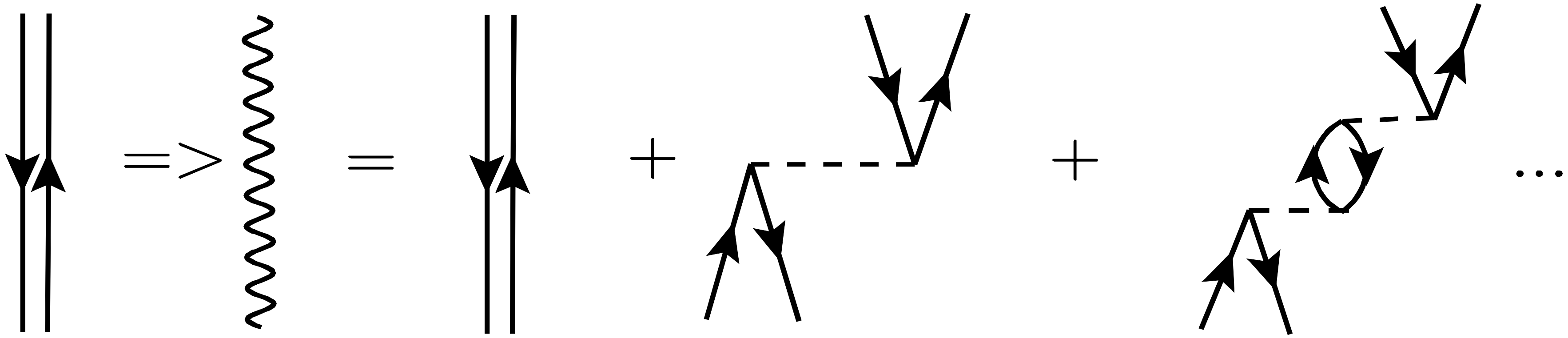
$$\Sigma_{\nu\gamma}\,[\mathcal{H}_{(\mu\alpha)(\nu\gamma)} - \mathcal{E}\,\mathcal{D}_{(\mu\alpha)(\nu\gamma)}]\,\mathcal{C}_{\nu\gamma} = 0$$

| | |
|---|---|
| $\mathcal{H} = \mathcal{A}\mathcal{D}$ | $\mathcal{D}_{(\mu\alpha)(\nu\gamma)} = <n,\gamma\|\, O_{\nu} O^{\dagger}_{\mu}\,\|n,\alpha>$ |

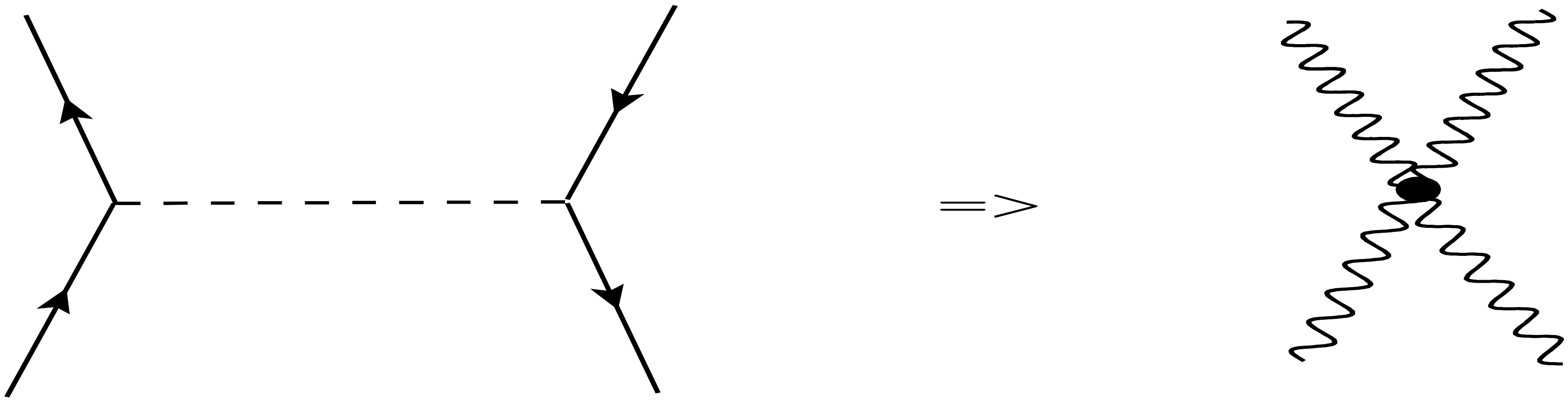
| 1-phonon: TDA | n-phonon: EofM |
|---|---|
| $A\, c = \hbar\omega\, c$ | $\mathcal{H} \mathcal{C} = \mathcal{E} \mathcal{D} \mathcal{C}$ <br> $\mathcal{H} = \mathcal{A} \mathcal{D}$ |
| $A_{(ph)(p'h')} = (\varepsilon_p - \varepsilon_h)\delta_{pp'}\delta_{hh'} + V_{ph'hp}$ | $\mathcal{A}_{(\mu\alpha)(\nu\gamma)} = (E_\mu + E_\alpha)\delta_{\alpha\gamma}\delta_{\mu\nu} + \mathcal{V}_{(\mu\alpha)(\nu\gamma)}$ |
| $\lvert\lambda\rangle = \Sigma_{ph}\, c^{\lambda}_{ph}\, a^{\dagger}_{p}\, a_h \lvert\rangle$ | $\lvert n;\beta\rangle = \Sigma_{\nu\alpha}\, C^{\beta}_{\nu\alpha}\, O^{\dagger}_{\nu} \lvert n-1;\alpha\rangle$ |

$$\varepsilon_p - \varepsilon_h \Rightarrow E_\mu$$

$$|ph\rangle \Rightarrow |\mu\rangle = \Sigma_{ph}\, c^{\mu}{}_{ph}\, a^{\dagger}{}_{p}\, a_h\, |\rangle$$



$$V_{ph'hp} \Rightarrow \mathcal{V}_{(\mu\nu)(\mu'\nu')}$$



# Generation of n-phonon states

$$\mathbf{A\,c} = \hbar\omega\ \mathbf{c} \Rightarrow |\lambda\rangle = \Sigma_{ph}\, \mathbf{c}^{\lambda}_{ph}\, a^{\dagger}_{p}\, a_{h}\, |\rangle$$

$$\mathcal{H}\,\mathcal{C} = \mathcal{E}\,\mathcal{D}\,\mathcal{C} \Rightarrow |n;\beta\rangle = \Sigma_{\nu\alpha}\, C^{\beta}_{\nu\alpha}\, O^{\dagger}_{\nu}\, |\, n-1;\alpha\rangle$$

n= 2, 3,.....

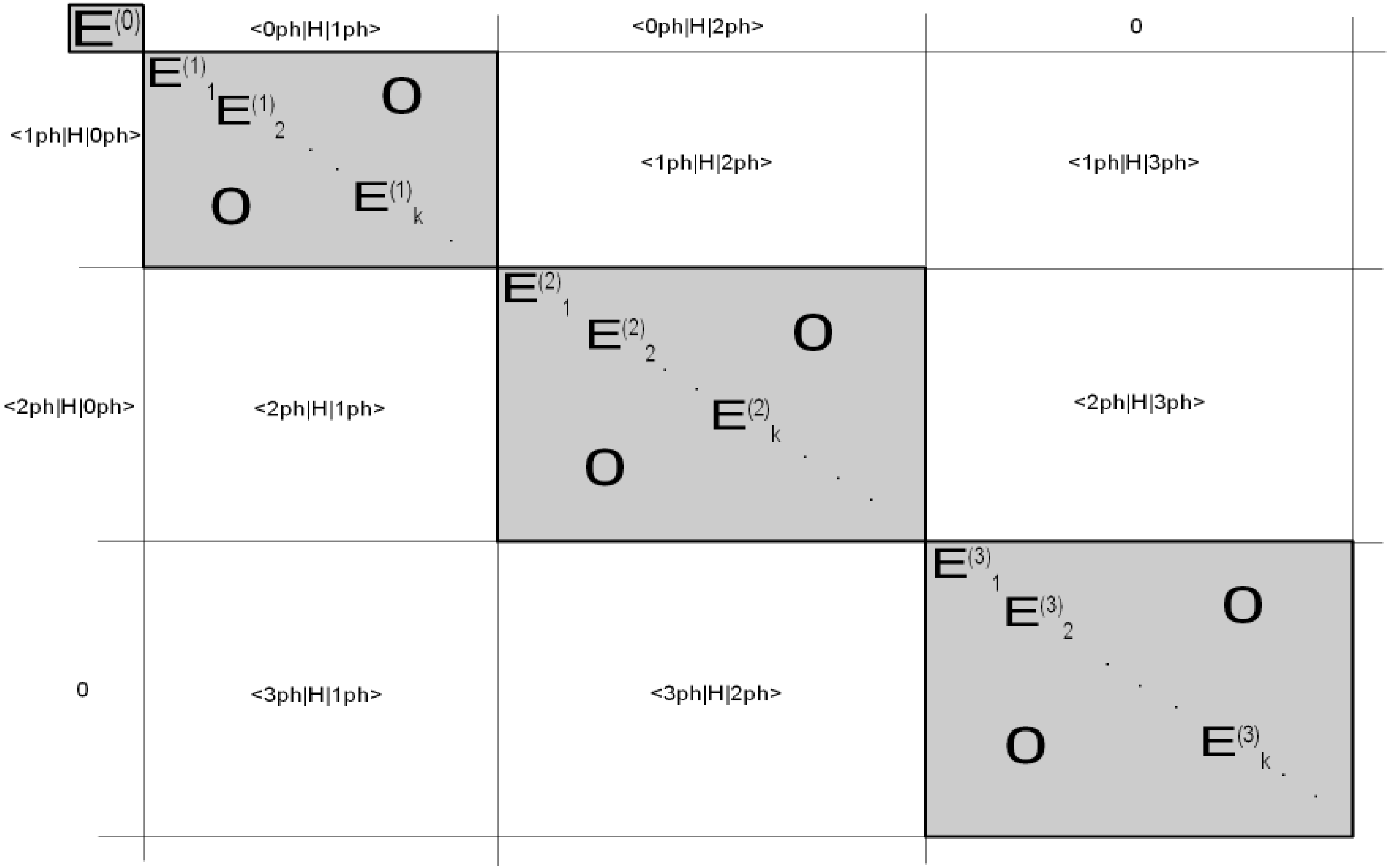
# Eigenvalue equations in phonon space

$$\langle n,\alpha | H | \Psi_\nu \rangle = E_\nu | \Psi_\nu \rangle$$

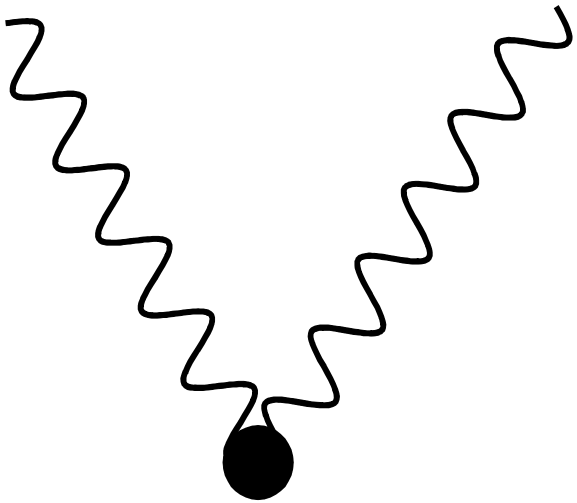
$$\sum^{\beta}_{n'} \left[ (E_\nu - E^{\alpha}_{n}) \delta_{\alpha\beta} \delta_{nn'} + V^{\alpha\beta}_{(n,n')} \right] C^{\nu}_{n'\beta} = 0$$

$$| \Psi_\nu \rangle = \sum_{n\alpha} C^{\nu}_{n\alpha} | n,\alpha \rangle$$

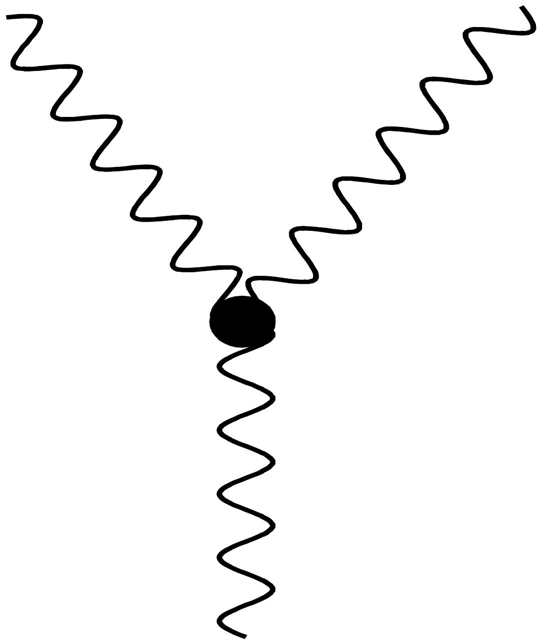
# **Hamiltonian** in the $|n-\alpha>$ basis (n =0,1,2,…..)



