ν We Never Knew You

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- Background.
- Examples of disagreements with theory.
  Every case examined disagrees.
- Discussion of proposed explanations.
- Future prospects.

Neutron-Width Data Disagree with the Porter-Thomas Distribution (ν=1).

In the beginning…

(of neutron resonance spectroscopy, circa 1950)

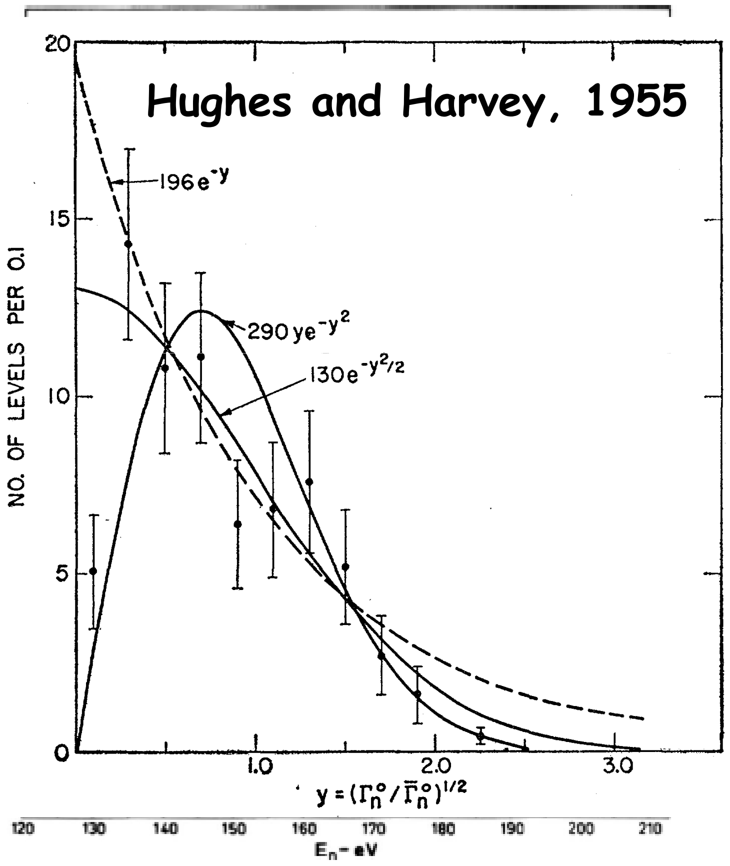
- Many narrow resonances observed.

  $\Gamma \ll D$ (long lived) ⇨ Compound nucleus model.

- Wide distribution of $\Gamma_n^0 = \Gamma_n / \sqrt{E_n}$.

  Early data agreed best with exponential distribution.

  Also theories from Porter and Thomas, and Bethe.


Hughes and Harvey, 1955

Consensus View from Last ~50 Years:

Reduced Neutron Widths (Γ_n^0) Follow a Porter-Thomas Distribution (PTD)

- PTD (1956) derived from 3 fundamental assumptions:

1) Time-reversal invariance holds ($\gamma_{\lambda c}$ real).

2) Single channel (elastic scattering) for neutrons.

3) Widths are "statistical".

Compound nucleus model, central-limit theorem ⇨

$\gamma_{\lambda c}$ Gaussian distributed with zero mean ⇨

Γ_n^0 follow a χ^2 distribution with one degree of freedom ($\nu = 1$).

Random Matrix Theory for the Gaussian Orthogonal Ensemble (RMT for the GOE)

- More formal footing.

- Broader predictions.

  Eigenvalue (spacings) as well as eigenvector (widths) fluctuations.

- Links to diverse systems and other fields.

  Atomic physics.

  Microwave billiards.

  Quantum chaos.

- Neutron resonance data routinely cited as proof of RMT.

All Neutron Data Disagree with RMT for the GOE

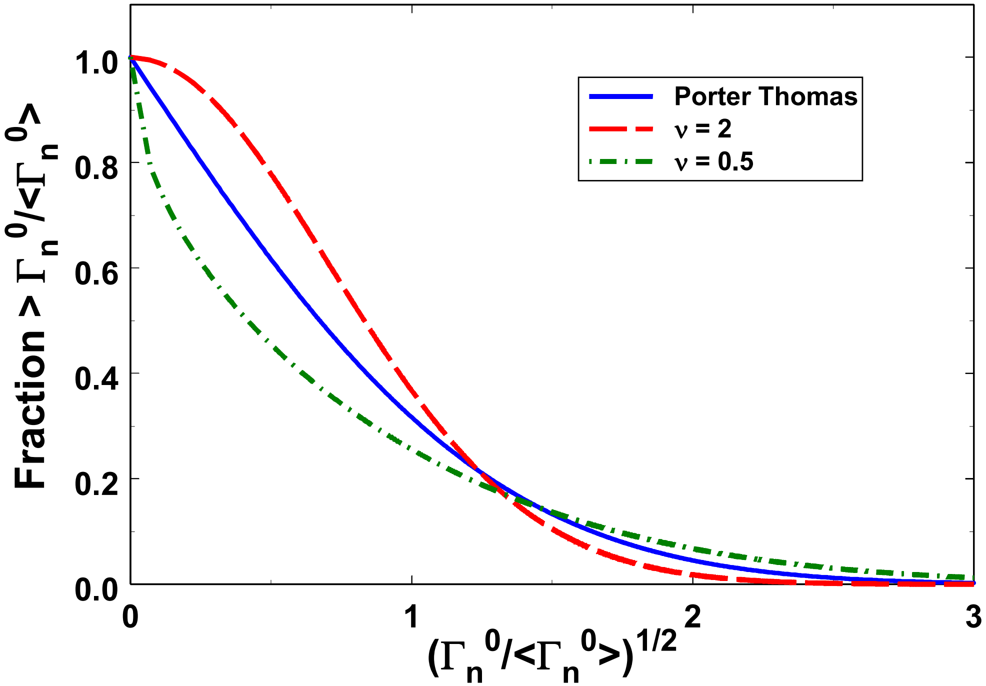
- Systematic problems with experiments:

  Small resonances missed (ν too large).

  p-wave contamination of *s*-wave data set (ν too small).

- When these problems are properly accounted for, is any remaining difference significant?

  Limited amount of data spread over a wide distribution.



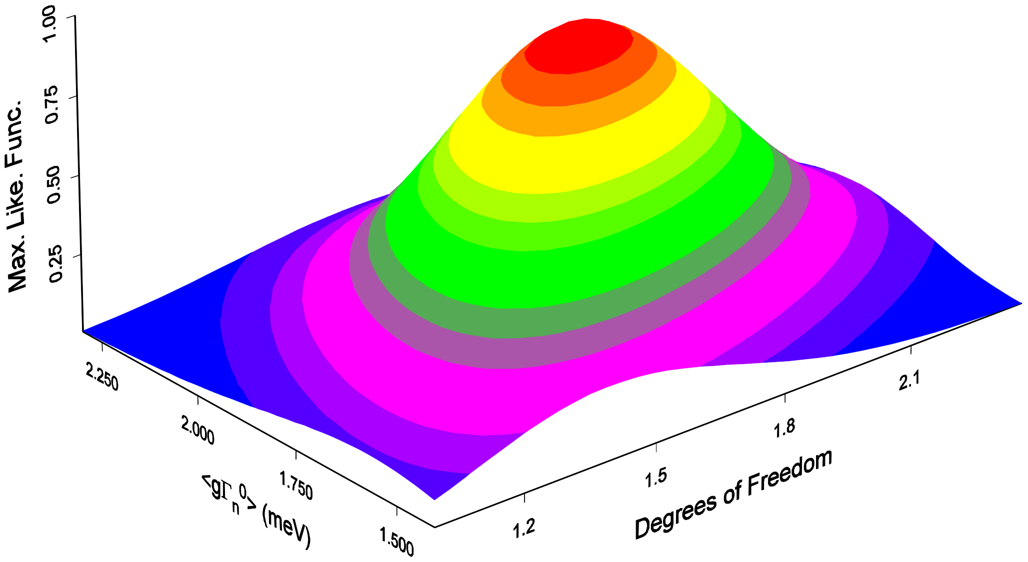
$$\boxed{\Gamma_n^0=\Gamma_n/\sqrt{E_n}}$$

Typical Test of the PTD

(“Lies, dammed lies, and statistics”)

- Assume Γ_n^0 are χ^2 distributed.
- Use maximum-likelihood (ML) technique to estimate ν and $<\Gamma_n^0>$.

