Excitation energy sorting in fission

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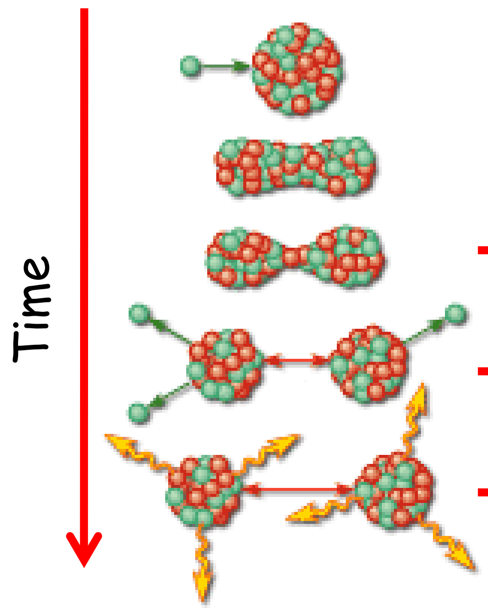
The fission process



→ Scission, deformed nuclei in contact through the neck

→ Fragments recover ground-state deformation
neutron emission

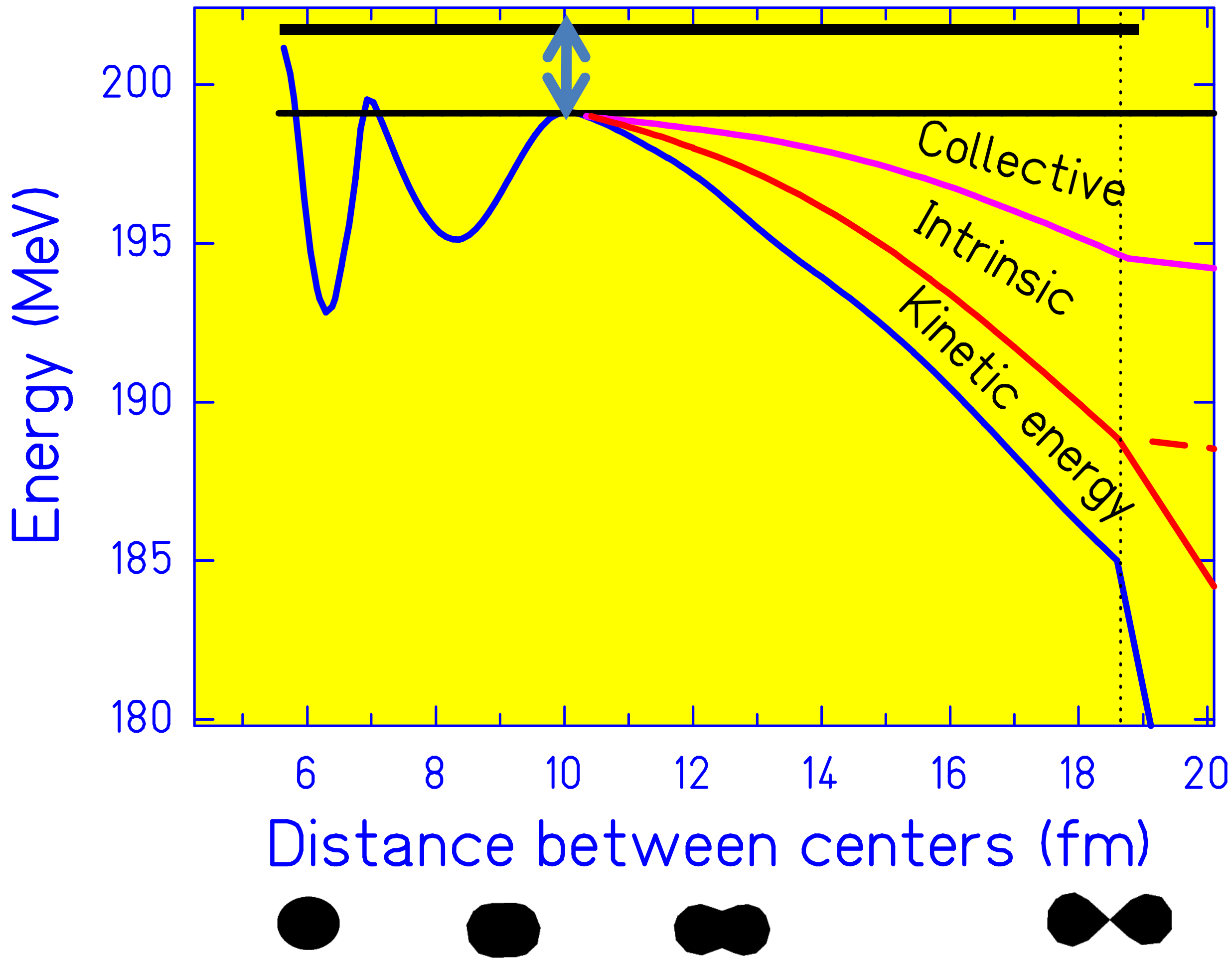
→ Gamma-ray emission

•Large scale collective motion ->Complex process

•Fragments acquire their individual properties (Shell-effects, Pairing…) well before scission