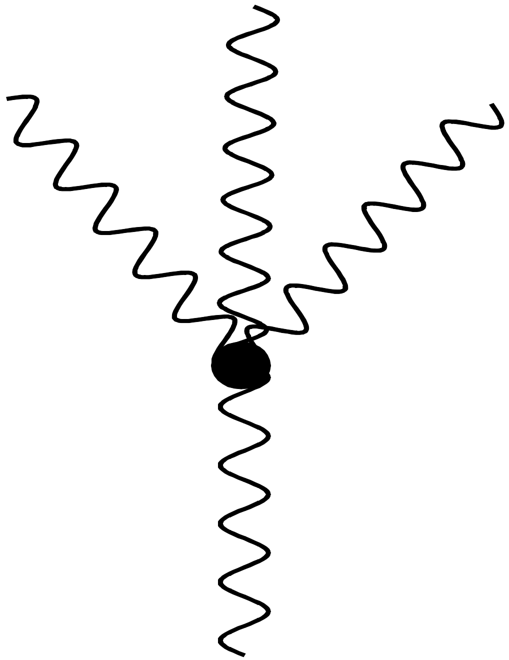
# Diagramatic:$V_{nn'}$



<2|V|0>

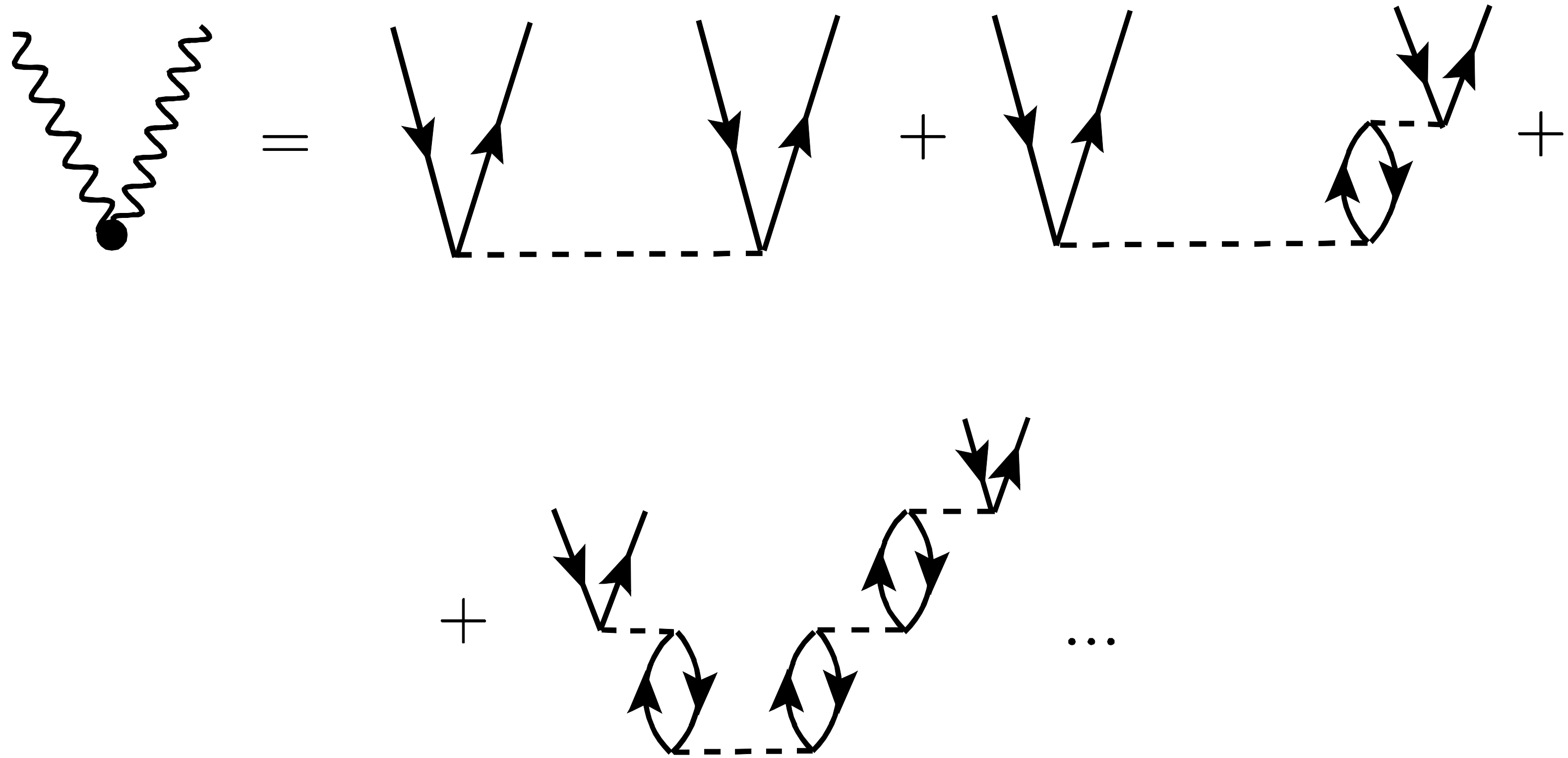


<2|V|1>



<3|V|1>

# Ground State Correlations (real!)



# EMPM : Numerical implementation

Hamiltonian

$$H = H_0 + V = \Sigma_i h_i + G_{bare} \quad ( \mathbf{V}_{\mathbf{BonnA}} \Rightarrow \mathbf{G}_{\mathbf{bare}})$$

$$h = t + h_{\mathbf{Nils}}$$

$$h = t + h_{\mathbf{HF}}$$

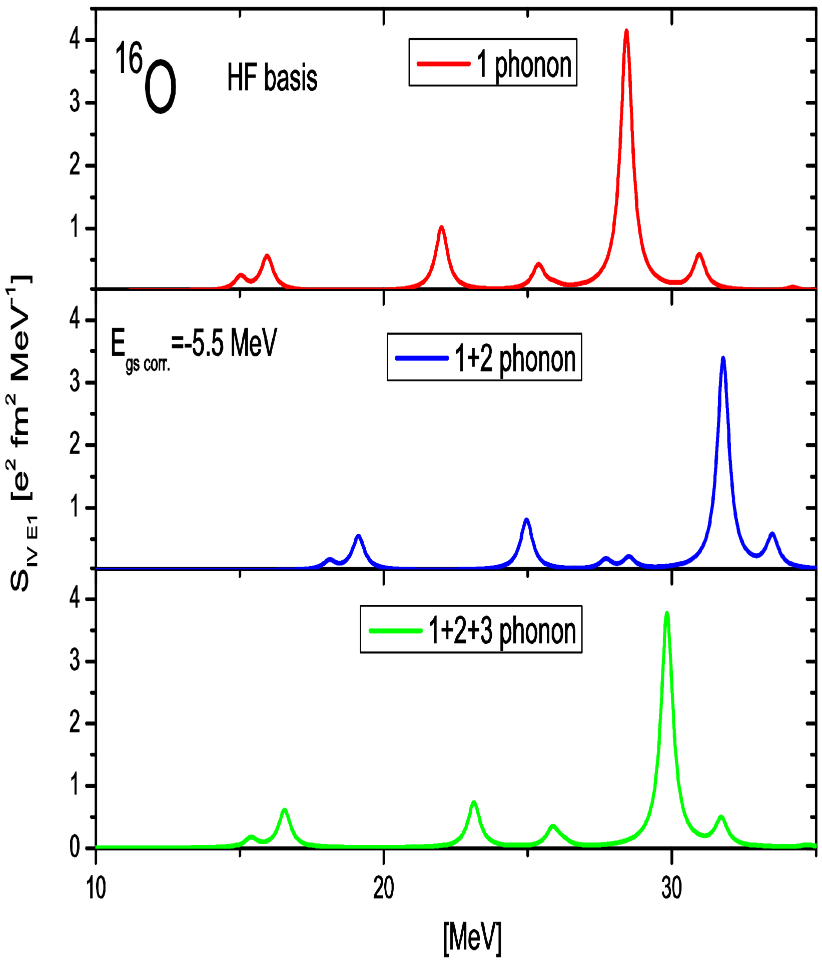
**Application to O isotopes**

**Phonon** space: up to **n=3**

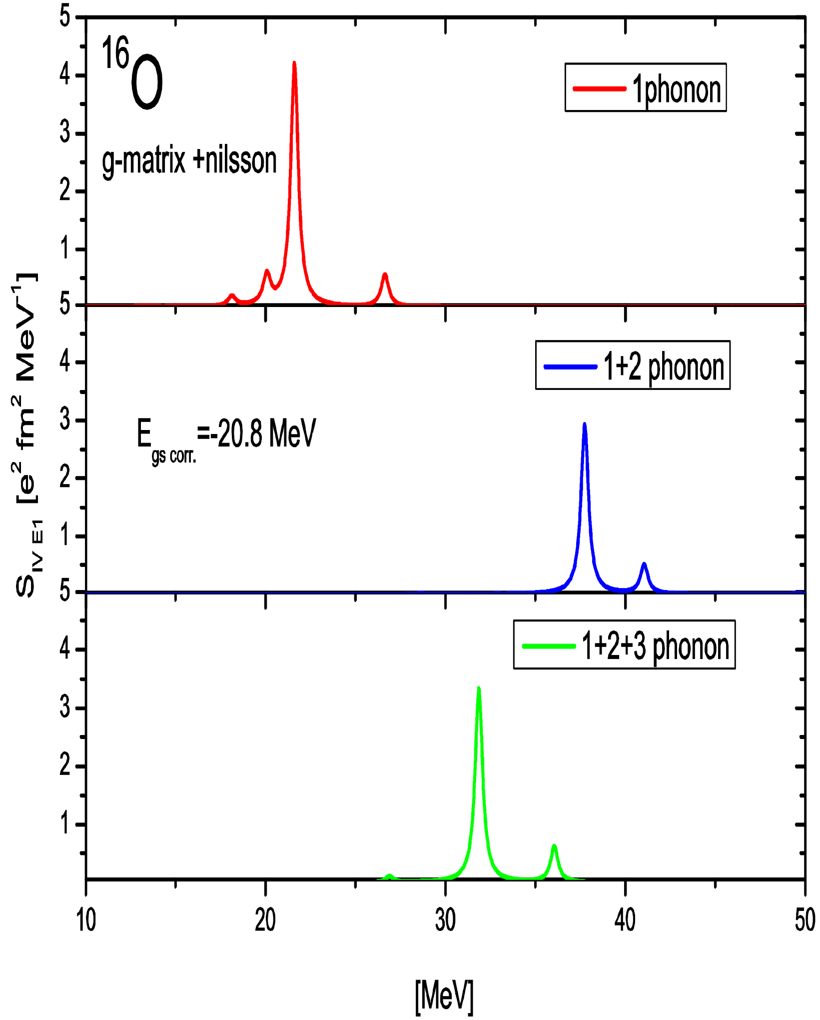
**SM** space

1. p-h of $1\hbar\omega$
2. p-h of $3\hbar\omega$ (with truncation of the n=3 phonon space)