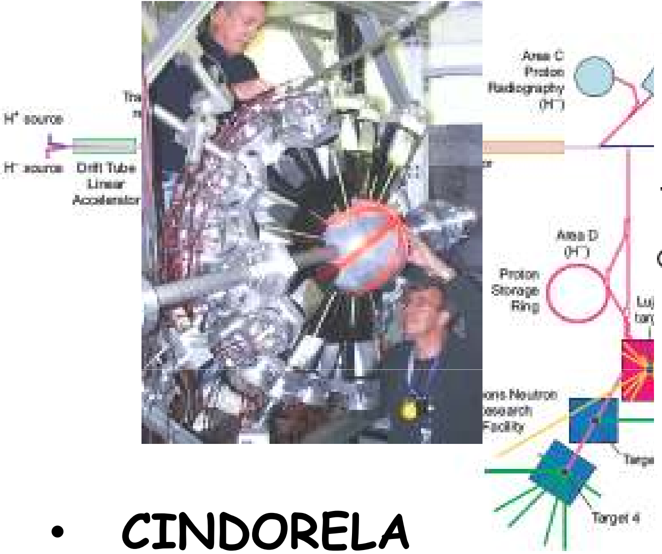
PTD has $\nu=1$.



LANSCE and ORELA White Neutron Sources
E_n via Time of Flight

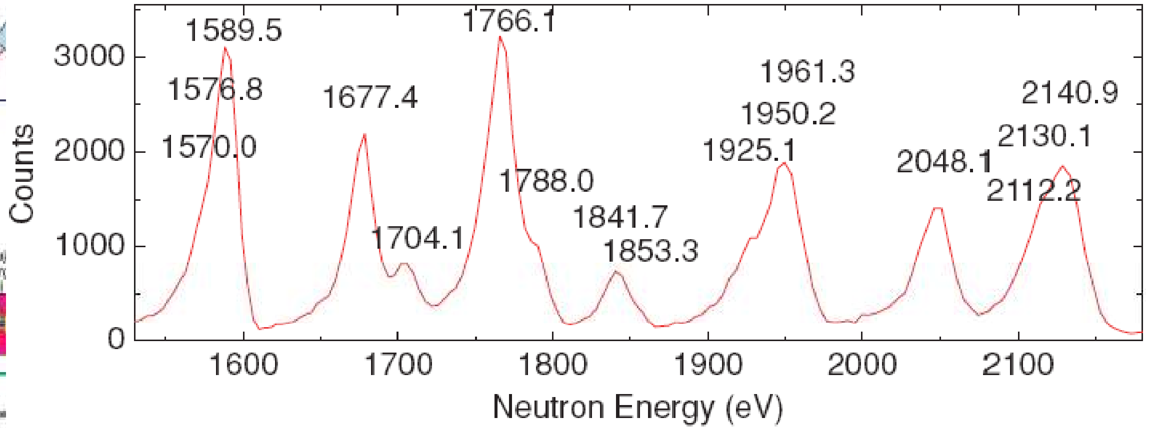
- DANCE @ LANSCE

LANSCE

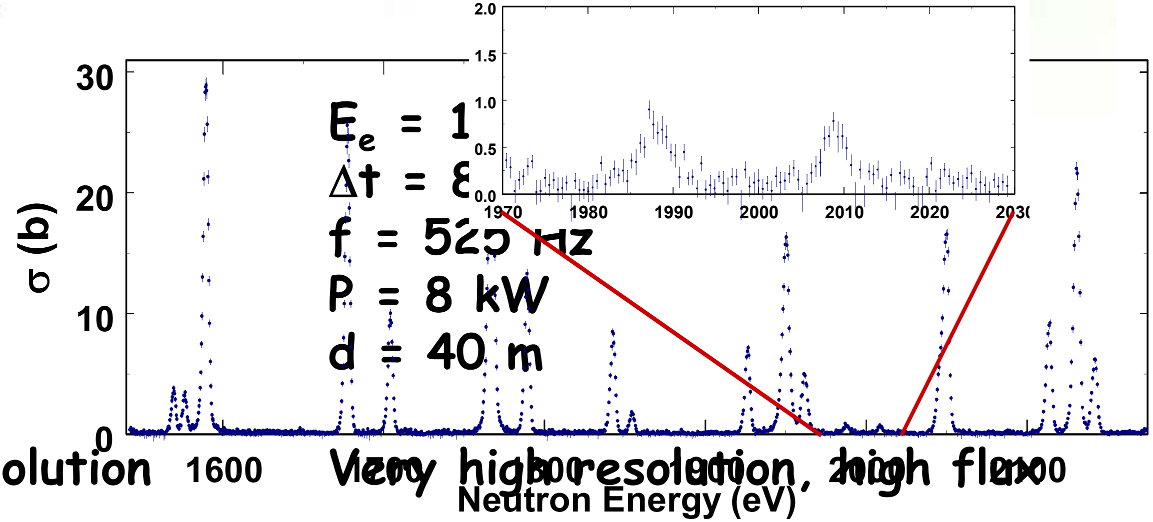


ORELA

Sheets *et al.*, Phys. Rev. C 76, 064317



- CINDORELA



E_e = 1

Δt = 8

f = 525 Hz

P = 8 kW

d = 40 m

Very high flux, poor resolution

Very high resolution, high flux

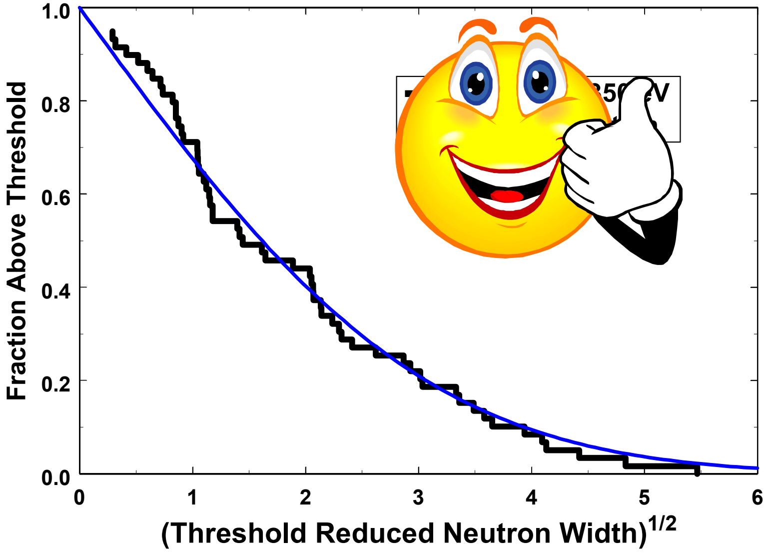
Data From DANCE:
Change from agreeing to disagreeing with PTD.

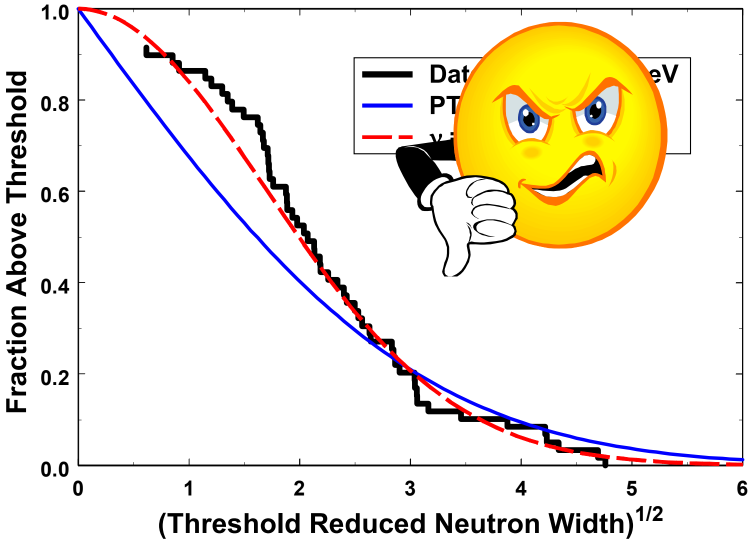
Koehler *et al.*, Phys. Rev. C 76, 025804 (2007)

^{147}Sm neutron resonances

1st 60 resonances: PTD OK.

2nd 60 resonances: PTD no good.





$\nu = 0.91 \pm 0.32$

$\nu = 3.19 \pm 0.83$

Previous Reports of Deviations from the PTD

- ^{232}Th+n.