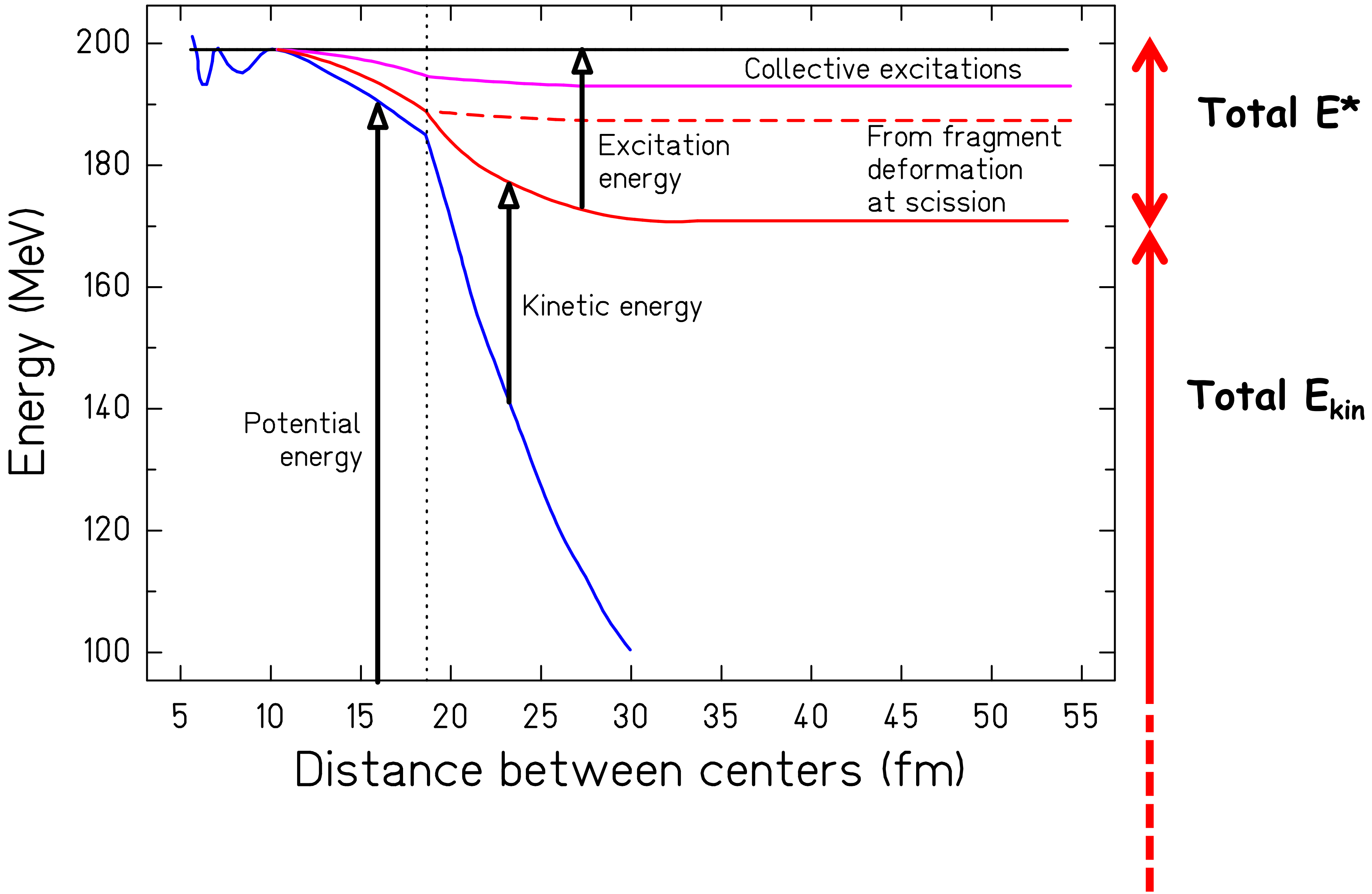
U. Mosel et al. PRC 4 (1971) 2185
W.D. Myers et al. NPA 612 (1997) 249
H.J. Krappe et al. NPA 690 (2002) 431

Energy considerations: situation at scission

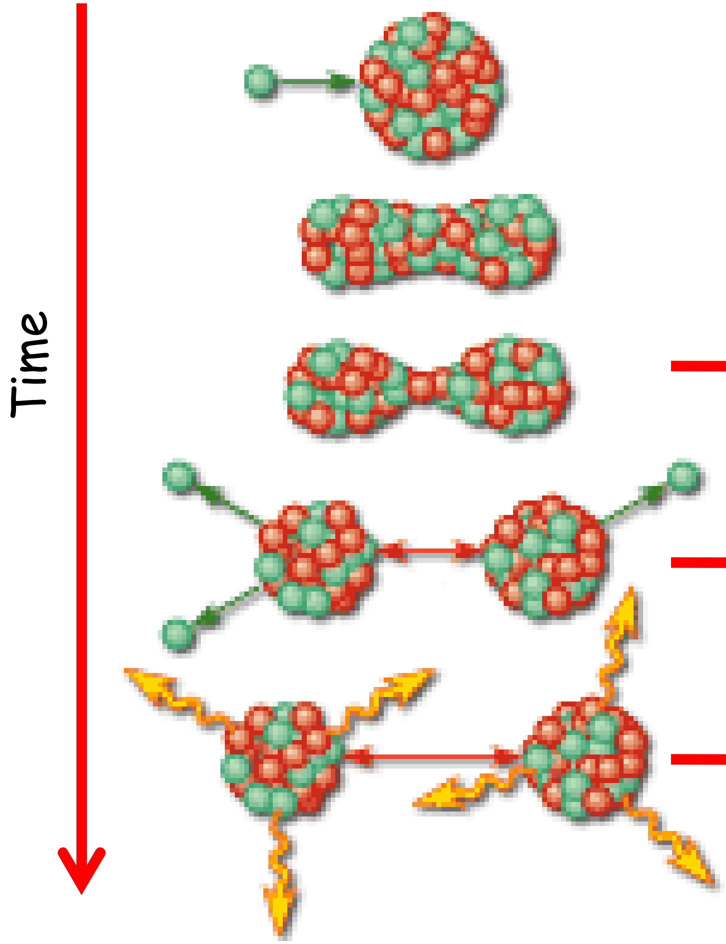


Increase of energy above the fission barrier mainly goes to intrinsic E*

Energy considerations: fully accelerated fragments



The contributions to total E* appear at different stages of the fission process!!