# $^{16}O$: E1 Strength Function

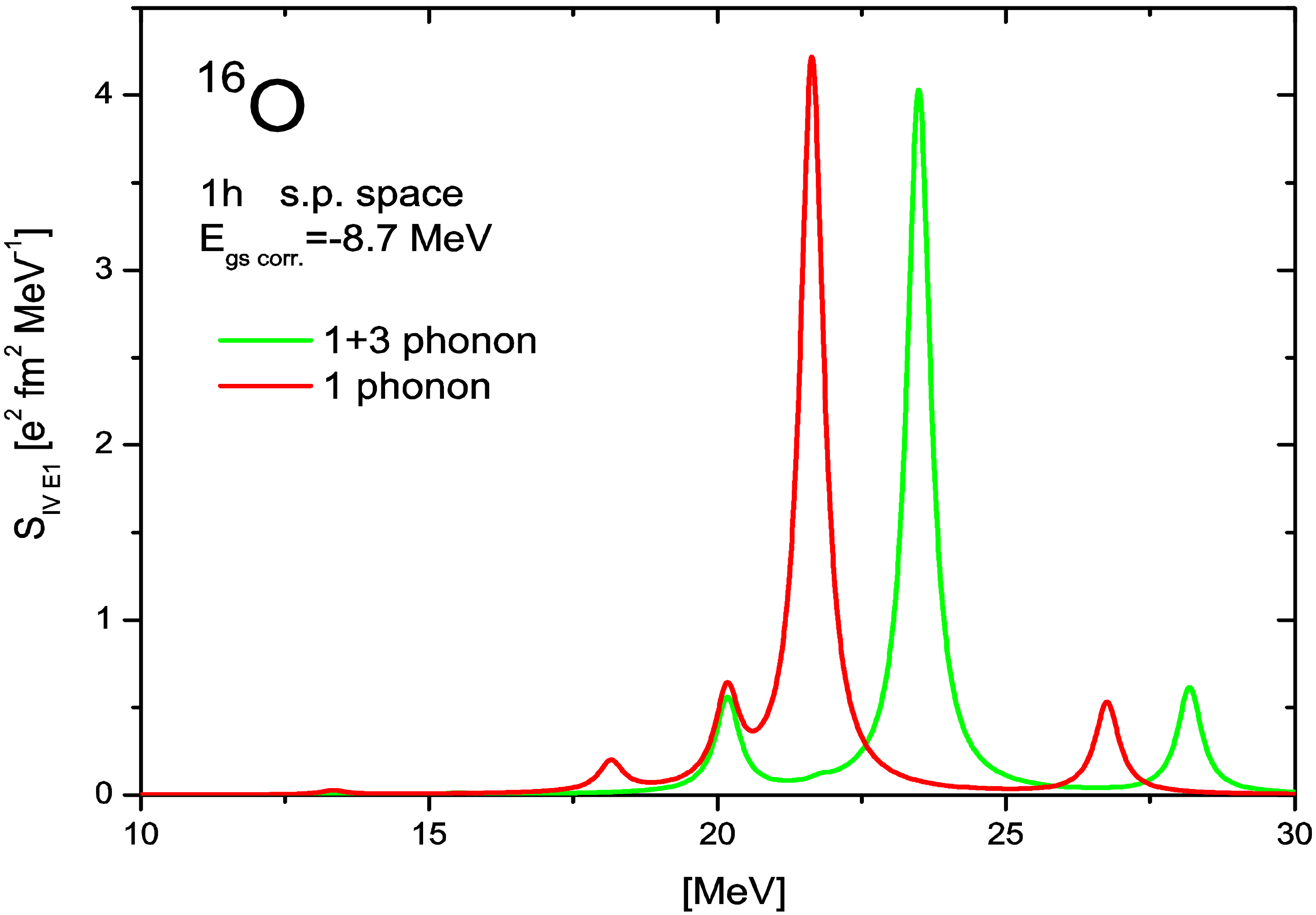



HF

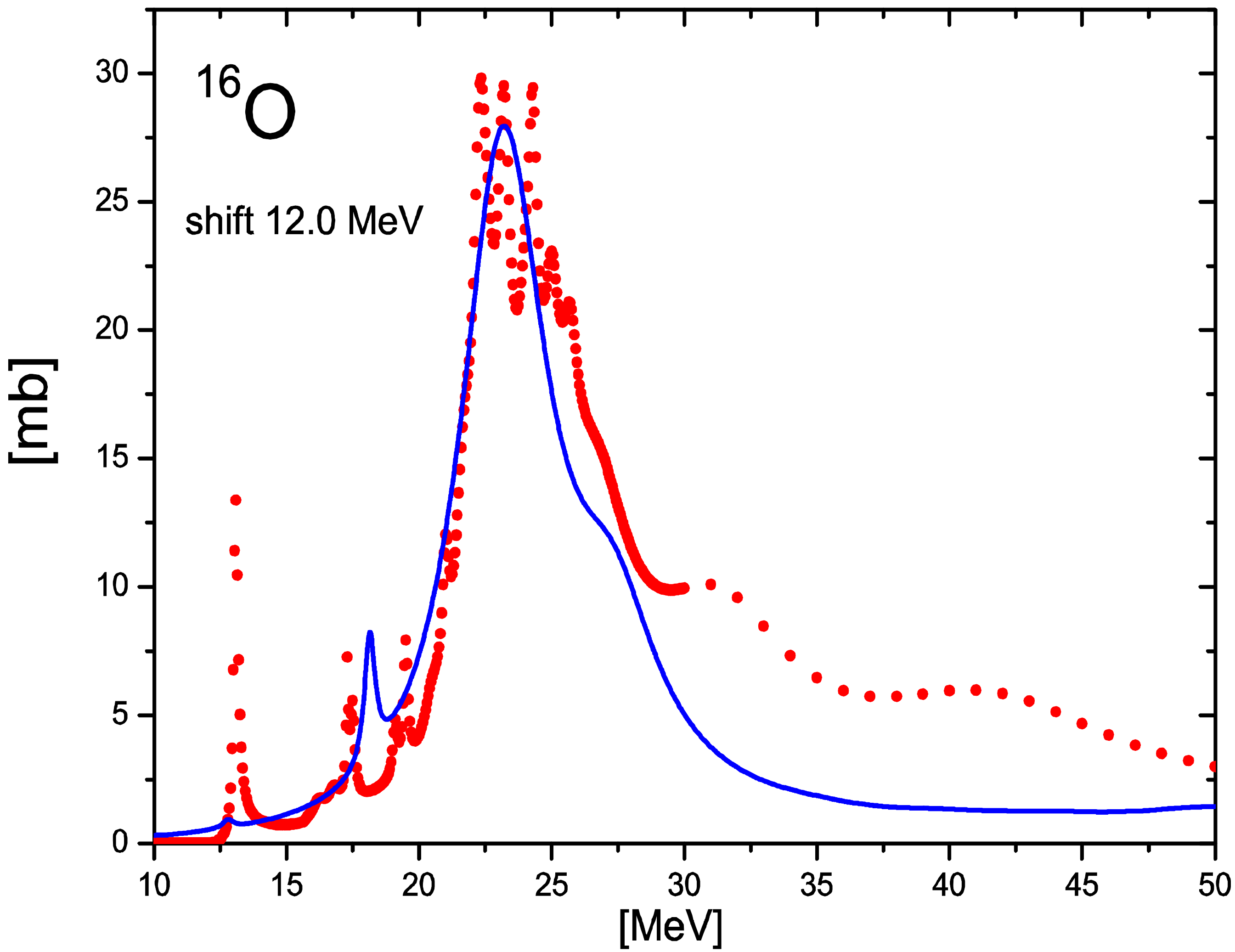



Nilsson

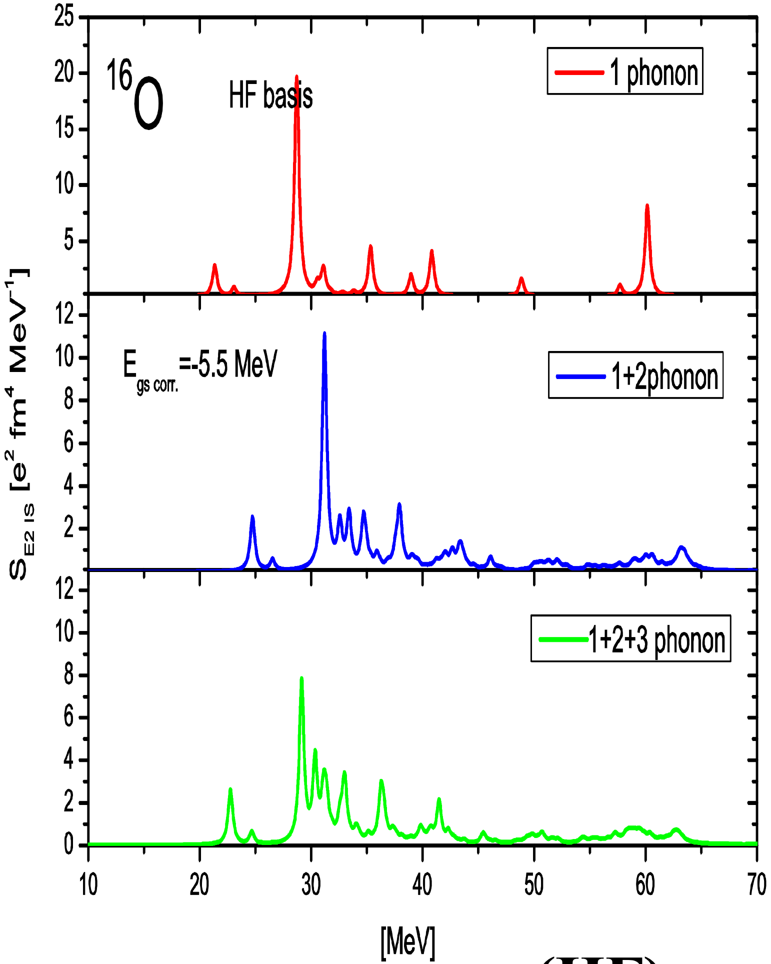
# GDR (1 $\hbar\omega$)



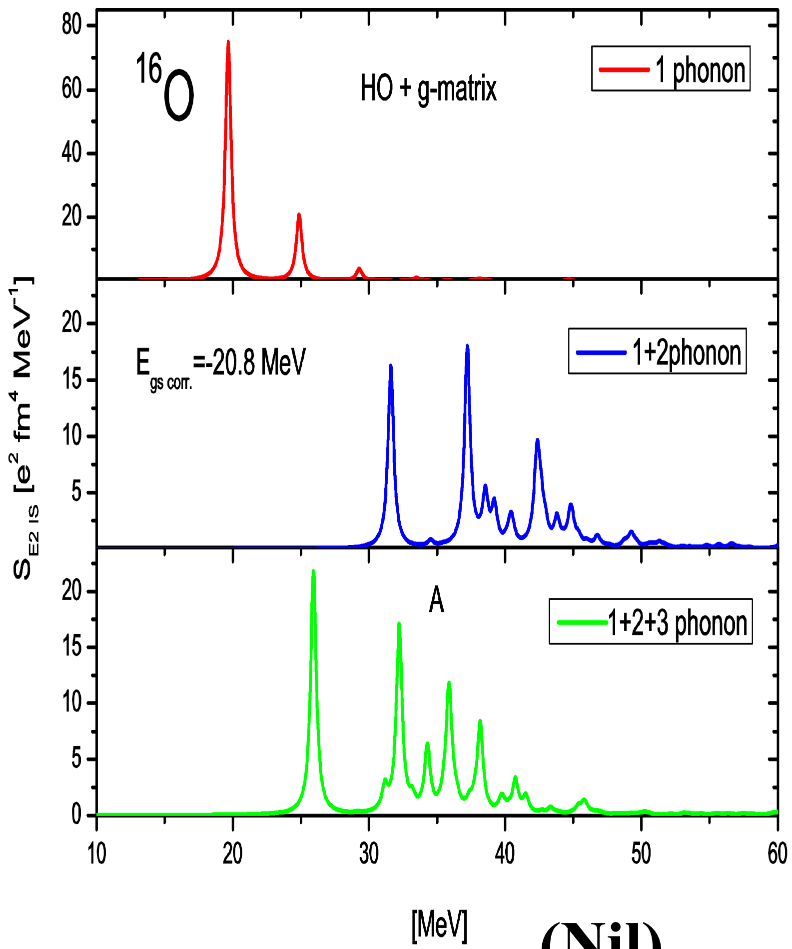
# GDR: Cross Section



# E2 Strength Function

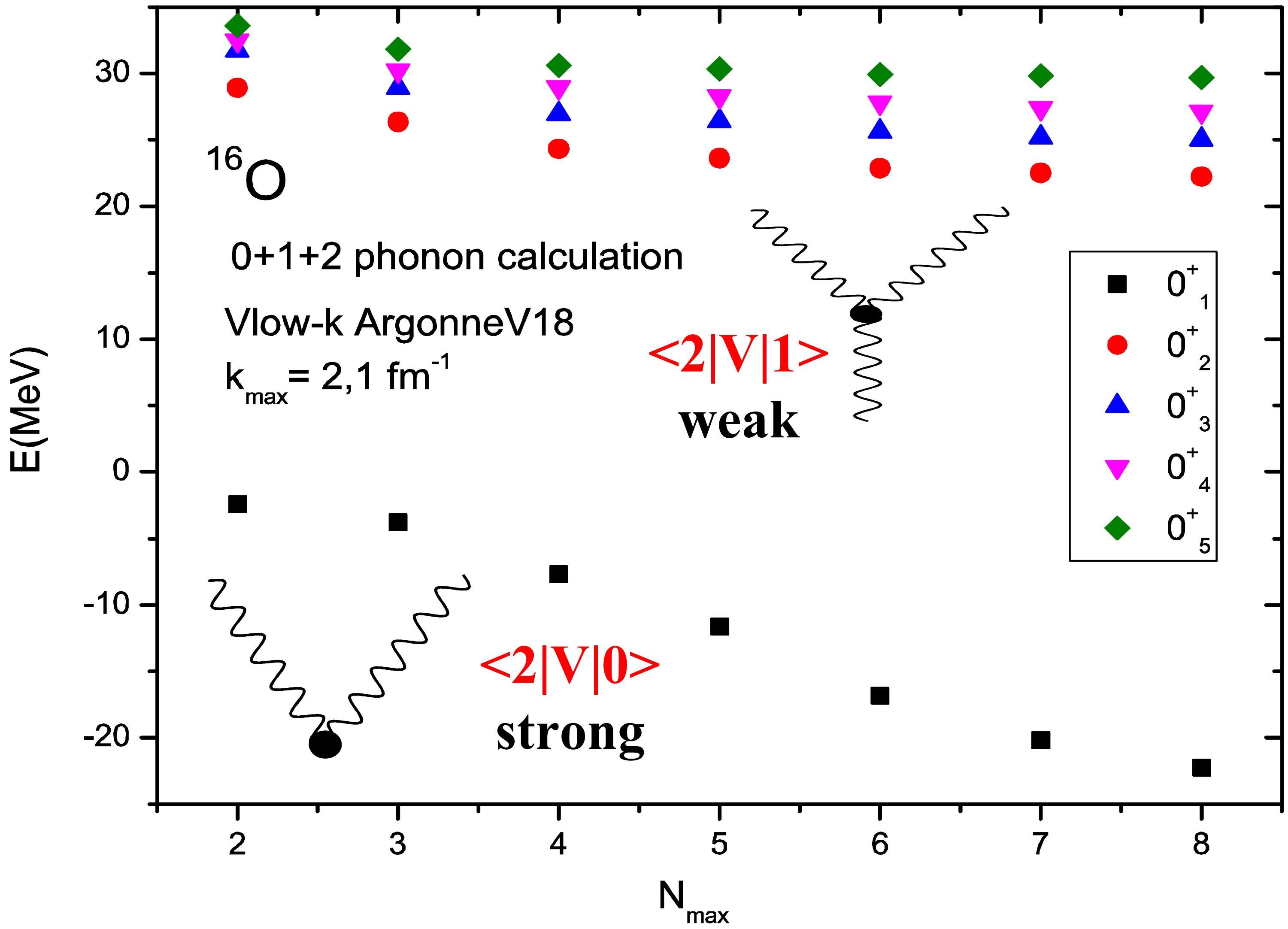



(HF)

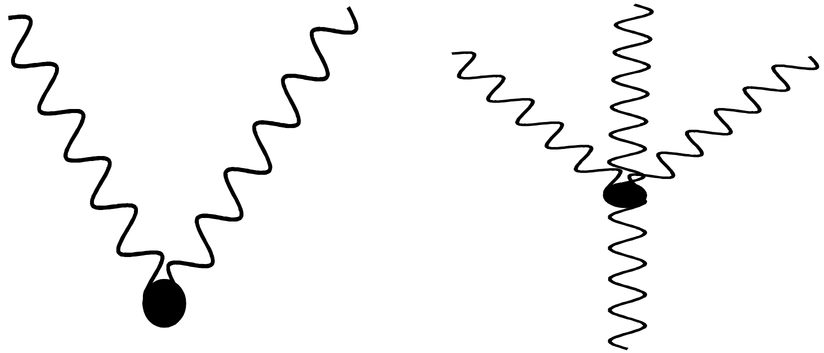



(Nil)