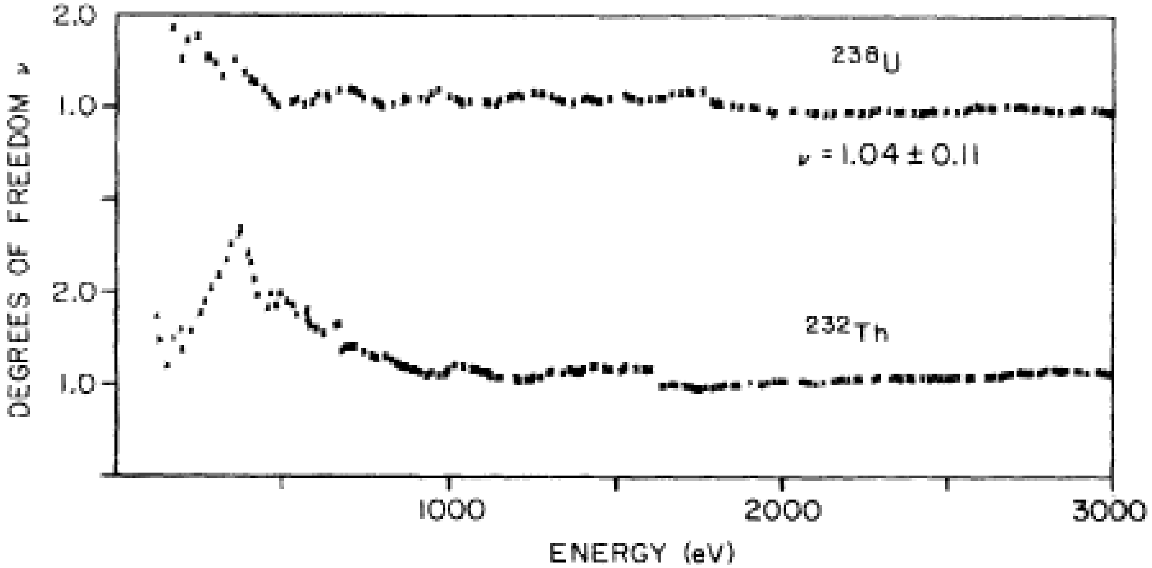
  ν=3.8±1.3, 1st 25 res.
  ν=0.83±0.68, 2nd 25 res.

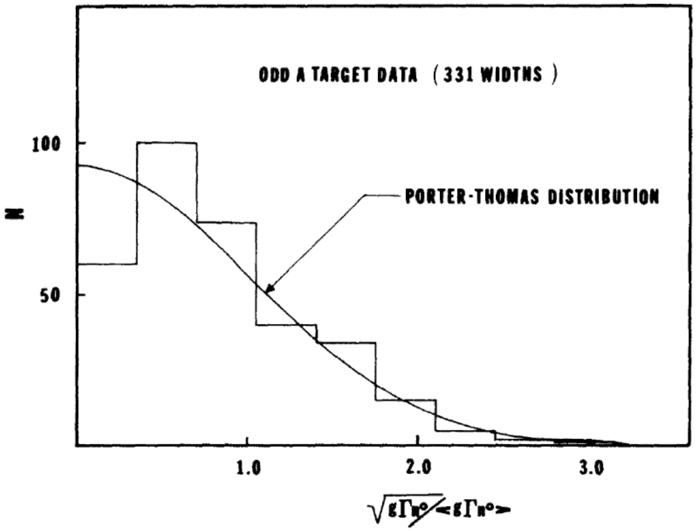
  Ribon, 1969.
  Forman *et al.*, 1971.
  Forman *et al.*, 1971.
  Rahn *et al.*, 1972.

- Five odd-A nuclides: ^{151}Sm, ^{163}Dy, ^{167}Er, ^{175}Lu, and ^{177}Hf.

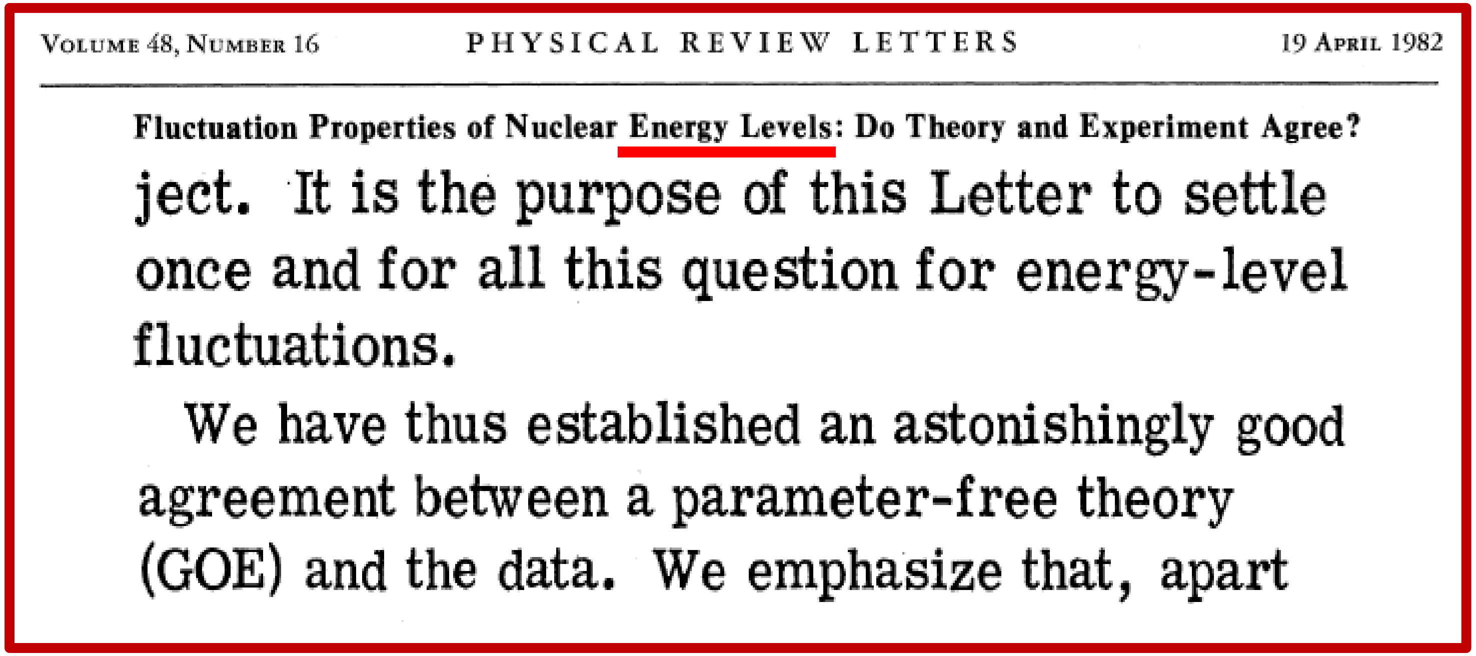
  "Striking" disagreement with the PTD even though Δ_3 agrees well with the GOE.

  Camarda, 1976.





Reduced Neutron Widths In the Nuclear Data Ensemble: Experiment and Theory Do Not Agree



VOLUME 48, NUMBER 16 PHYSICAL REVIEW LETTERS 19 APRIL 1982

Fluctuation Properties of Nuclear Energy Levels: Do Theory and Experiment Agree?

ject. It is the purpose of this Letter to settle once and for all this question for energy-level fluctuations.

We have thus established an astonishingly good agreement between a parameter-free theory (GOE) and the data. We emphasize that, apart

The Impact of Haq *et al*.

- **Often cited as providing striking confirmation of random matrix theory (RMT) predictions for the Gaussian orthogonal ensemble (GOE).**

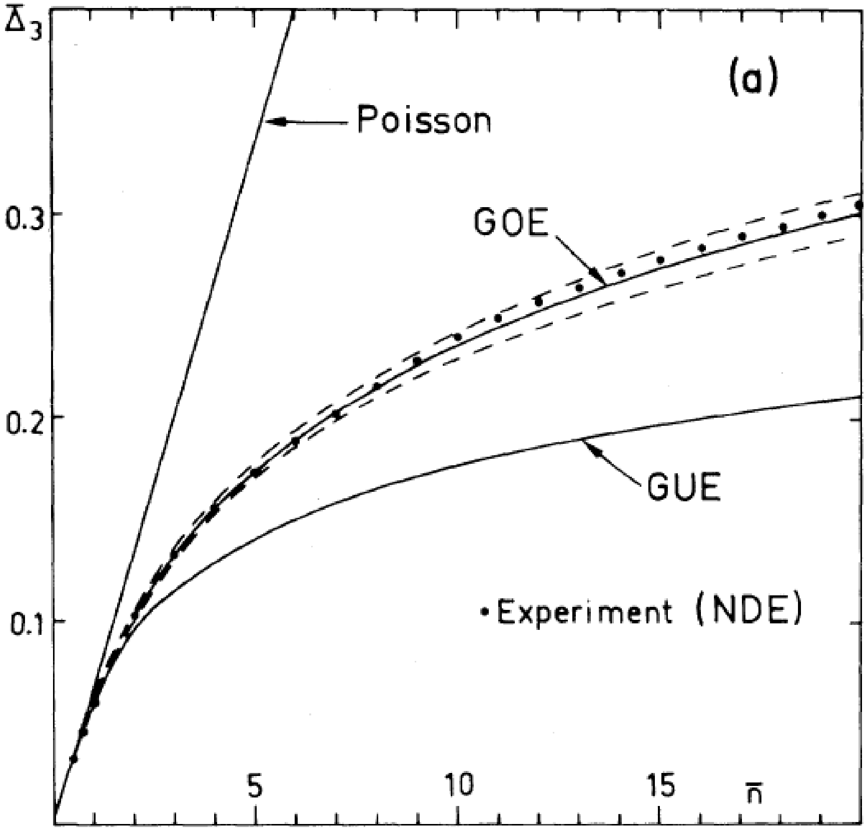
- **New neutron resonance data hardly ever used anymore to test RMT for the GOE.**
  Despite almost 40 years of improvements in detectors, neutron sources, and analysis codes, new data don't agree with GOE as well as data used by Haq *et al*.