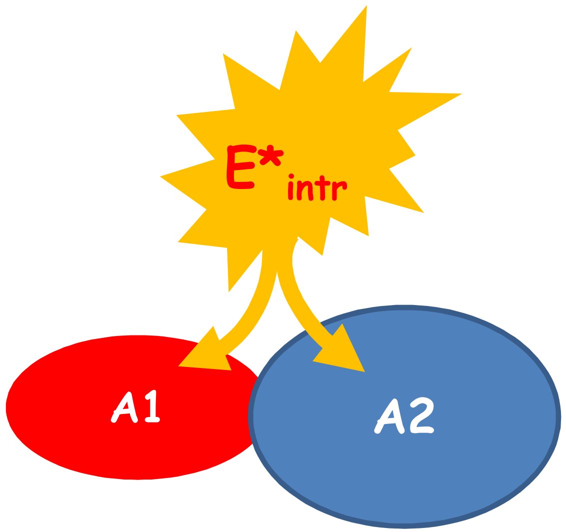
E^*_{intr} has to be shared between the fragments before scission (neutron emission)

E_{def} is transformed into E* after scission (neutron emission)

E_{coll} is transformed into E* after scission (gamma emission)

Only the E^*_{intr} is statistically shared between the fragments!!!!!
(Most models consider that E* total is statistically shared…)

Configuration near scission:
two moderately excited nuclei in contact :



How do they share E^*_{intr}?
Transfer of E* between the nuclei until thermal equilibrium

Statistical mechanics

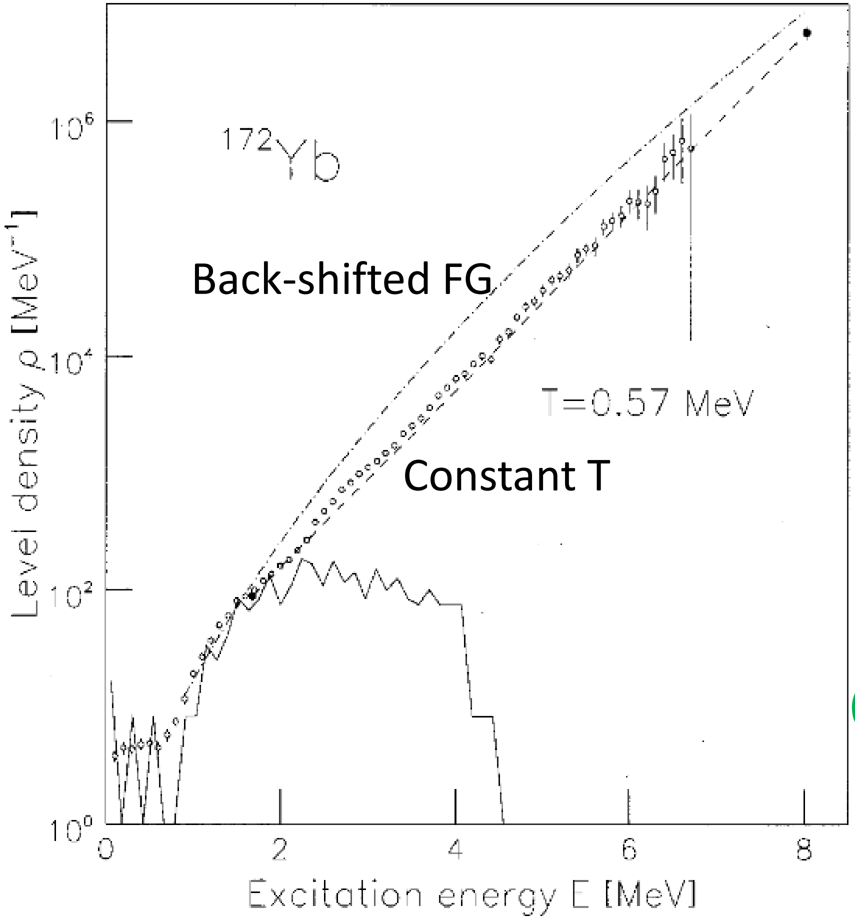
Statistical equilibrium → Each state of the complete system is occupied with the same probability!

$$< E_L^* >= \frac{\int_{E=0}^{E=E_{\mathrm{int}\,r}^*} E\rho_L(E)\rho_H(E_{\mathrm{int}\,r}^* - E)dE}{\int_{E=0}^{E=E_{\mathrm{int}\,r}^*} \rho_L(E)\rho_H(E_{\mathrm{int}\,r}^* - E)dE}$$