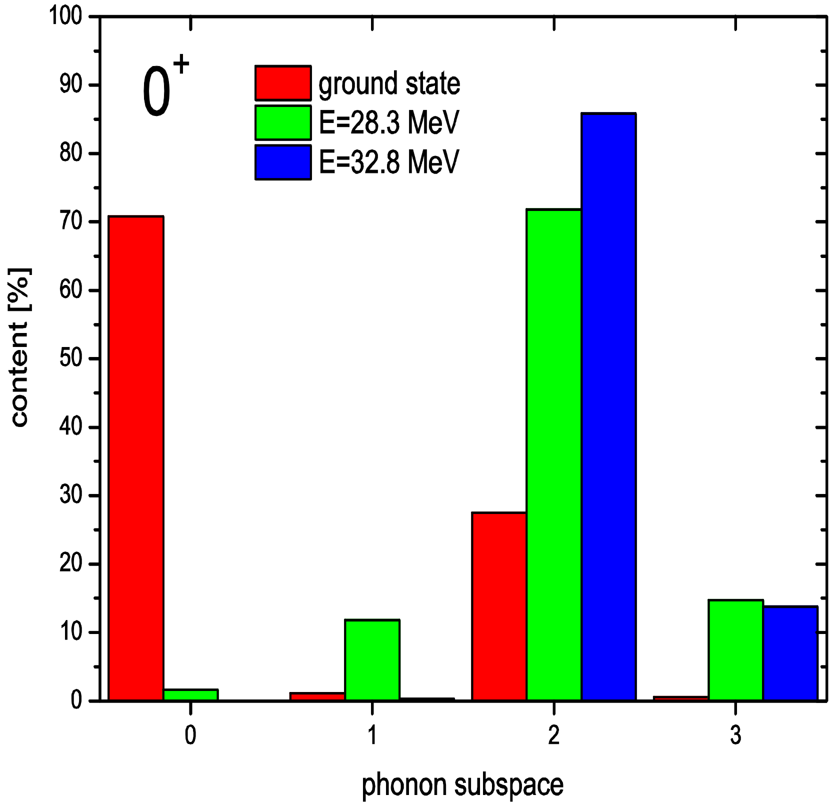
# Ground state



# 3-phonons needed but not enough

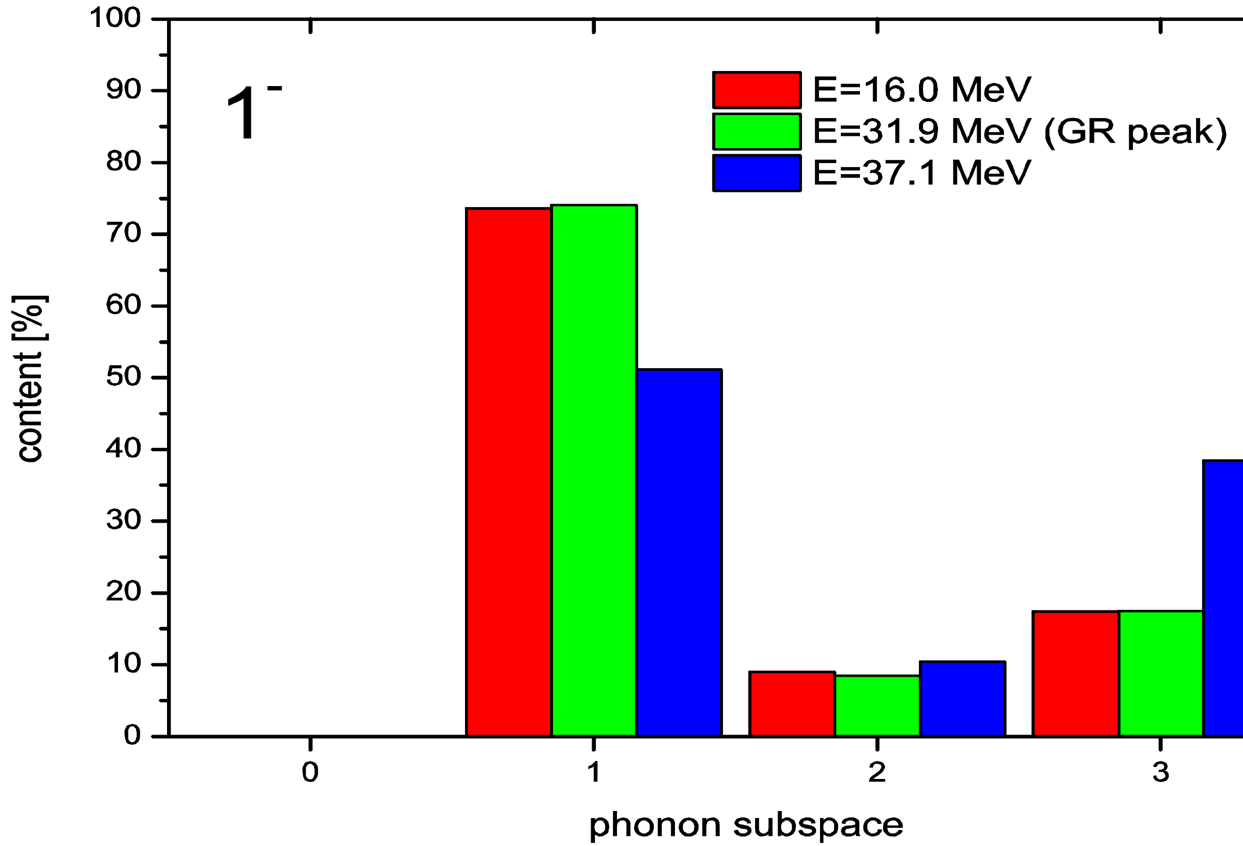


$<n|V|n-2>$
strong



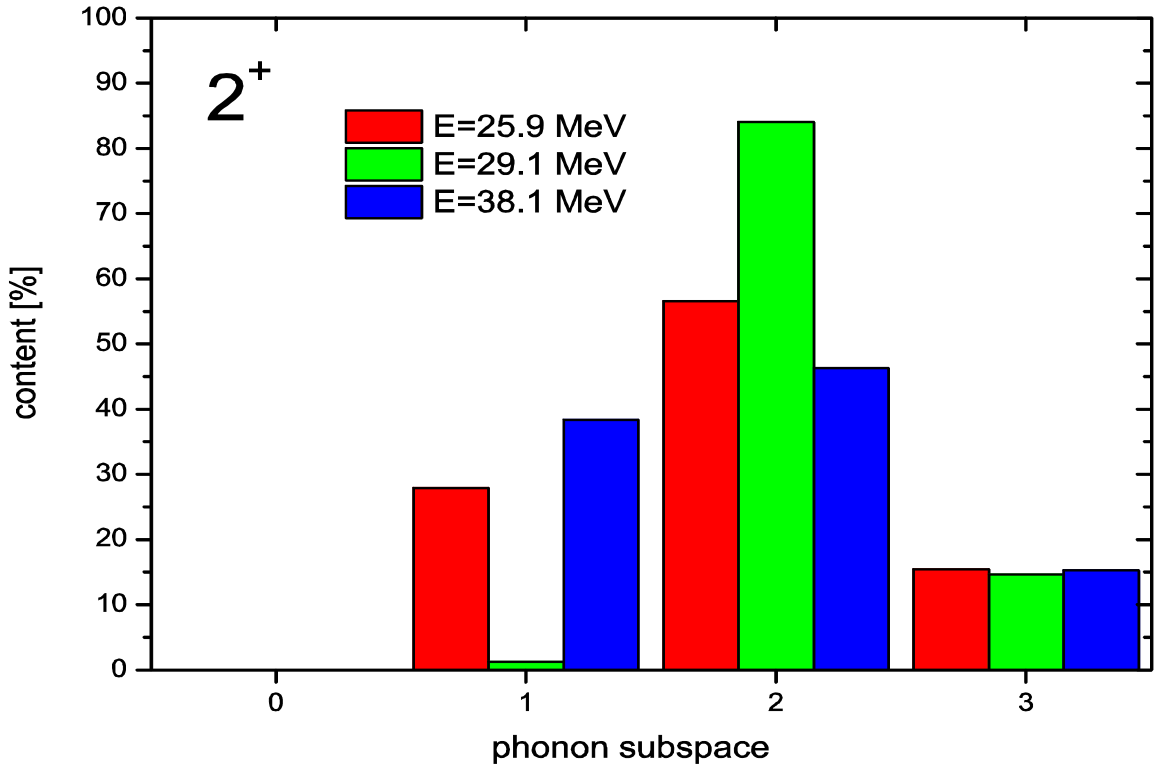

4-phonons needed

# Phonon content: $1^-$




3-phonon maybe enough but large p-h space

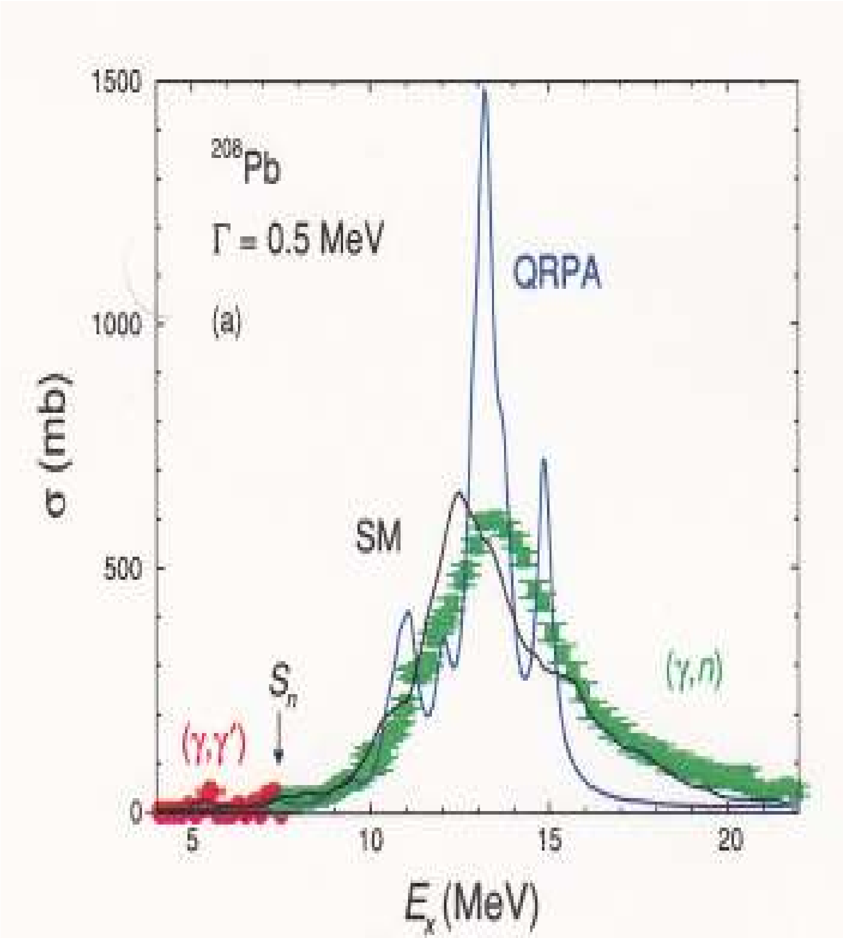
# Phonon content: **2**$^+$




**4-phonons** needed also for $2^+$

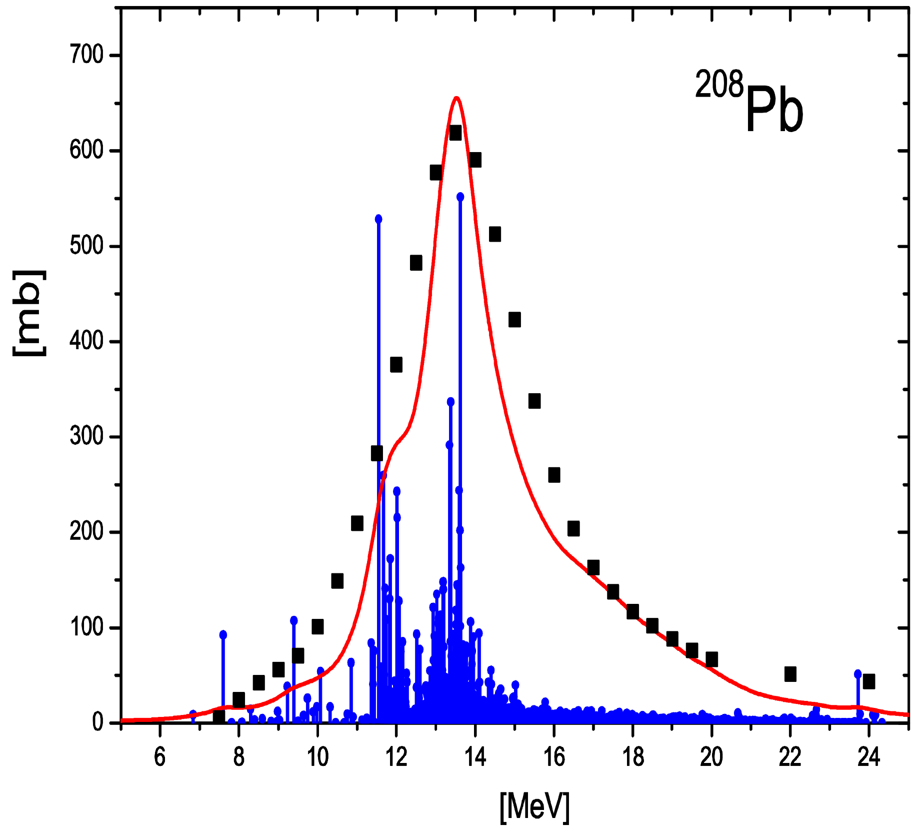
# Application to **heavy** nuclei (**easy** up to **two-phonons**)

## GDR in $^{208}$Pb



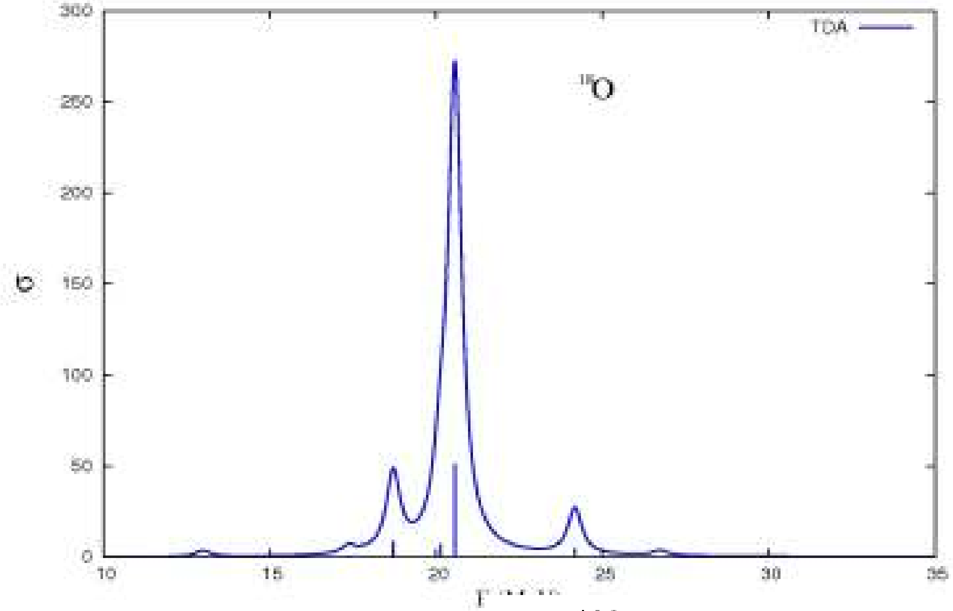

SM

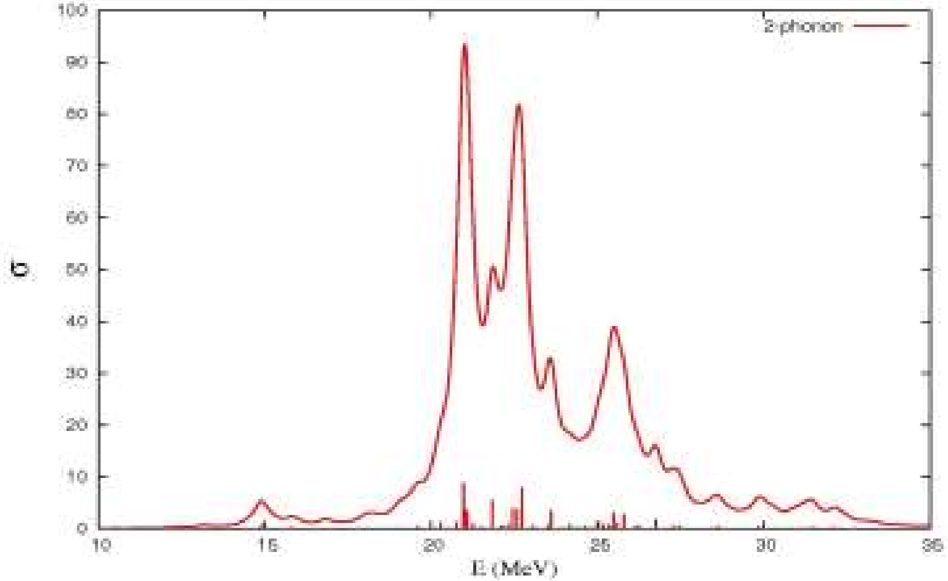
(R. Schwengner..A. Brown PRC81(2010))