- **Instead, theory used to correct the data for missed or misassigned resonances.**
  What is so special about data used by Haq *et al*.?
  Why are new data not as good?

Data used by Haq *et al*.: The Nuclear Data Ensemble (NDE)

- Set of 1407 energies of (mostly) neutron and proton resonances in 27 different nuclides.
  Uncertainty dominated by limited sample size.
  Slightly different NDE in O. Bohigas, R. Haq, and A. Pandy, in *Nuclear Data for Science and Technology*, p. 809 (1983).

- Found remarkably good agreement between theory and experiment for Δ_3 ensemble average (a weighted average of averages).



Advantages of Using Widths Rather Than Spacings

- Experimental effects easily incorporated into analysis.

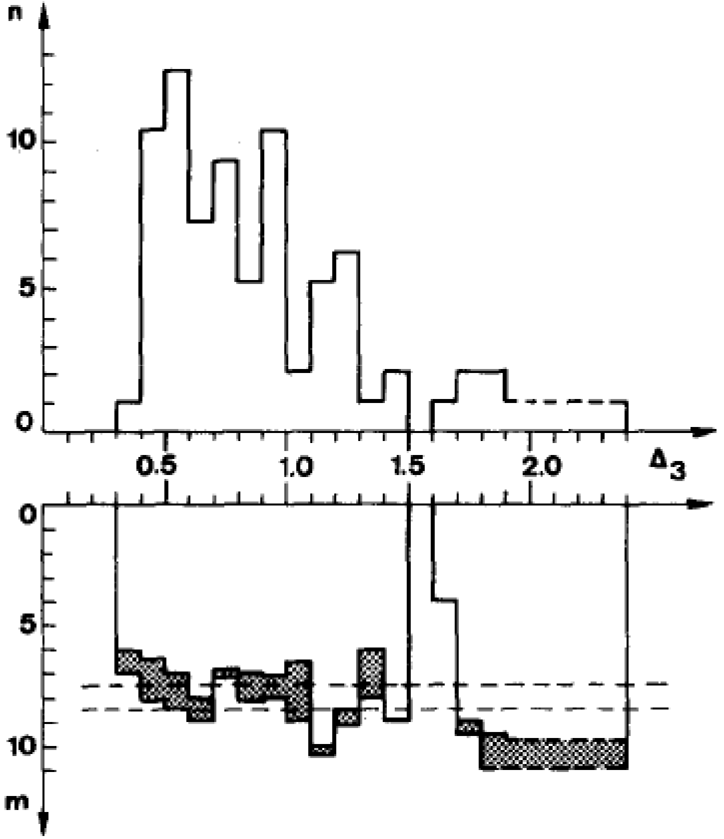
  Know how widths are missed. Number missed depends on ν.

- Use of maximum-likelihood technique is straightforward.

  PTD is χ^2 with $\nu=1$.

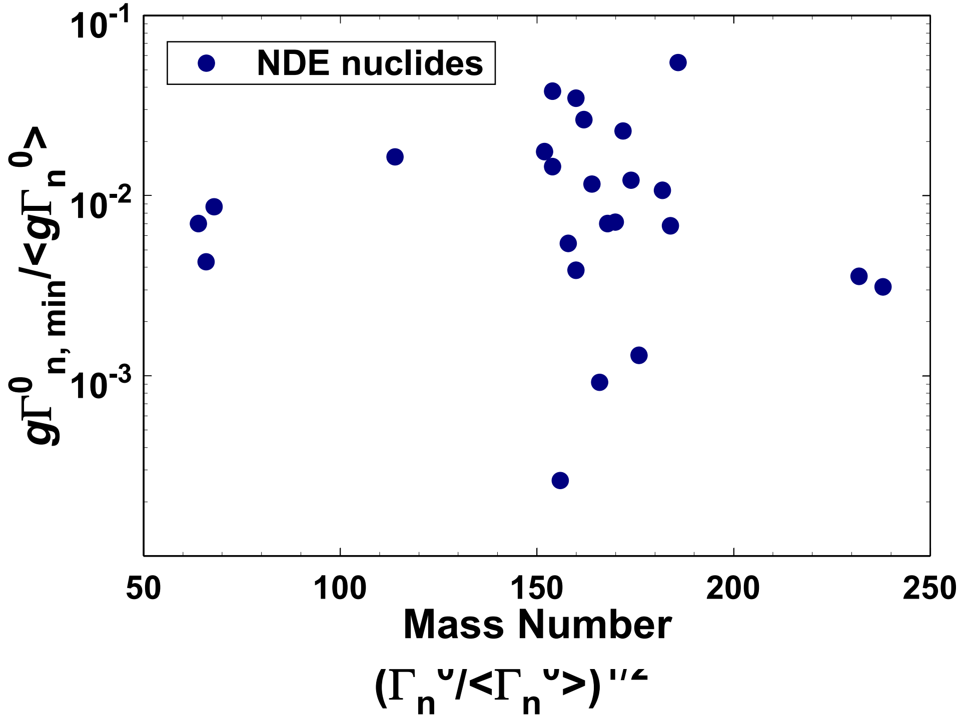
- Widths provide a more reliable and sensitive test of theory.

  "It follows that a measurement of resonance energies alone is not a very powerful tool for testing the statistical model of spectra fluctuations, while a much more reliable analysis can be performed if also neutron widths are measured." – Coceva and Stephanon, Nucl. Phys. 315, 1 (1979).



Problems With the NDE Neutron Widths

1. Apparent thresholds vary by orders of magnitude.
2. More resonances missed at higher energies.
3. Serious *p*-wave contamination.
   131/1245 = 10.5 % overall.
   As much as 35 % in some cases.



NDE is neither complete nor pure.

Systematic errors result from analyzing NDE as a single group.

Must analyze each nuclide separately, and then combine.

Threshold must be used to account for missed resonances and to eliminate *p*-waves.

Improved ML Analysis of NDE Neutron-Width Distribution

- Use threshold proportional to E_n.

  $\Gamma_n^0/<\Gamma_n^0> \geq T\ E_n/E_{max}$

- Analyze each nuclide separately.

  Minimizes systematic errors due to missing small resonances and *p*-wave contamination.

  Eliminates *p*-waves equally effectively at all energies.

  Close to shape of experimental sensitivity.

  Maximizes statistical precision.

Results: ML Analysis of NDE with Minimum Thresholds

- T chosen for each nuclide so that threshold just below all data.

  If NDE agrees with PTD, expect $\nu \geq 1$ because experiment thresholds might not be perfectly "black".

- Weighted average for 24 nuclides (1245 widths):

  $\nu = 0.801 \pm 0.052$.

  3.8 std. dev. smaller than PTD.

- Interesting "new" physics?

  More likely: Due to *p*-wave contamination.
  Either way, contradicts good agreement reported for Δ_3.

Results: Cleansing the NDE of *p* waves

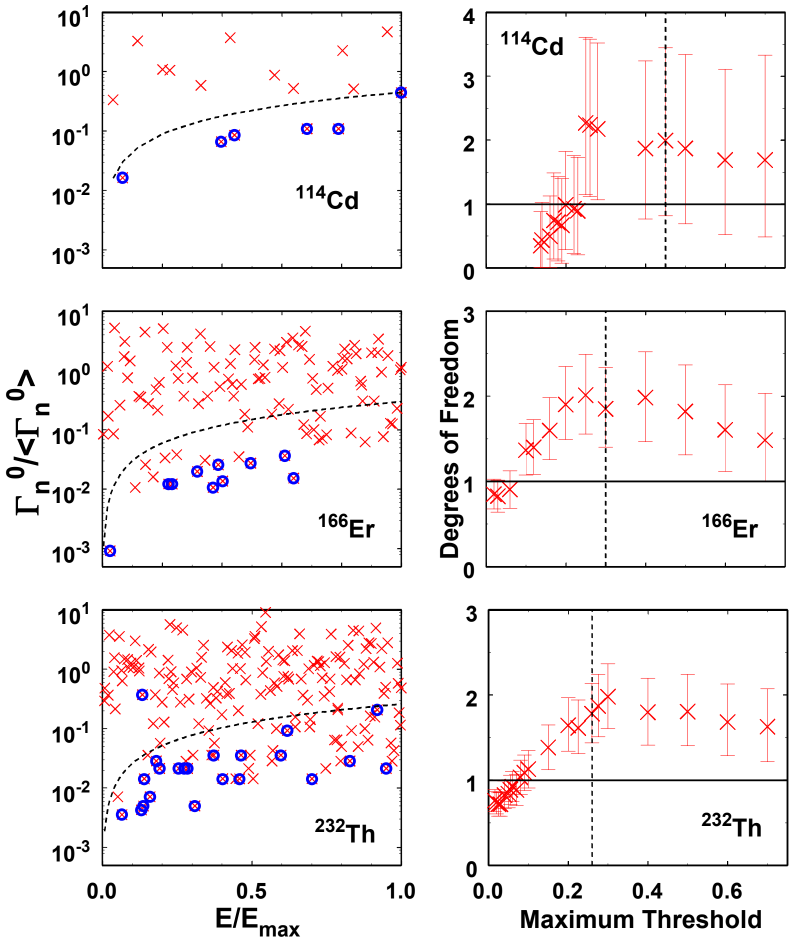
- **Used higher thresholds, T.**
- **Studied ν as function of T.**
- **For many NDE nuclides:**

  ν initially increases with T, then levels out.