with $E_{\mathrm{int}\,r}^* = E_L^* + E_H^*$

Average E* in each fission fragment at statistical equilibrium

Level density of warm nuclei in fission fragment mass region



Guttormsen et al. 2001

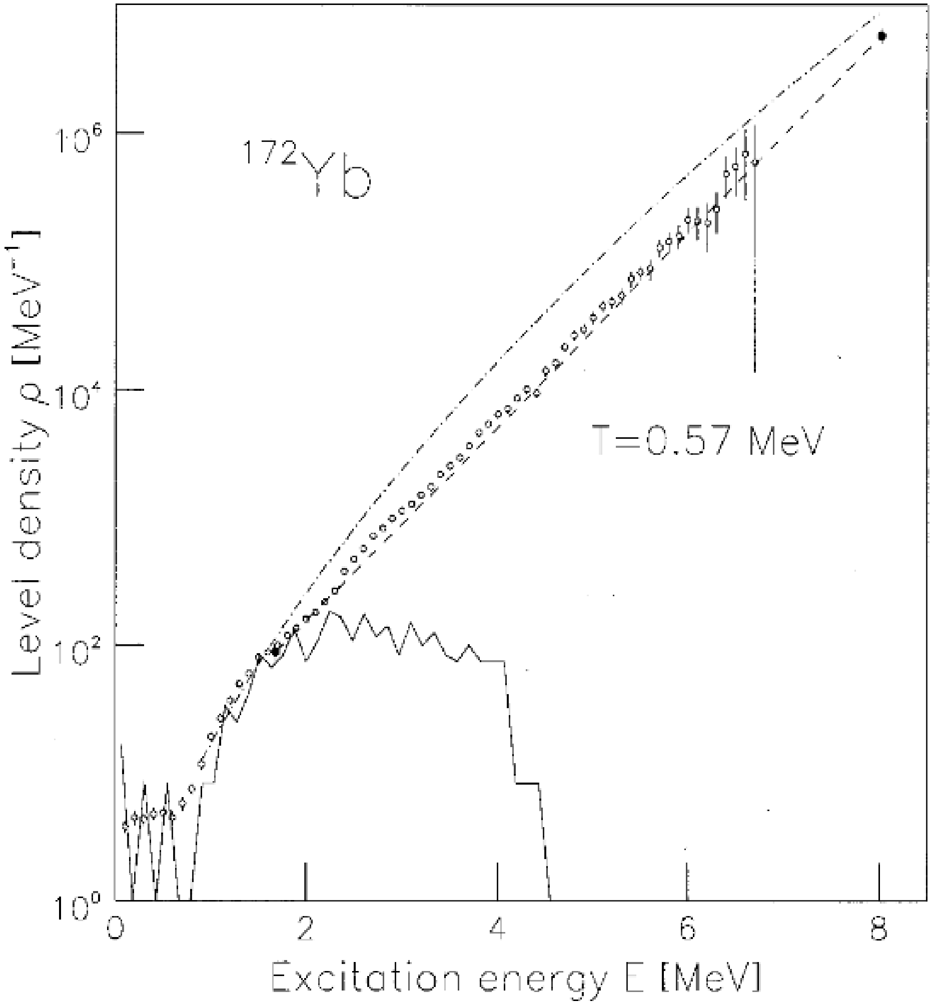
Rather exact constant-temperature behaviour:

$\rho(E^*) \propto \exp(E^*/T)$

Constant temperature up to 20 MeV!!

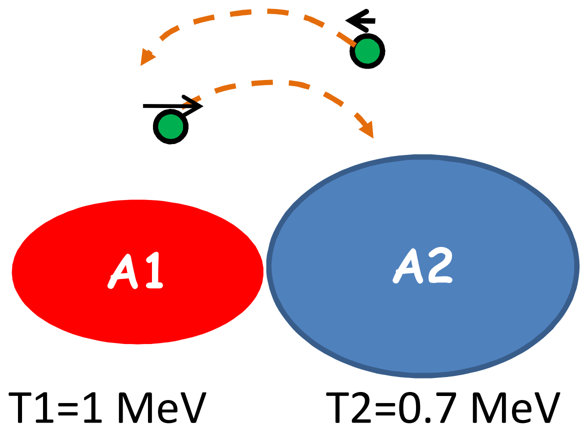
(Voinov et al. PRC 79 (2009) 031301 (R))

Can we really talk about constant temperature?



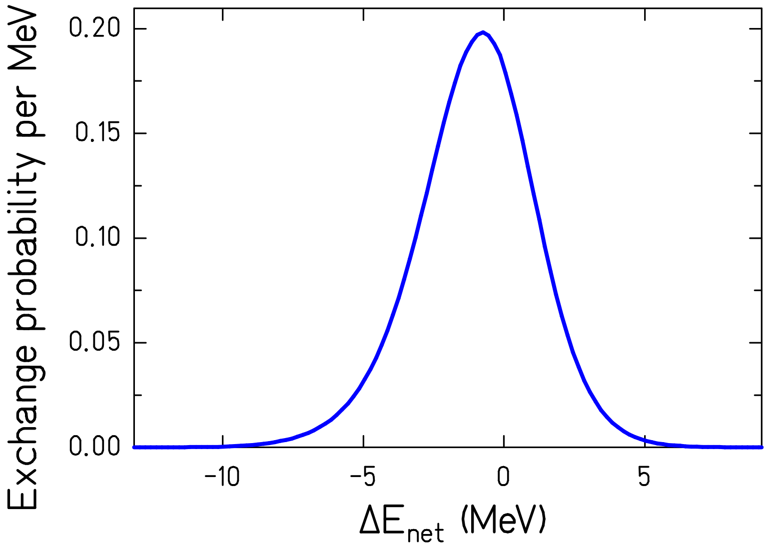
Fluctuations and structural effects are present in particular at the lowest E*