EMPM

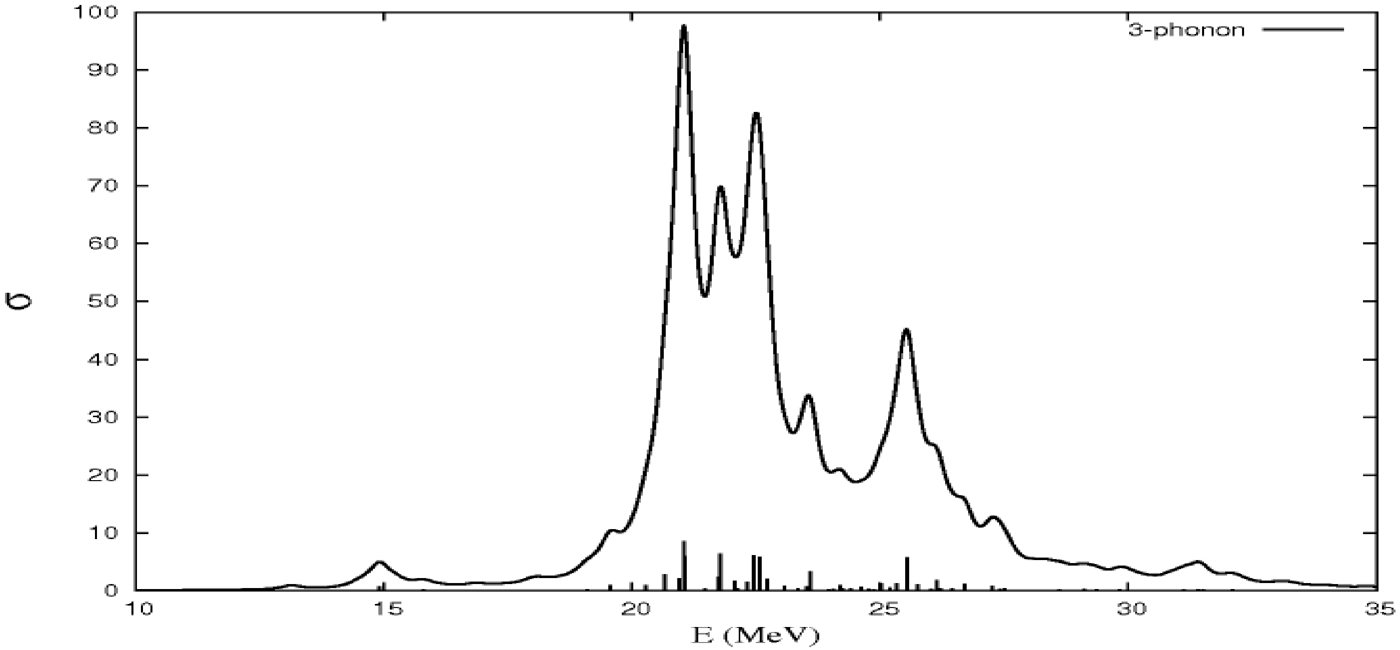
# **Open** shell nuclei: $^{18}$O



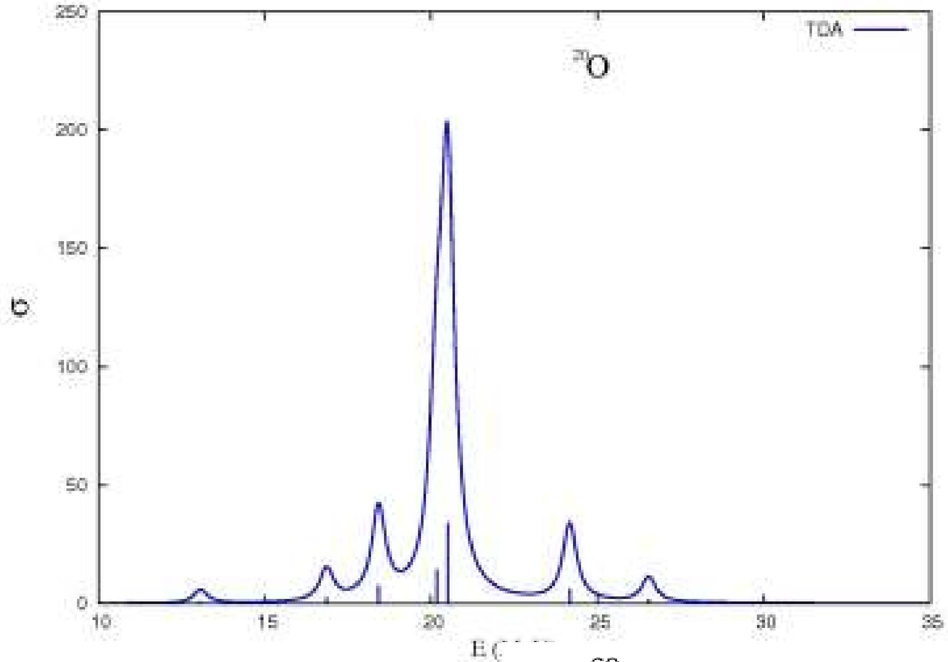



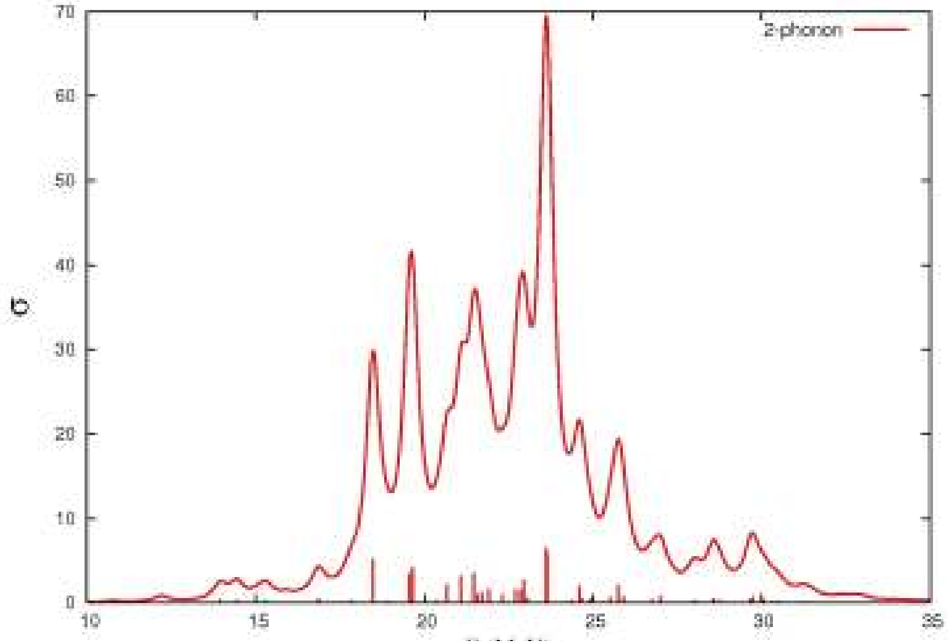

**E1**

cross section



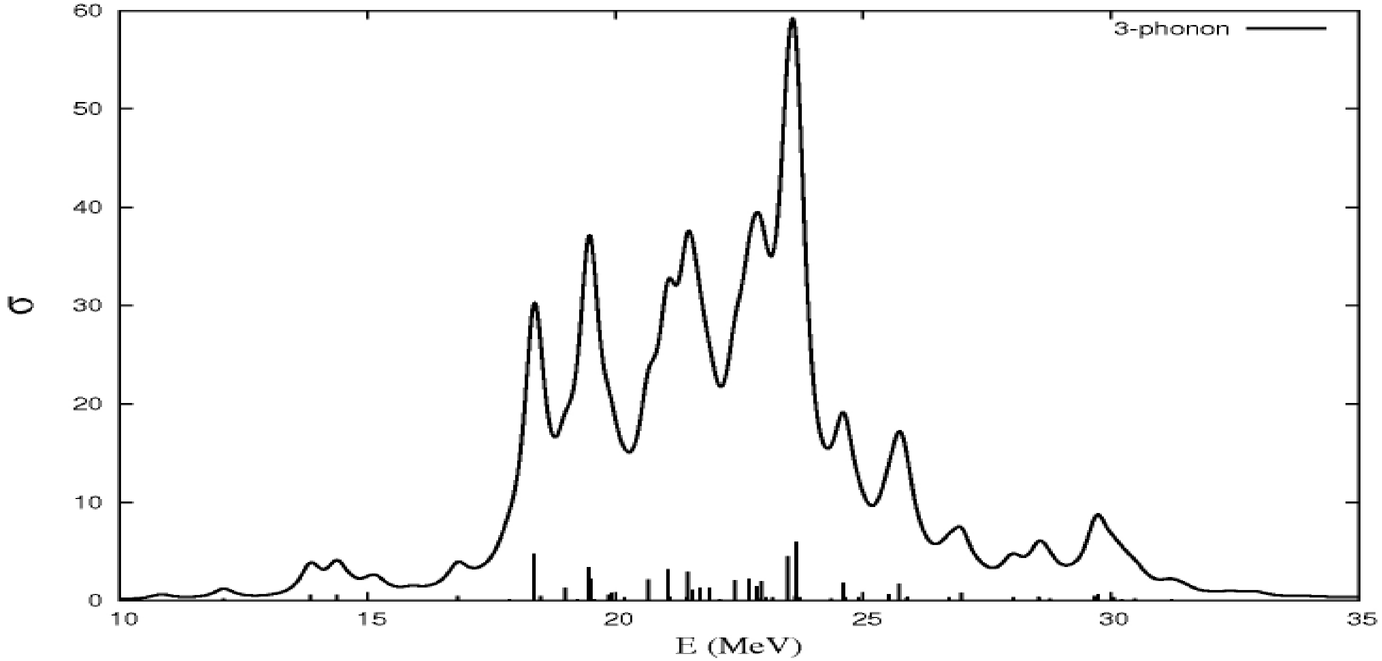
# **Open** shell nuclei: $^{20}$O



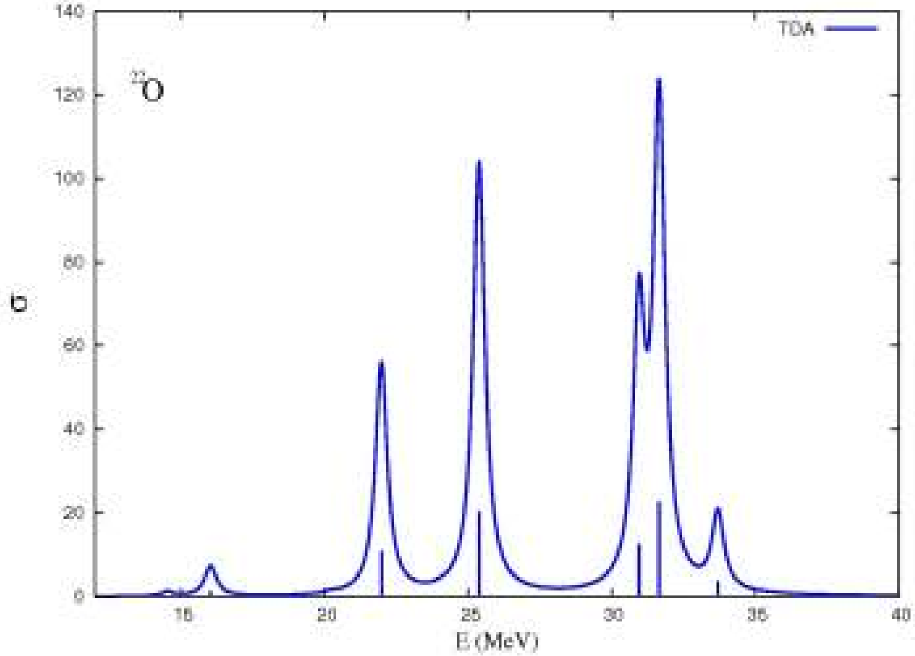



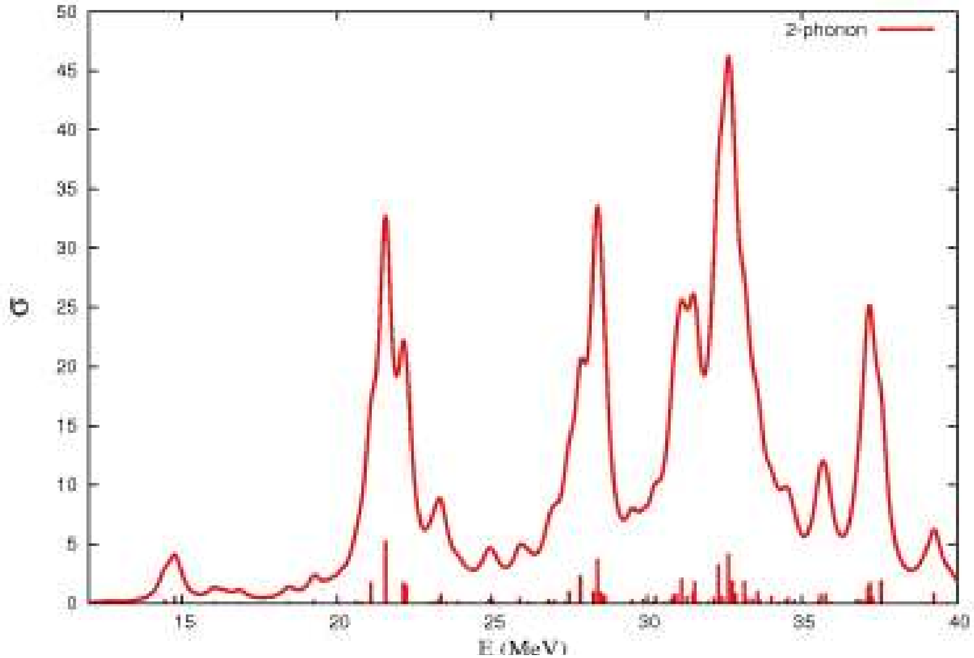

**E1**

cross section