  Expected if *p*-wave contamination.

- **“*p* free” weighted average (978 widths):**

  $\nu = 1.217 \pm 0.092$



NDE neutron widths do not agree with the GOE.

Why Does the NDE Agree So Well With GOE Spacings?

- **Data selected (at least in part) to agree with theory.**

  "The criterion for inclusion in the NDE is that the individual sequences be in general agreement with the GOE." (Bohigas *et al*.).

- **For all but three of NDE nuclides, separation of *s*- from *p*-wave resonances accomplished using measures derived from the GOE.**

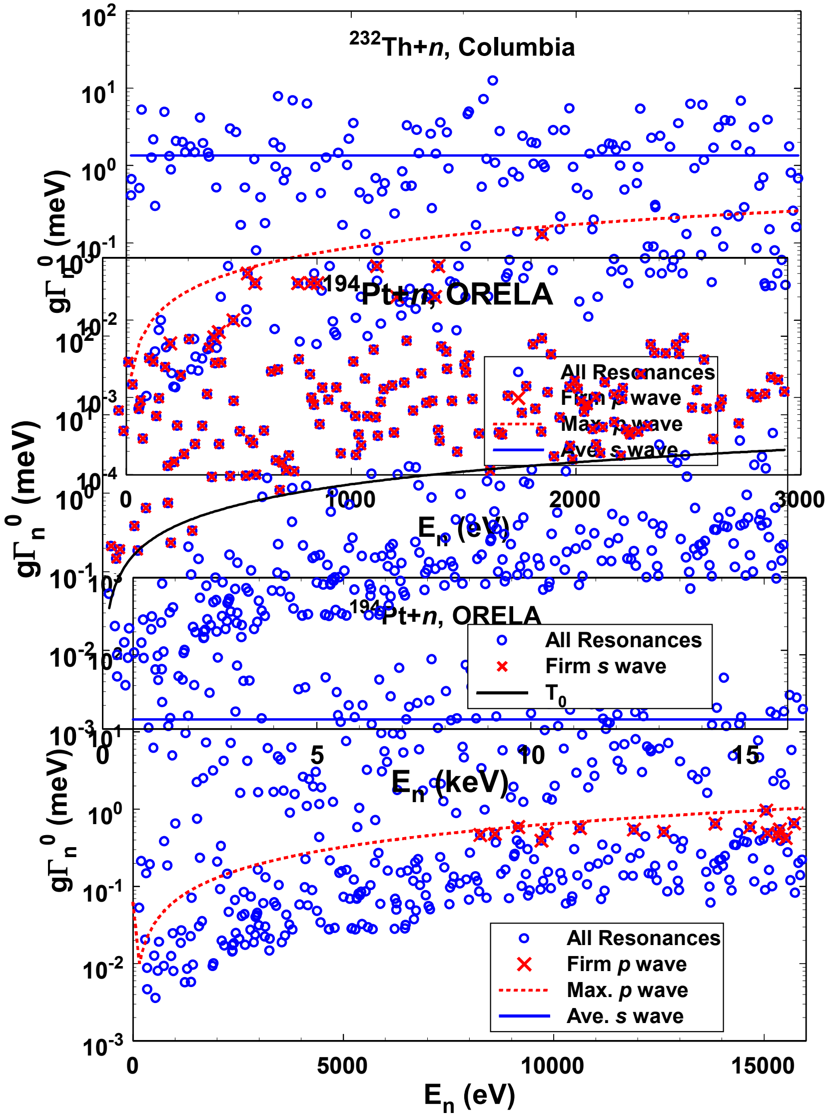
  "…no specific tests for *s* vs *p* levels, so there may be errors in these assignments." (Liou *et al*., ^{166,168,170}Er).

- **It is possible to find a subset of the observed resonances which agrees with GOE spacings.**

  Incompleteness compensated by impurity.

Testing the PTD Using ^{192,194,196}Pt+n ORELA Data

- Better separation of *s*- from *p*-wave resonances.
  Pt: $S_0/S_1 \approx 10$).
  ^{232}Th: $S_0/S_1 \approx 0.5$
- Better data.
  Neutron capture and total cross sections for ^{192,194,195,196,nat}Pt
- Better analysis.
  Simultaneous *R*-matrix analysis (SAMMY).
  Many firm *s*-wave assignments <u>independent of RMT</u>.
  Extra statistical tests to ensure veracity of ML results.



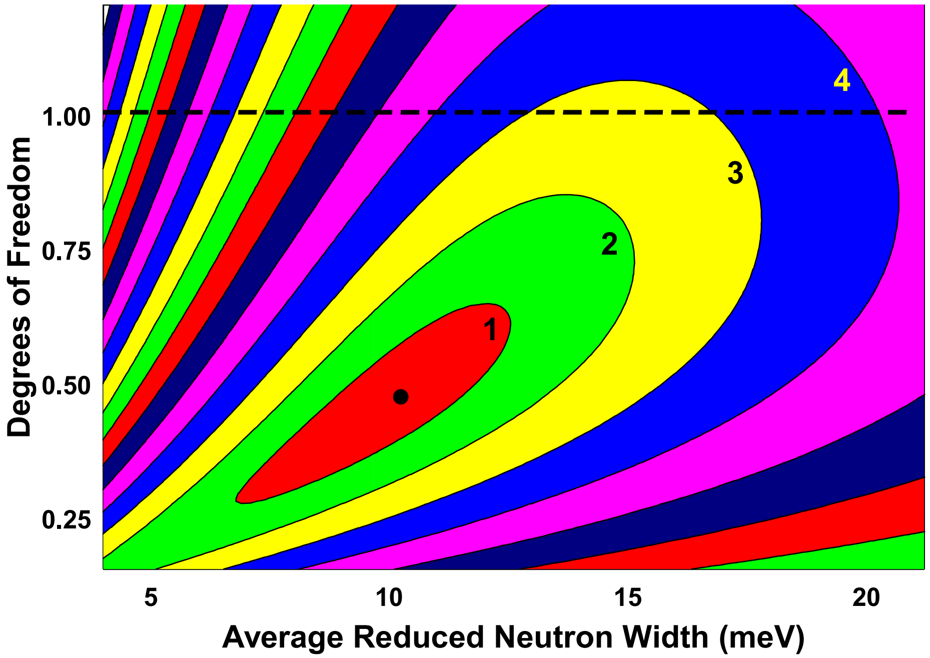
^{192,194,196}Pt+n Results

- Resonance parameters.

  830 resonances.
  318 firm *s* wave.

- Maximum-Likelihood (ML) analysis.