Microscopic view of energy transfer



Heat is mainly transferred via nucleon exchange through the neck region

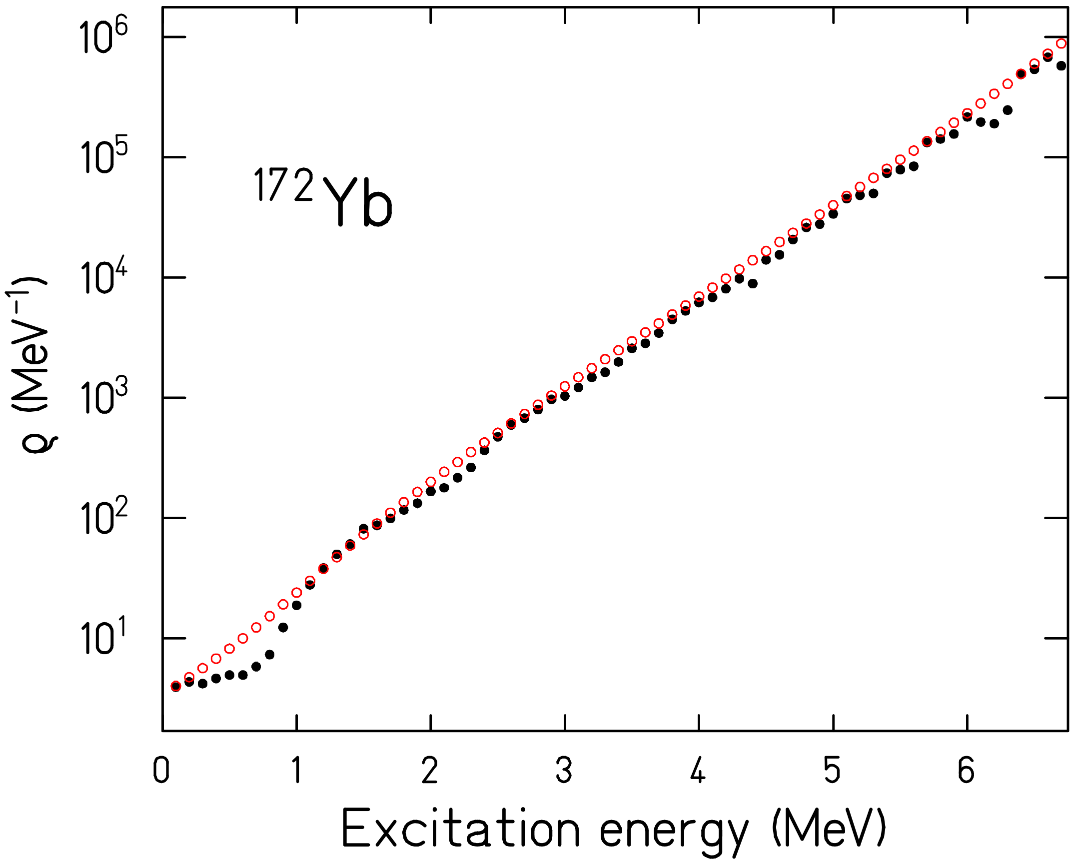
Simple model calculation



Probability distribution of the change ΔE of the excitation energy of the light nucleus after one nucleon exchange.

Energy transfer occurs in large steps of about 0.96 MeV and is subject to strong fluctuations σ ~ 3 MeV!!

Thermodynamics of two nuclei in contact
Can we really talk about constant temperature?



<u>**Thermal averaging:**</u>

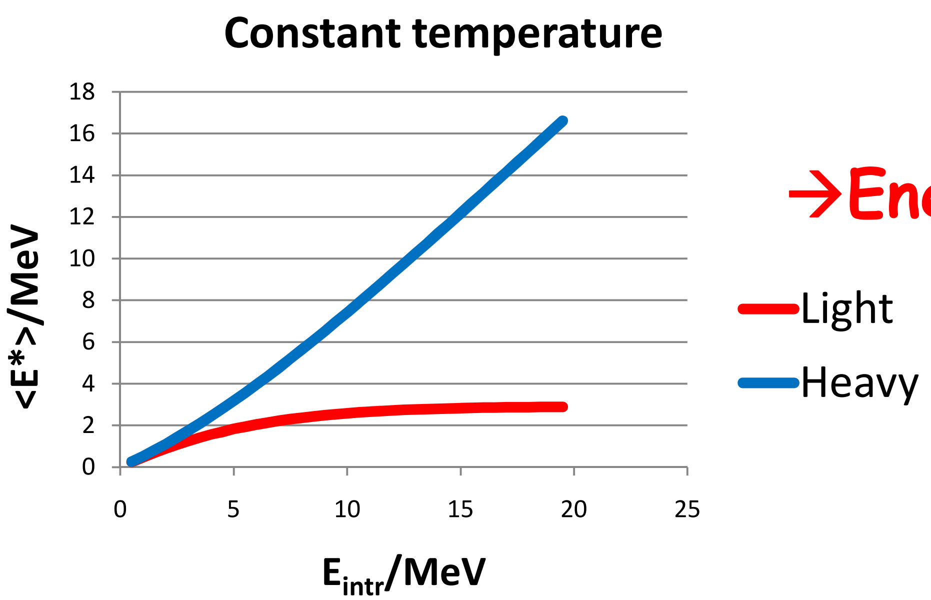
The effective level density is the result of folding the real level density with the previous function

All the structures are washed out down to E*=0!