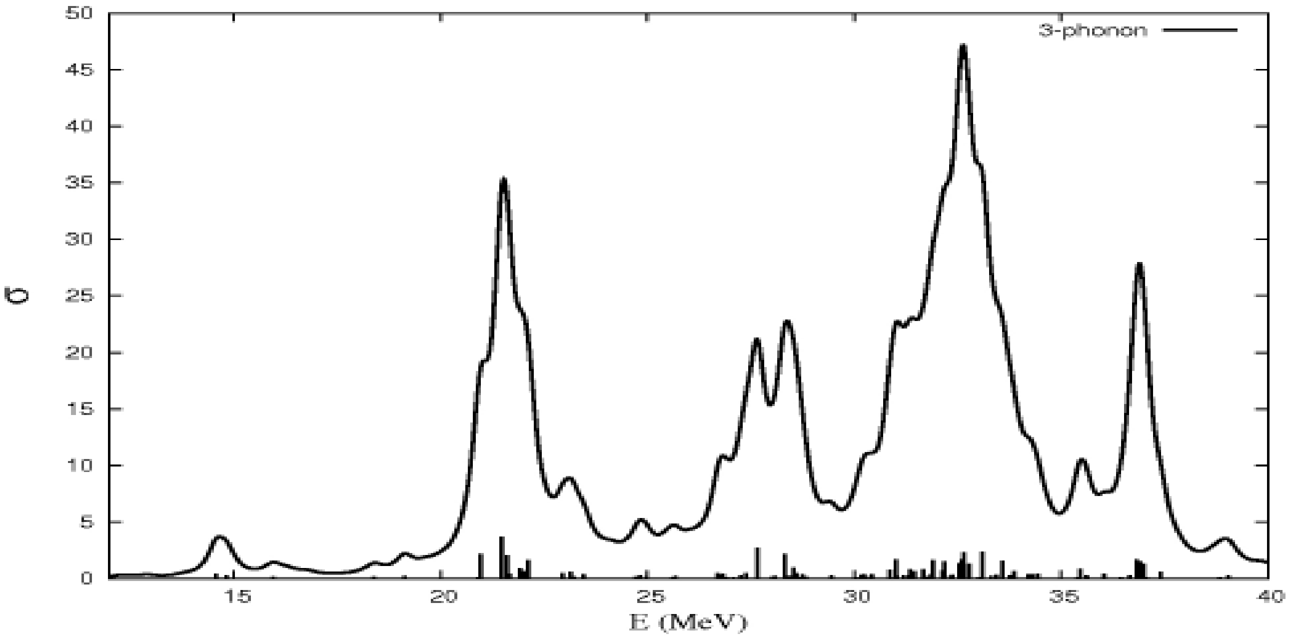


# **Open** shell nuclei: $^{22}O$

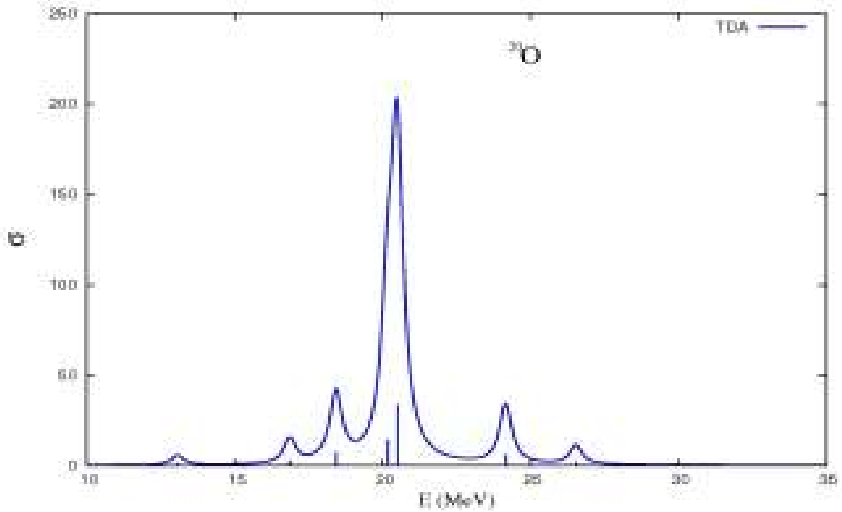


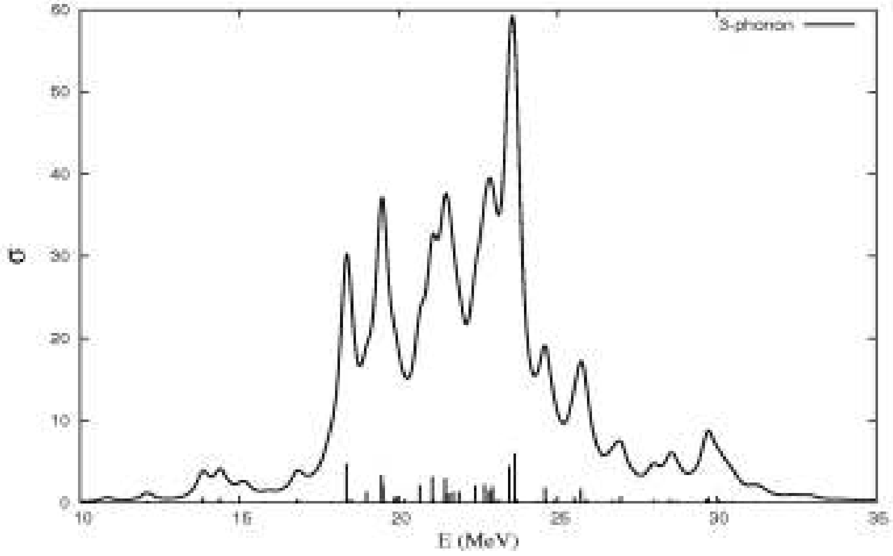



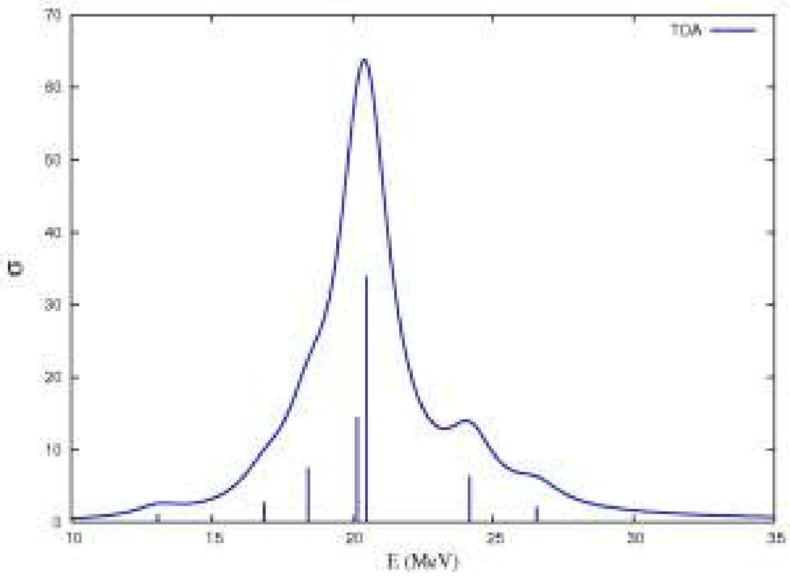

**E1**
cross section

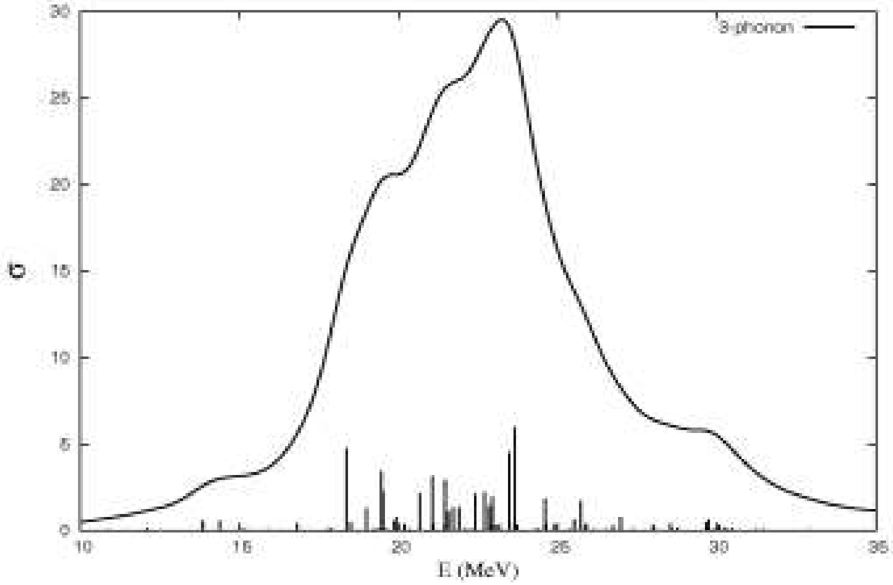


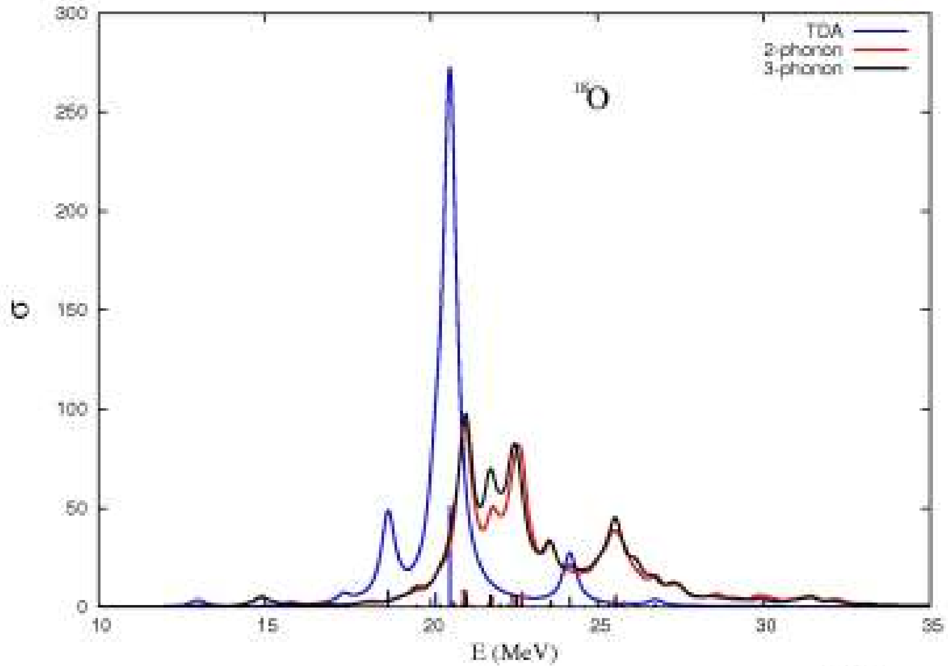
# **Open** shell nuclei: $^{20}O$

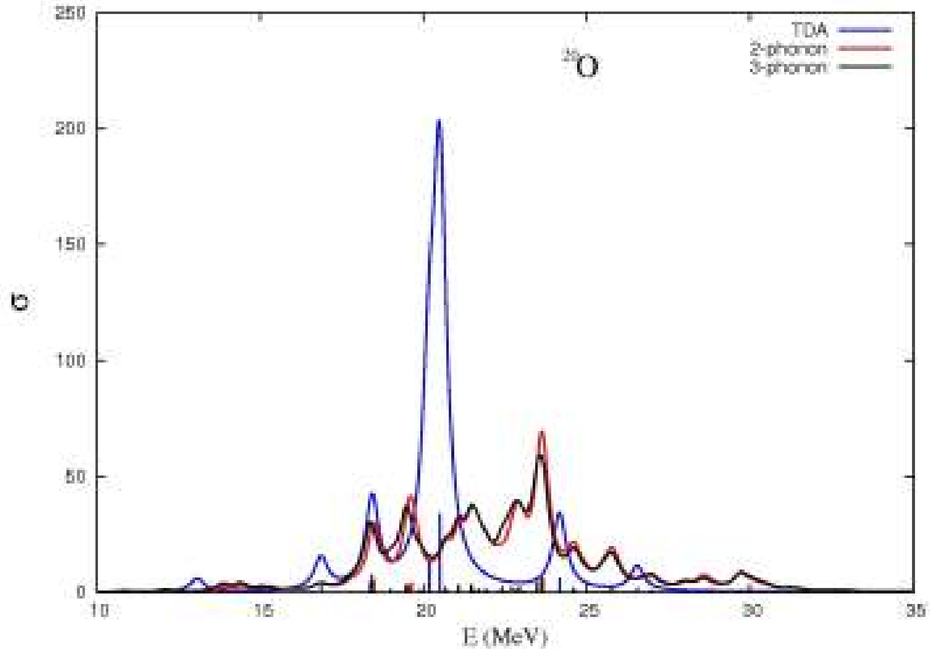