  ^{192}Pt: ν = 0.57±0.16
  ^{194}Pt: ν = 0.47±0.19
  ^{196}Pt: ν = 0.60±0.28

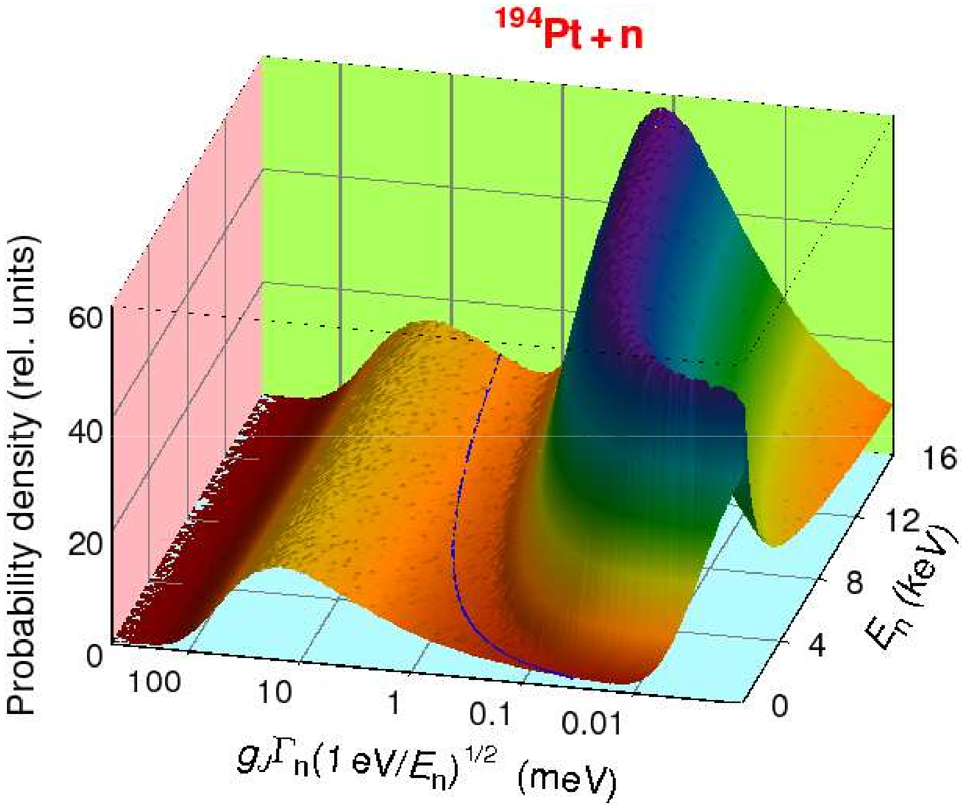


$$z\left(\nu,\mathrm{E}[\Gamma^0_{\lambda\mathrm{n}}]\right)=2^{\frac{1}{2}}\left[\ln L_{\mathrm{max}}-\ln L\left(\nu,\mathrm{E}[\Gamma^0_{\lambda\mathrm{n}}]\right)\right]^{\frac{1}{2}}$$

^{192,194,196}Pt+n Results

- Additional calculations to determine confidence level (CL) for rejecting PTD.
  Monte Carlo simulation to determine CL as function of $\langle \Gamma_n^0 \rangle$.
  Two new statistics to limit range of $\langle \Gamma_n^0 \rangle$.
  Confirmed ML results.

- Auxiliary ML analysis to verify that *p*-wave contamination is negligibly small (0.069 for ^{192}Pt, 0.0047% for ^{194}Pt).



<u>PTD rejected at >99.997% confidence level</u>

Phys. Rev. Lett. 105, 072502 (2010)

PTD Scorecard

- ^{232}Th:
  ν = 3.8±1.3 to 0.83±0.68.

- ^{147}Sm:
  ν = 0.91±0.32 to 3.19±0.83.

- NDE (“*p* free”)
  ν = 1.217 ± 0.092.

- ^{192,194}Pt:
  PTD rejected at >99.997% confidence level.

  ^{192}Pt: ν = 0.57±0.16
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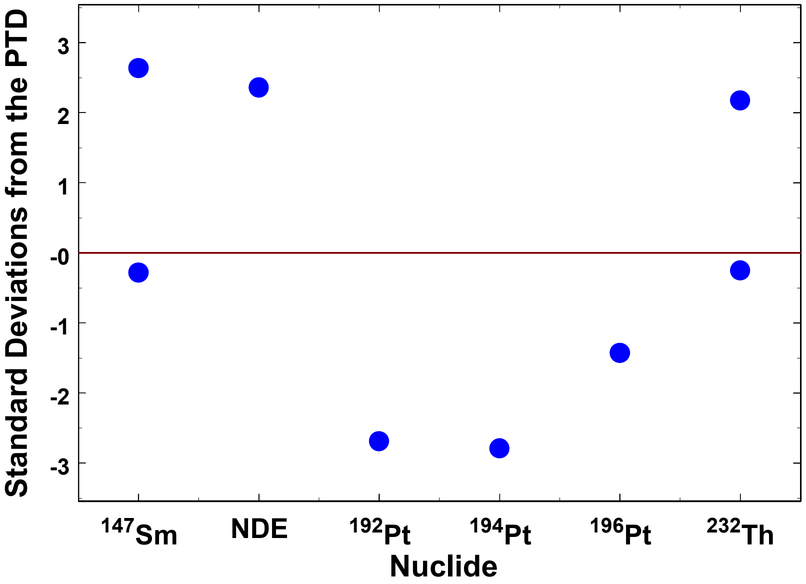


Possible explanation (ν≈0.5)?

- Did Bethe get it right way back in 1955?

- Hughes and Harvey, Phys. Rev. C 99, 1032 (1955).

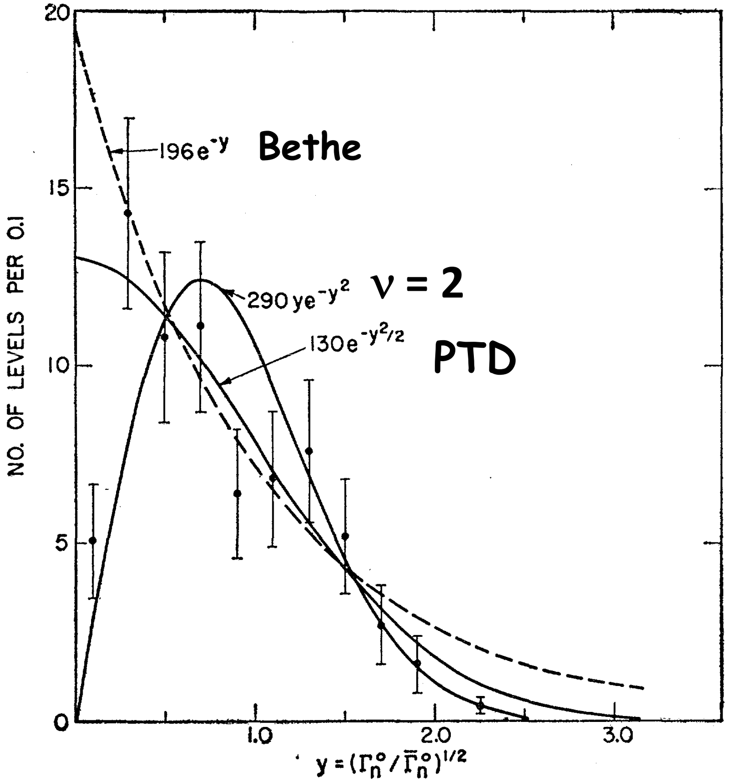
  Compared their NDE to 3 theories.

  Theory attributed to Bethe (private communication) is broader than PTD.

- Bethe's distribution published in Peaceful Uses of Atomic Energy:

  "Since there is no theory of the statistical distribution of reduced neutron width, we feel free to assume a purely empirical formula…"

  "This [the PTD] also has some slight theoretical foundation…"



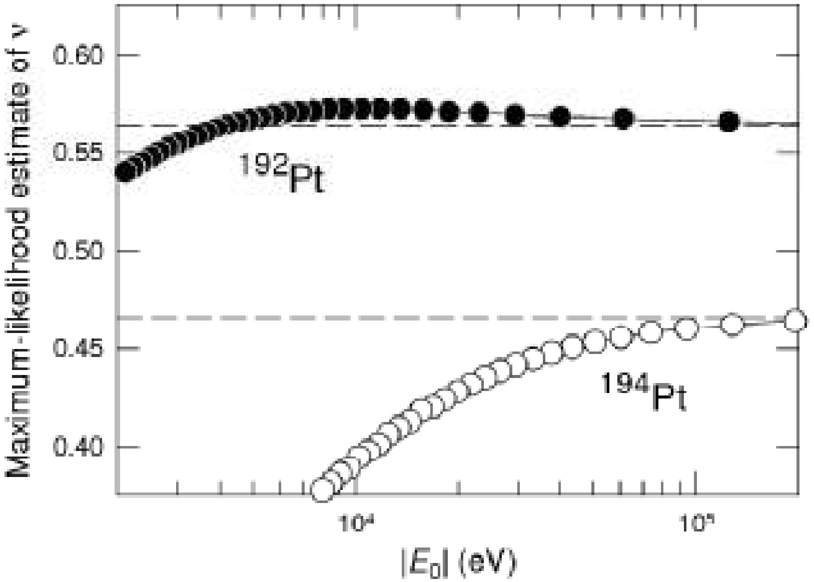
~~Possible explanation?~~

- **Alternate transformation of Γ_n to Γ_n^0 for nuclides near peaks of *s*-wave strength function.**

  Weidenmüller, Phys. Rev. Lett. 105, 232501 (2010).