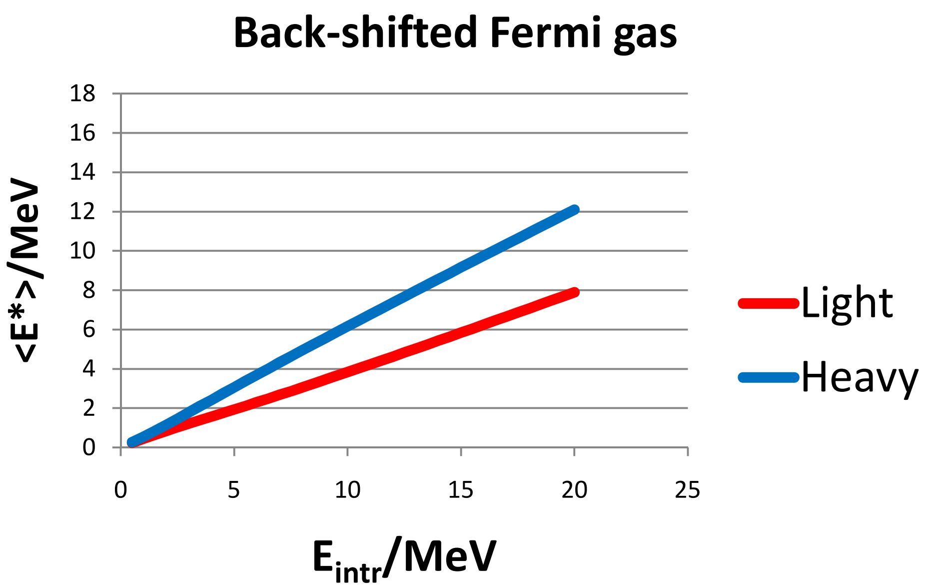
K.-H. Schmidt and B. Jurado, Phys. Rev. C 83 (2011) 014607

Partition of E*intr

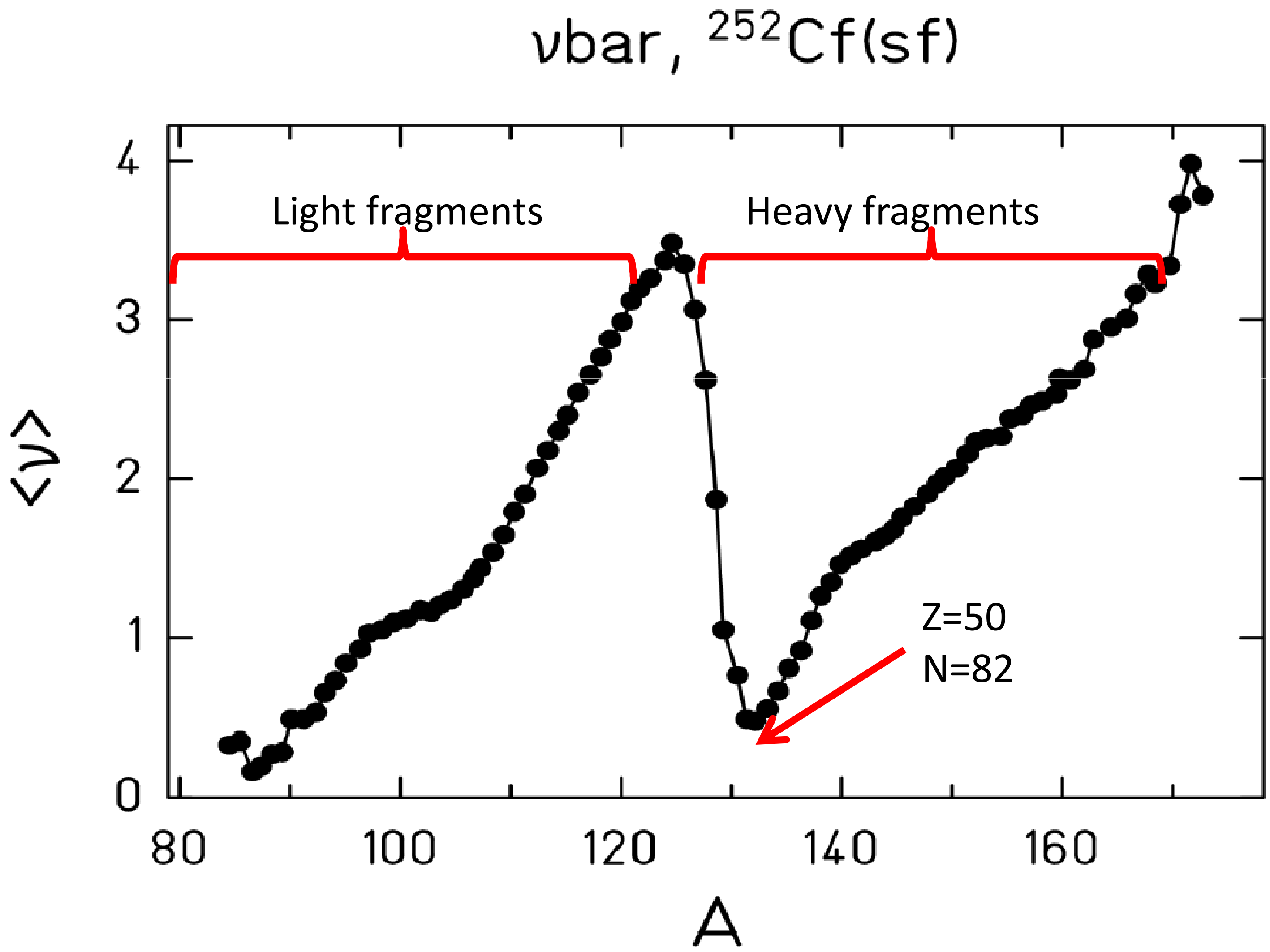
A_{light}=97
A_{heavy}=140



→Energy sorting!!!