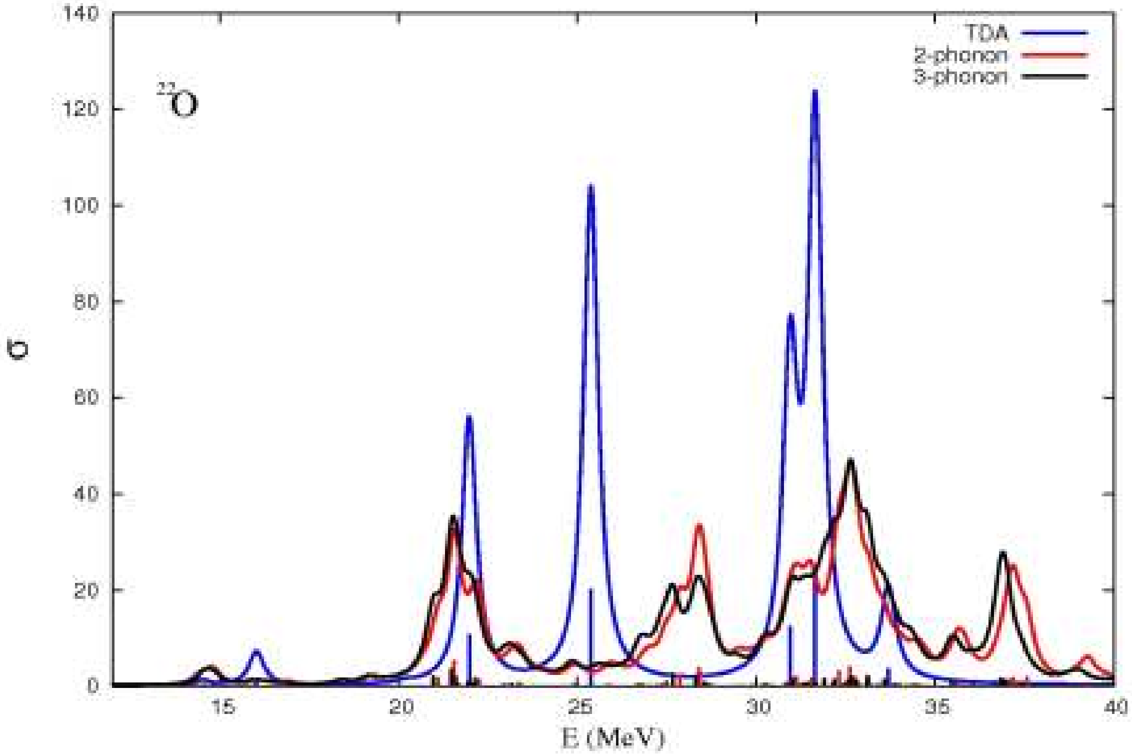



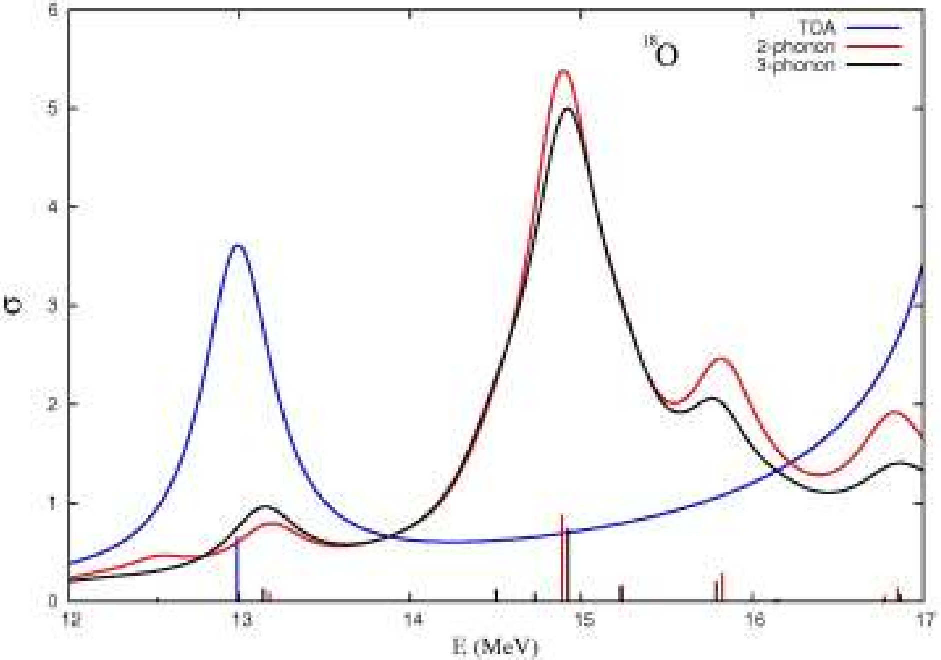





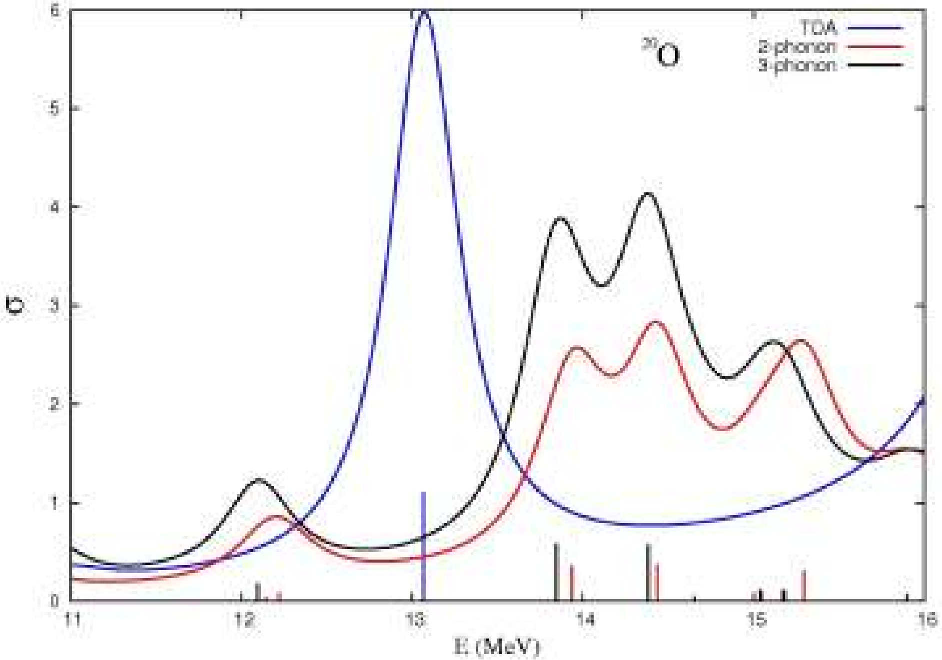



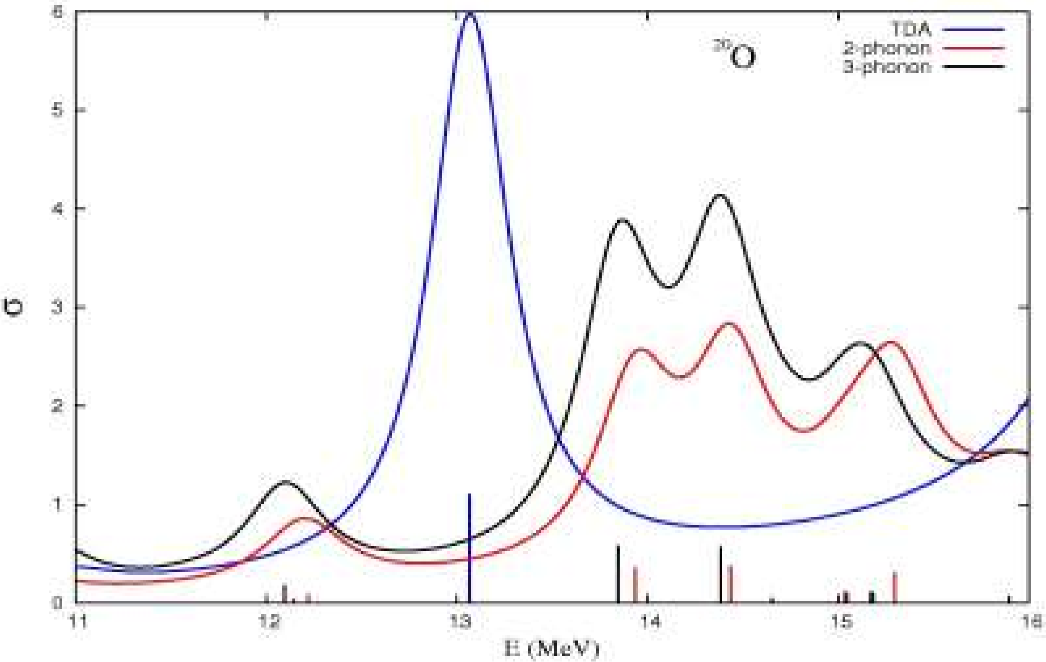

Overview
E1 cross section
in $^{18\text{-}22}$O



TDA
2-phonon
3-phonon
$^{18}O$
σ
E (MeV)

TDA
2-phonon
3-phonon
$^{20}O$
σ
E (MeV)

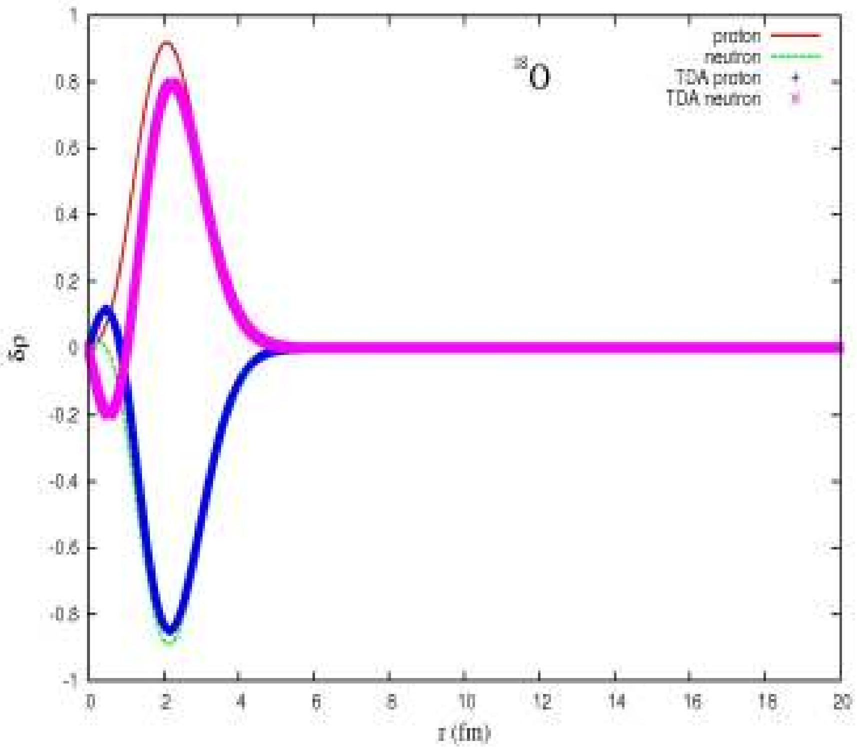



TDA
2-phonon
3-phonon
$^{20}O$
σ
E (MeV)