$$\Gamma_n^{0'} = C \times \Gamma_n \frac{E + |E_0|}{\sqrt{E}}$$

- **Worsens disagreement for ^{192,194}Pt.**
- **Broadens distribution unless $\langle\Gamma_n^0\rangle$ decreasing with *E*.**
- **Cannot explain, e.g. ^{232}Th.**

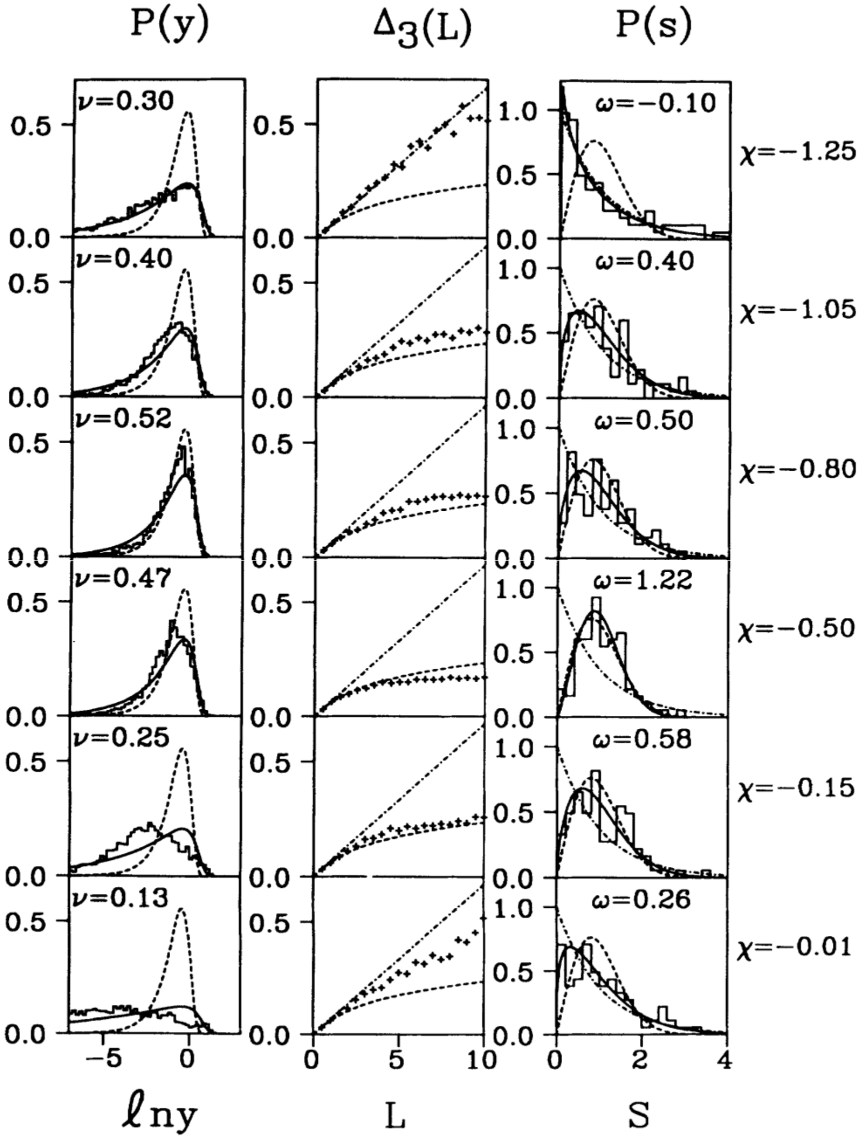


Possible explanation (ν≈0.5)?

- Might be signature of collective effect (e.g., Y. Alhassid and A. Novoselsky, Phys. Rev. C 45, 1677 (1992)).

  Model calculations for low excitations yielded transition strength distributions with ν<1 as system became more collective.

  But why would highly excited states in ^{193,195}Pt be collective?



Possible explanation?

- External mixing of resonance states via the continuum causes deviations from "complete randomness".

  Kleinwächter and Rotter, Phys. Rev. C 32, 1742 (1985) and subsequent papers.

- Continuum shell model.

- Resonances appear isolated ($<\Gamma_n> << D_0$), but they actually are coupled.

  "The narrow compound nucleus resonances in heavy nuclei are the result of a dynamical phase transition. They are characterized by *essential collective aspects of the interplay between the constituent particles and not by a combination of one-body problems."* (Rotter, J. Mod. Phys. 1, 303 (2010).

- Expected to broaden the width distribution.

Possible explanation?

- Correlations between parent and daughter nuclear systems result in deviations of decay width statistics from the PTD

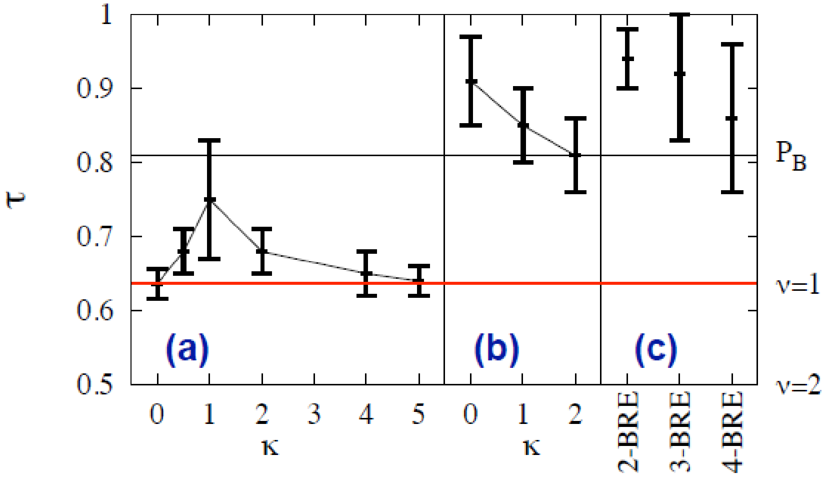
  Volya, Phys. Rev. C 83, 044312 (2011).

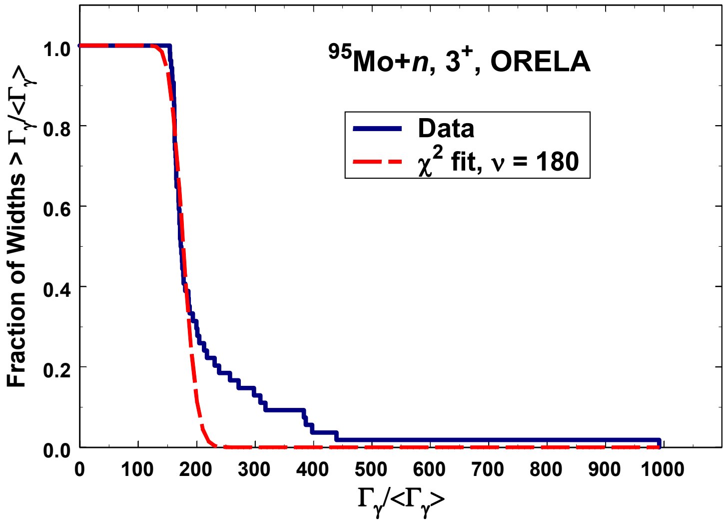
- Continuum shell model.

- Different from compound nucleus model.

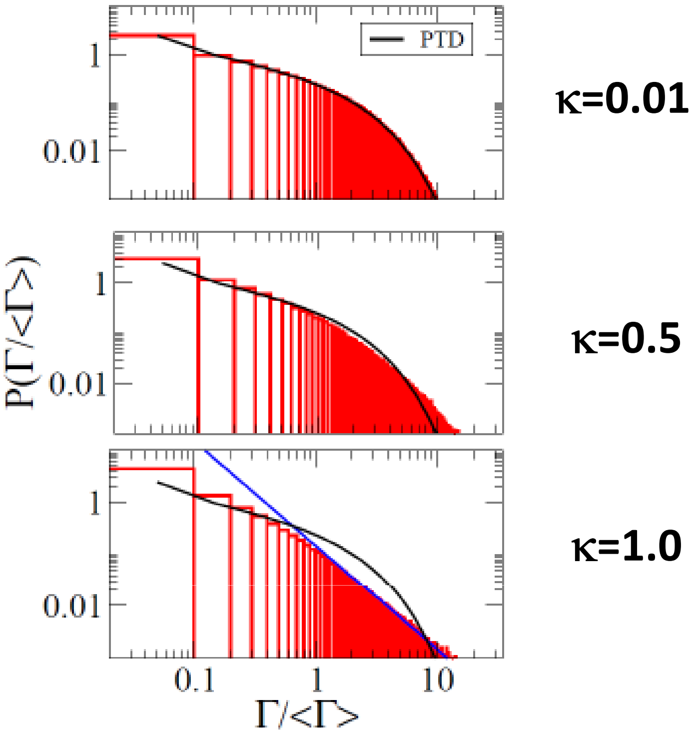
  "…the two-body or other low-rank Hamiltonian does not lead to dynamical mixing of states strong enough for the decaying system to lose all memory of its creation."