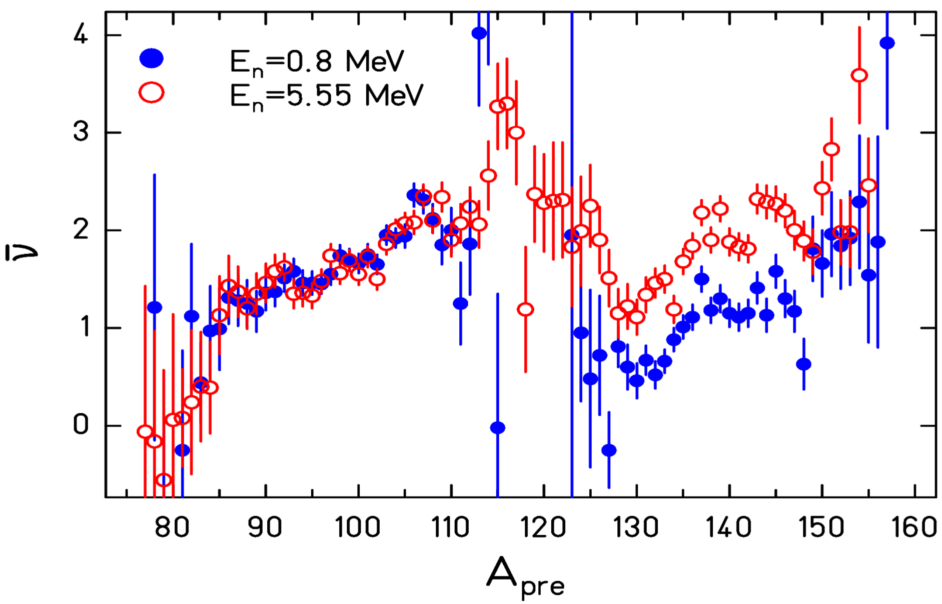


Neutron yields vs. fragment mass



Neutron yields in fission

^{237}Np(n,f) (Naqvi et al., PRC 34 (1986) 218)

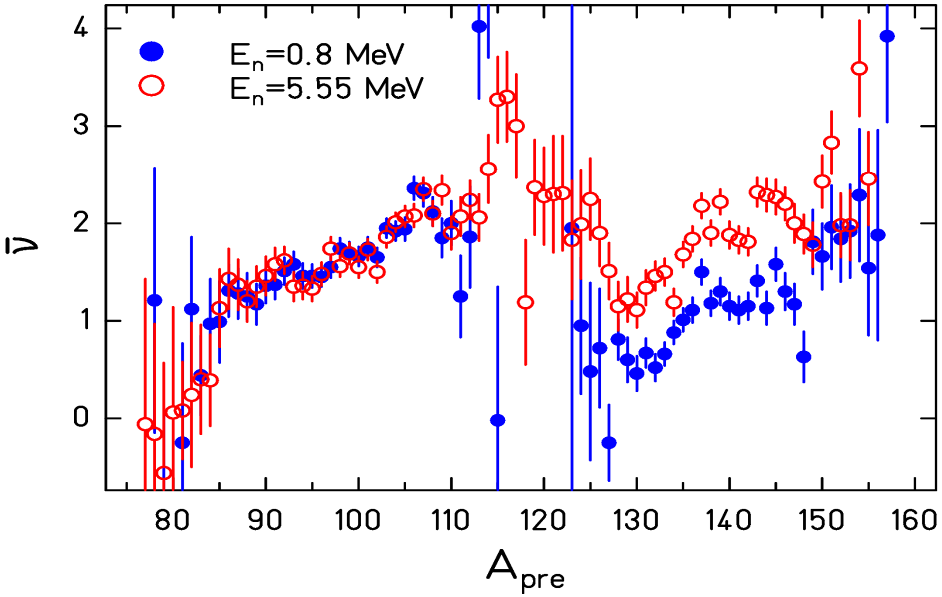


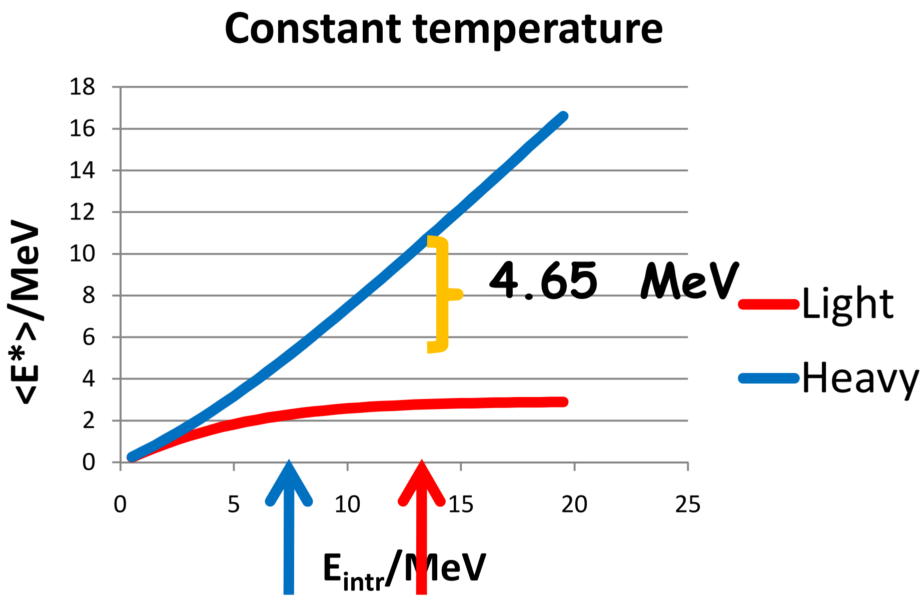
Only the neutron yields of heavy fragments increase!!!