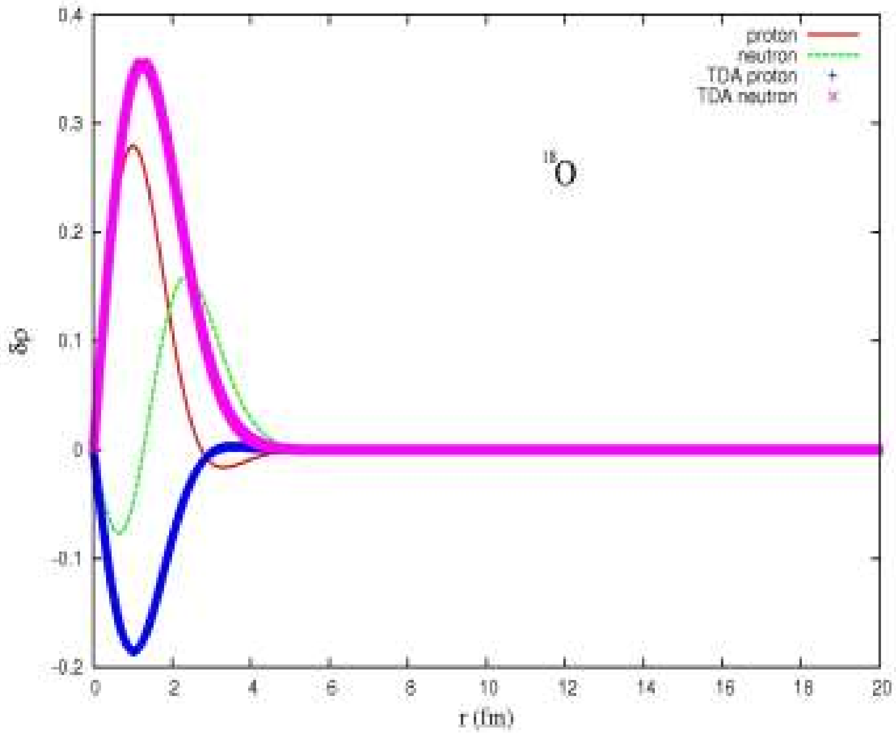

Pygmy

# E1 transition density: $^{18}O$

**GDR**

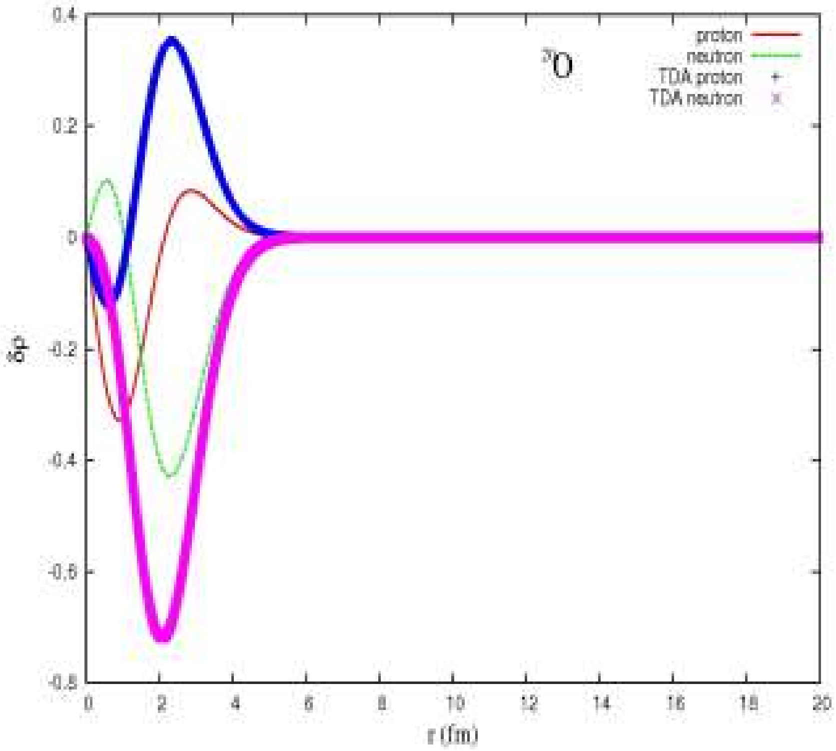



**Pygmy**

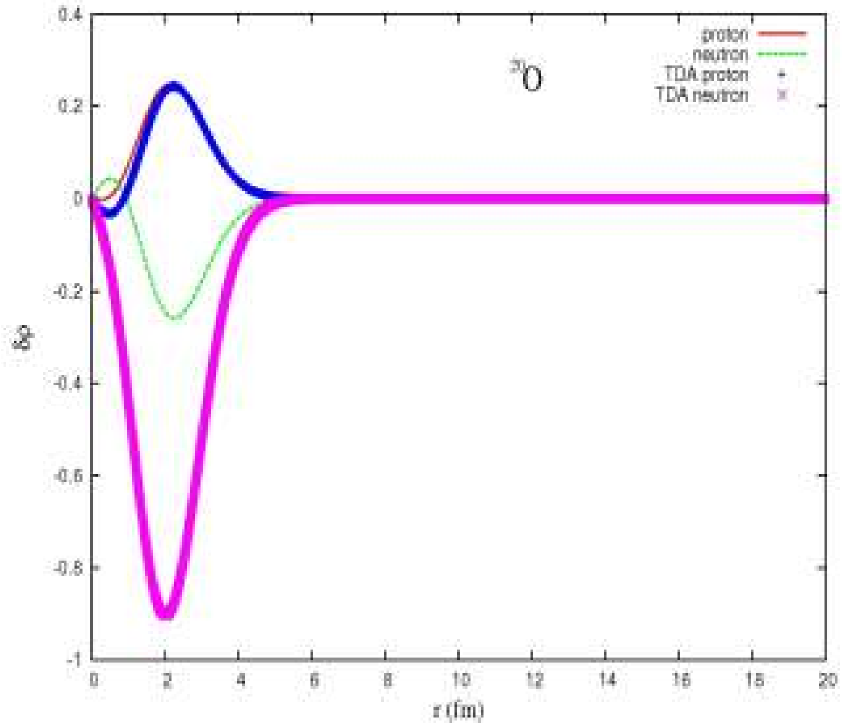


# E1 transition density: $^{20}O$

GDR

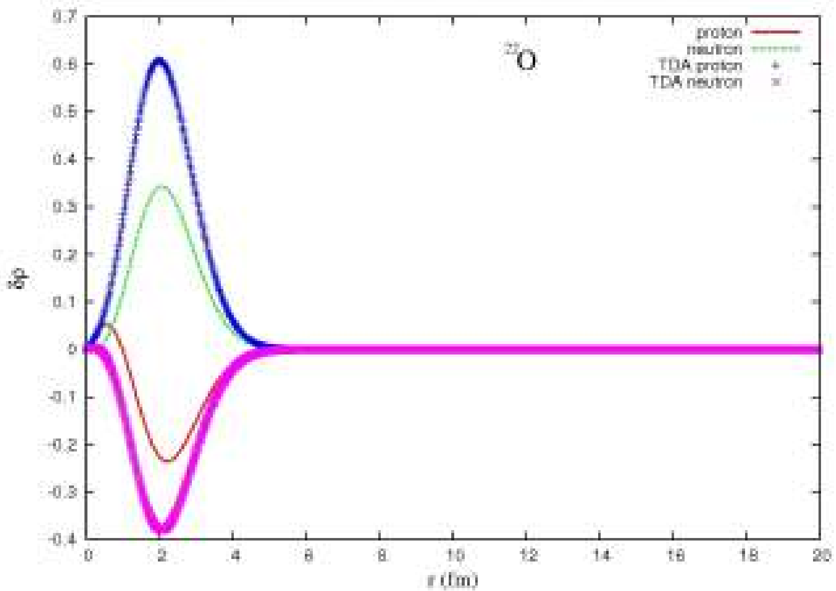



Pygmy

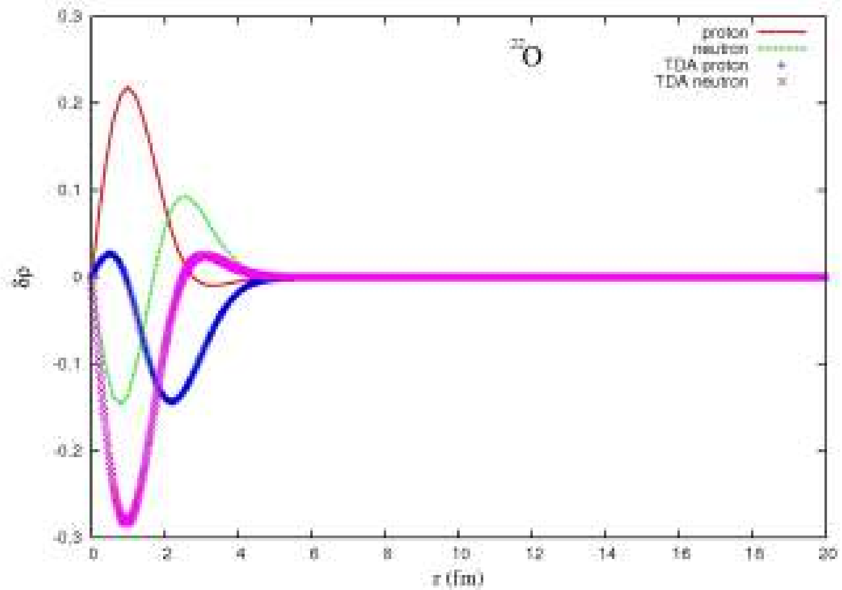


# E1 transition density: $^{22}O$

GDR




Pygmy



# Concluding remarks

- The **EMPM** gives an
**exact formulation** of the **Nuclear Eigenvalue Problem**
in a **microscopic phonon** basis for an **arbitrary** number **n** of **phonons**

- **Actual implementation :**

**- n arbitrarily large** in **small** configuration space
- **n=3** in **fairly large configuation** space (but **n=4** is in progress)
- **n=2** in very large configuration space

**Final goal**:
generate **TDA phonons** in **very large** configuration **space**
and **select** only the **relevant** ones so as to allow a

**Truncation** of the **phonon space**

# THANK YOU

# $^{16}O$ as theoretical lab

**Structure** of $^{16}$**O**: A theoretical challenge

Pioneering work: First excited $0^+$ as deformed 4p-4h excitations

G. E. Brown, A. M. Green, Nucl. Phys. 75, 401 (1966)

(TDA) IBM (includes up to 4 TDA Bosons)

H. Feshbach and F. Iachello, Phys. Lett. B 45, 7 (1973); Ann. Phys. 84, 211 (194)

SM up to 4p-4h and 4 ħω

W.C. Haxton and C. J. Johnson, PRL 65, 1325 (1990)

E.K. Warbutton, B.A. Brown, D.J. Millener, Phys. Lett. B293,7(1992)

No-core SM (NCSM) Huge space!!!

Symplectic No-core SM (SpNCSM) a promising tool for cutting the SM space

T. Dytrych, K.D. Sviratcheva, C. Bahri, J. P. Draayer, and J.P. Vary, PRL 98, 162503 (2007)

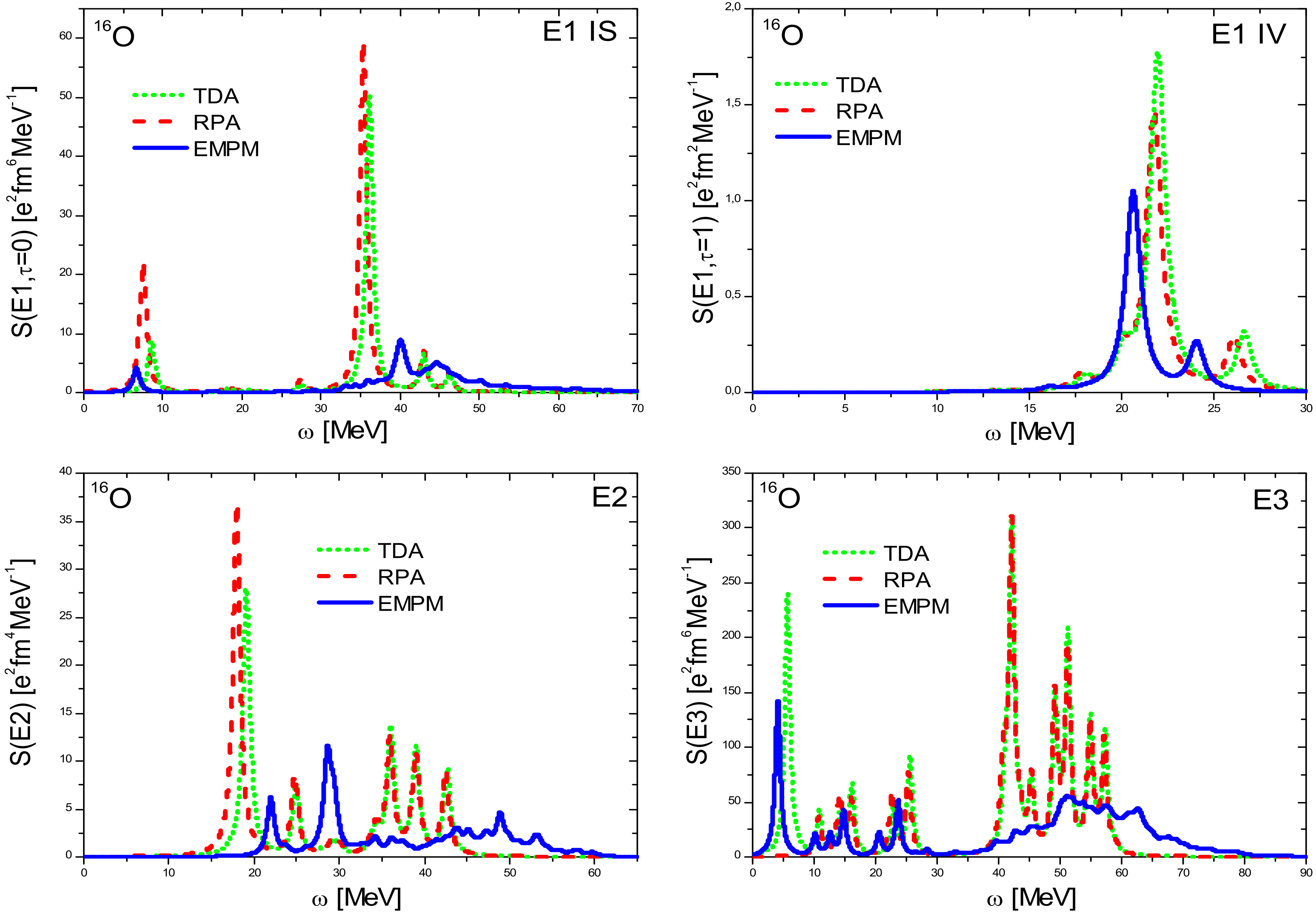
Self-consistent Green function (SCGF)

(extends RPA so as to include dressed s.p propagators and coupling to two-phonons)

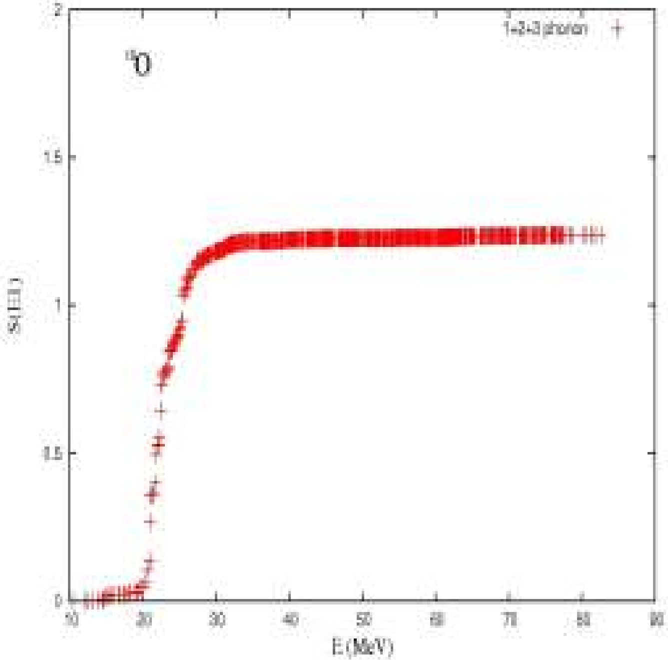
C. Barbieri and W.H. Dickhoff, PRC 68, 014311 (2003);

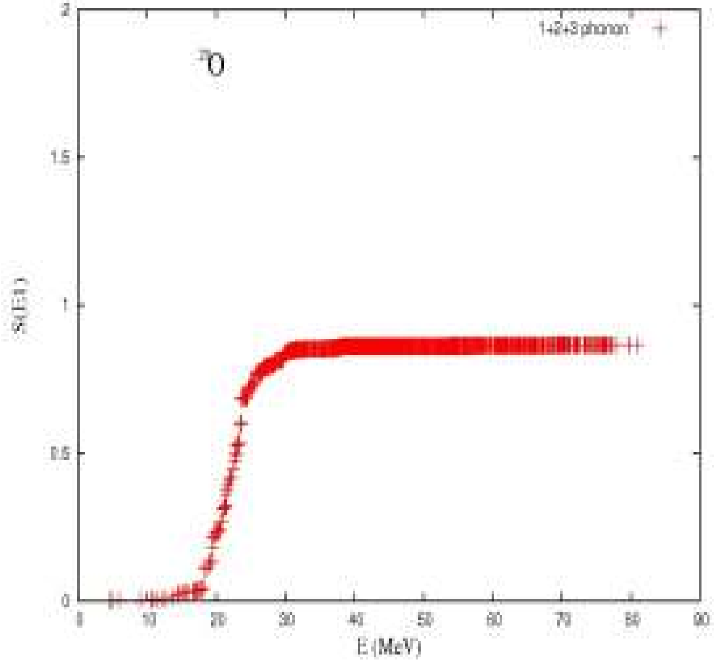
W.H. Dickhoff and C. Barbieri, Pro. Part. Nucl. Phys. 25, 377 (2004)

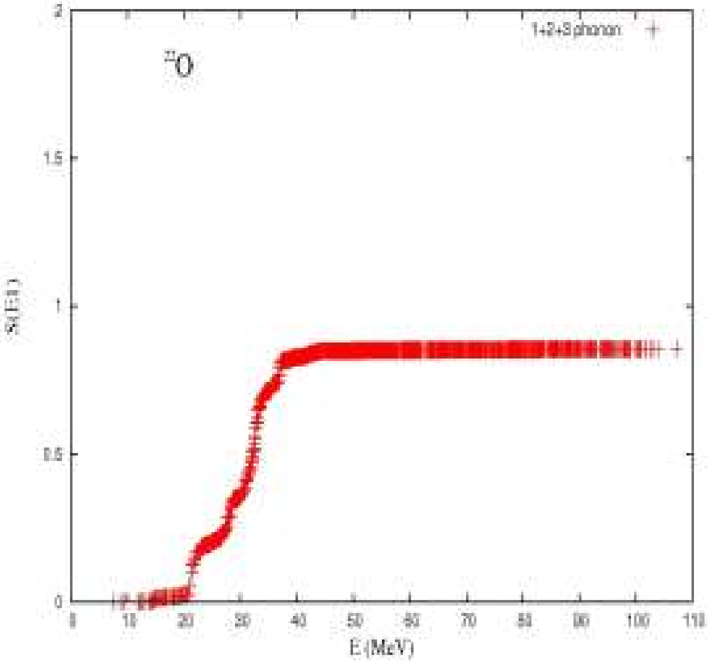
# Eλ response



16O
E1 IS
TDA
RPA
EMPM
S(E1,τ=0) [e2fm6MeV-1]
ω [MeV]
16O
E1 IV
TDA
RPA
EMPM
S(E1,τ=1) [e2fm2MeV-1]
ω [MeV]
16O
E2
TDA
RPA
EMPM
S(E2) [e2fm4MeV-1]
ω [MeV]
16O
E3
TDA
RPA
EMPM
S(E3) [e2fm6MeV-1]
ω [MeV]


1+2+3 phonon
O
S(E1)
E (MeV)


1+2+3 phonon
O
S(E1)
E (MeV)


1+2+3 phonon
O
S(E1)
E (MeV)

# Running Sum