- Deviation from PTD also expected for electromagnetic transitions.





~~Possible explanation?~~



- **Continuum coupling causes deviations from the PTD.**

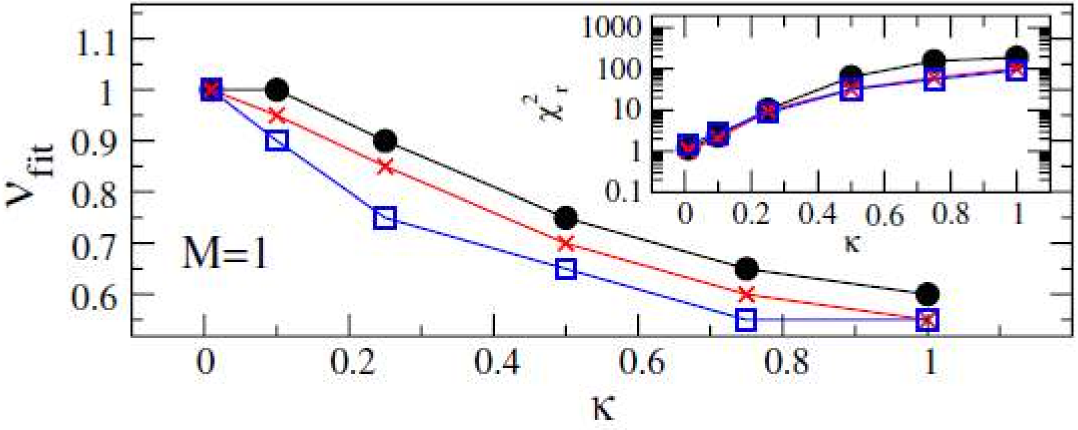
  Celardo *et al.*, Phys. Rev. Lett. 106, 042501 (2011).

- **Coupling strength calculated from our data is too small to cause our observed deviation from the PTD.**

$$\kappa \approx \frac{\pi}{2} S_0 \sqrt{E}$$

κ<0.04 => ν>0.95

($\nu \approx$ 0.5 for ^{192,194}Pt)



Conclusions

- Porter-Thomas distribution (RMT for the GOE) shown to be incorrect in several cases.
- Most famous "proof" of RMT for the GOE (the NDE) is fatally flawed.
- Best case so far: Γ_n^0 data for ^{192,194,196}Pt.

  PTD excluded at 99.997% confidence level.
  $\nu \approx 0.5$ (PTD has $\nu = 1$).
  Several models proposed, some excluded by the data.
- Other cases (e.g., ^{147}Sm and ^{232}Th), ν changes from 1 to ≈2.

Future Prospects

- Need high quality neutron capture and total cross sections.

  New resonance parameters should be used to test theory instead of using theory to correct data.

- Need careful R-matrix analysis.

  Very important to indicate which J^{π} assignments are firm (independent of theory being tested).

- New techniques for determining J^{π}'s should be extremely valuable (and are not too difficult to implement).

Collaborators

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Impact On Applications
(e.g., Astrophysics, Nuclear Energy)

- **Shape of neutron-width distribution affects calculated cross sections and important parameters for applications.**

  ν=1 assumed throughout nuclear statistical model.

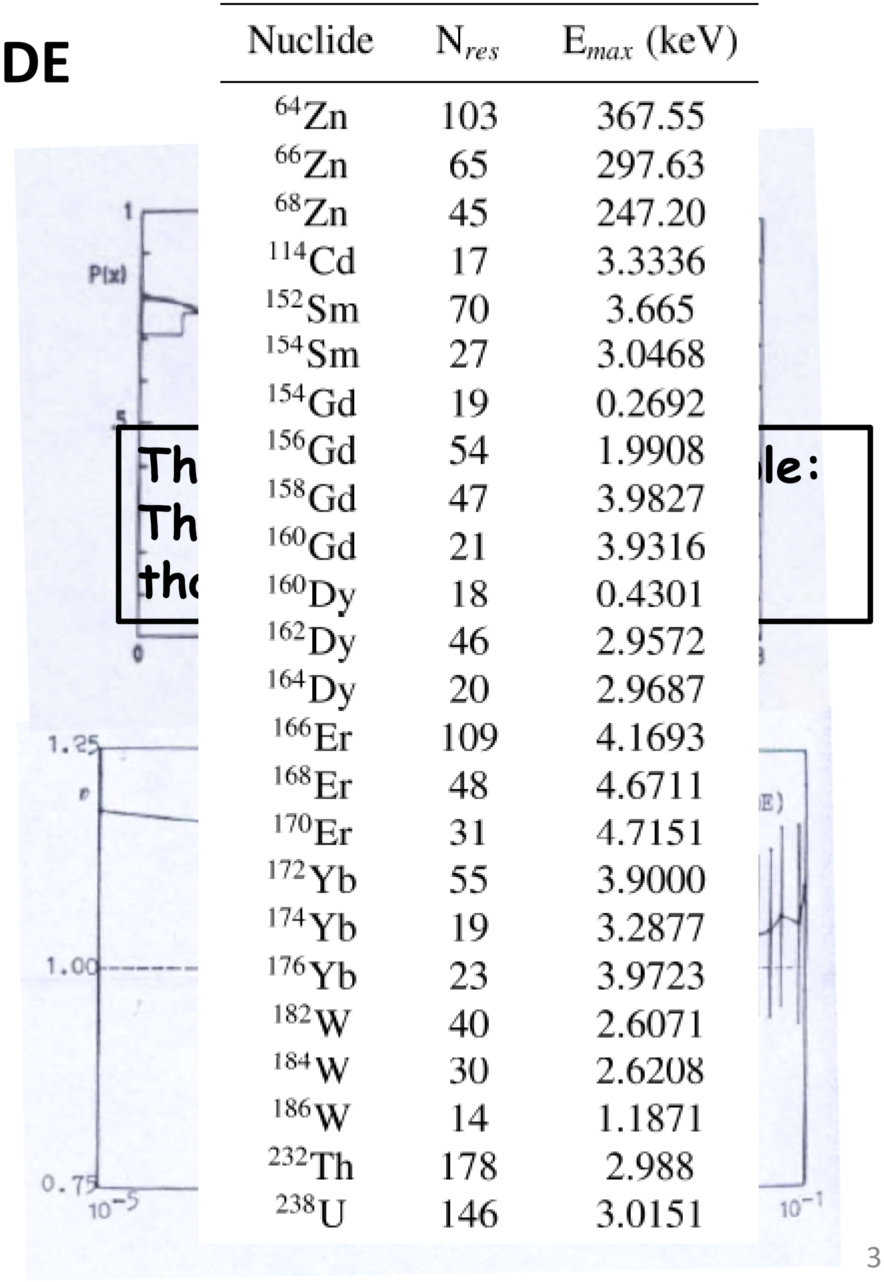
  Width fluctuation correction depends on ν.
  e.g., for $\Gamma_{c'}/\Gamma_c=1$, $S_{cc'}=\nu/(\nu+1)$.

  Self shielding correction for reactors, etc. vary with ν.

Neutron Widths in the NDE

- Bohigas *et al*. used the maximum-likelihood technique to analyze 1182 Γ_n^0 ’s in the NDE, all as one group.
  Found good agreement between the data and PT.

- Γ_n^0 ’s for subset of 1245 resonances available.
  All but ^{64,66,68}Zn (ORELA) and ^{156}Gd (GELINA) from Columbia University group, published in the 1970’s.



Nuclide	N_{res}	E_{max} (keV)
^{64}Zn	103	367.55
^{66}Zn	65	297.63
^{68}Zn	45	247.20
^{114}Cd	17	3.3336
^{152}Sm	70	3.665
^{154}Sm	27	3.0468
^{154}Gd	19	0.2692
^{156}Gd	54	1.9908
^{158}Gd	47	3.9827
^{160}Gd	21	3.9316
^{160}Dy	18	0.4301
^{162}Dy	46	2.9572
^{164}Dy	20	2.9687
^{166}Er	109	4.1693
^{168}Er	48	4.6711
^{170}Er	31	4.7151
^{172}Yb	55	3.9000
^{174}Yb	19	3.2877
^{176}Yb	23	3.9723
^{182}W	40	2.6071
^{184}W	30	2.6208
^{186}W	14	1.1871
^{232}Th	178	2.988
^{238}U	146	3.0151

Possible explanations?

- ~~TRIV and unknown (e.g. inelastic) extra neutron channel ruled out.~~

  ~~Lead to $\nu>1$, but $\nu<1$ observed.~~

- Widths not statistical.

  But typical nonstatistical signatures absent in data (e.g., steps in $\Sigma\Gamma_n^0$ vs. E_n).