Observation also found for ^{233}U, ^{238}U and proton-induced fission

Unexplained up to now!

Neutron yields in fission at diff. energies: Signature of energy sorting





The increase in emitted neutrons corresponds to an increase in E^*_{heavy} of 4.8 ± 0.2 MeV

→Can only be explained with energy sorting!!!

K.-H. Schmidt, B. Jurado, PRL 104 (2010) 212501

Conclusions

✓The scission configuration offers a unique opportunity to observe the behavior of two warm nuclei set in contact:

→Transfer of energy occurs in large steps subject to important fluctuations

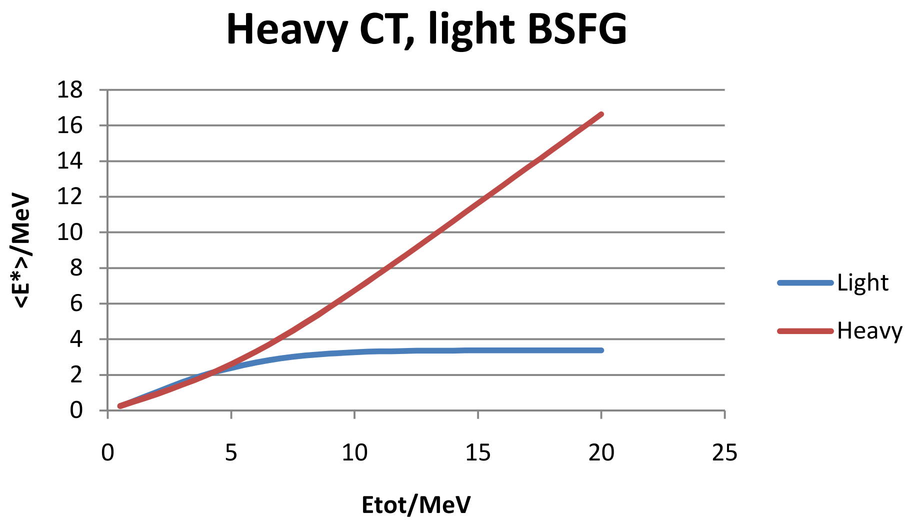
Thermal averaging → justifies constant temperature down to E*=0

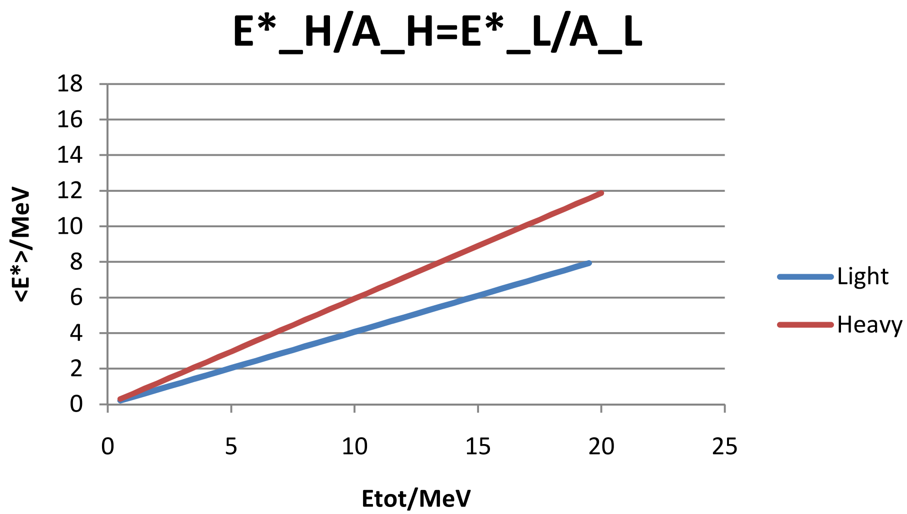
→**Excitation energy sorting →** Most of E* is found in the heavy fragment

✓Energy sorting → **Clearly reflected by number of prompt neutrons vs. A**

An increase of E* translates into an increase of υ for the heavy fragment only .

This observation remained unexplained up to now!!!


Heavy CT, light BSFG
18
16
14
12
10
8
6
4
2
0
0
5
10
15
20
25
Etot/MeV
<E*>/MeV
Light
Heavy


E*_H/A_H=E*_L/A_L
18
16
14
12
10
8
6
4
2
0
0
5
10
15
20
25
Etot/MeV
<E*>/MeV
Light
Heavy

Back-shifted Fermi gas according to RIPL3

$$\rho(E) = \frac{1}{12\sqrt{2}\sigma a^{1/4}(E-\Delta)^{5/4}} e^{2\sqrt{a(E-\Delta)}}$$

$$\sigma(E)^2 = 0.01389\,A^{5/3}\,\frac{\sqrt{E-\Delta}}{\sqrt{a}}$$

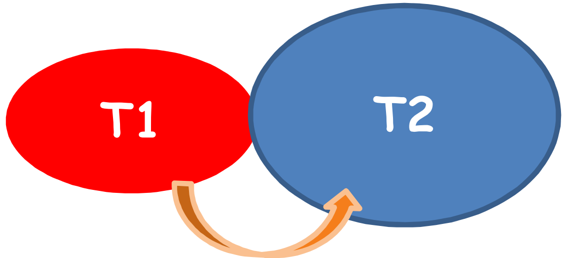
$$a = 0.0692559\,A + 0.282769\,A^{2/3}$$

$$\Delta = n\frac{12}{\sqrt{A}} + 0.173015$$

n =-1 → o-o
n =0 → e-o
n =1 → e-e

Behaviour of two moderately excited nuclei in contact:

T1 > T2



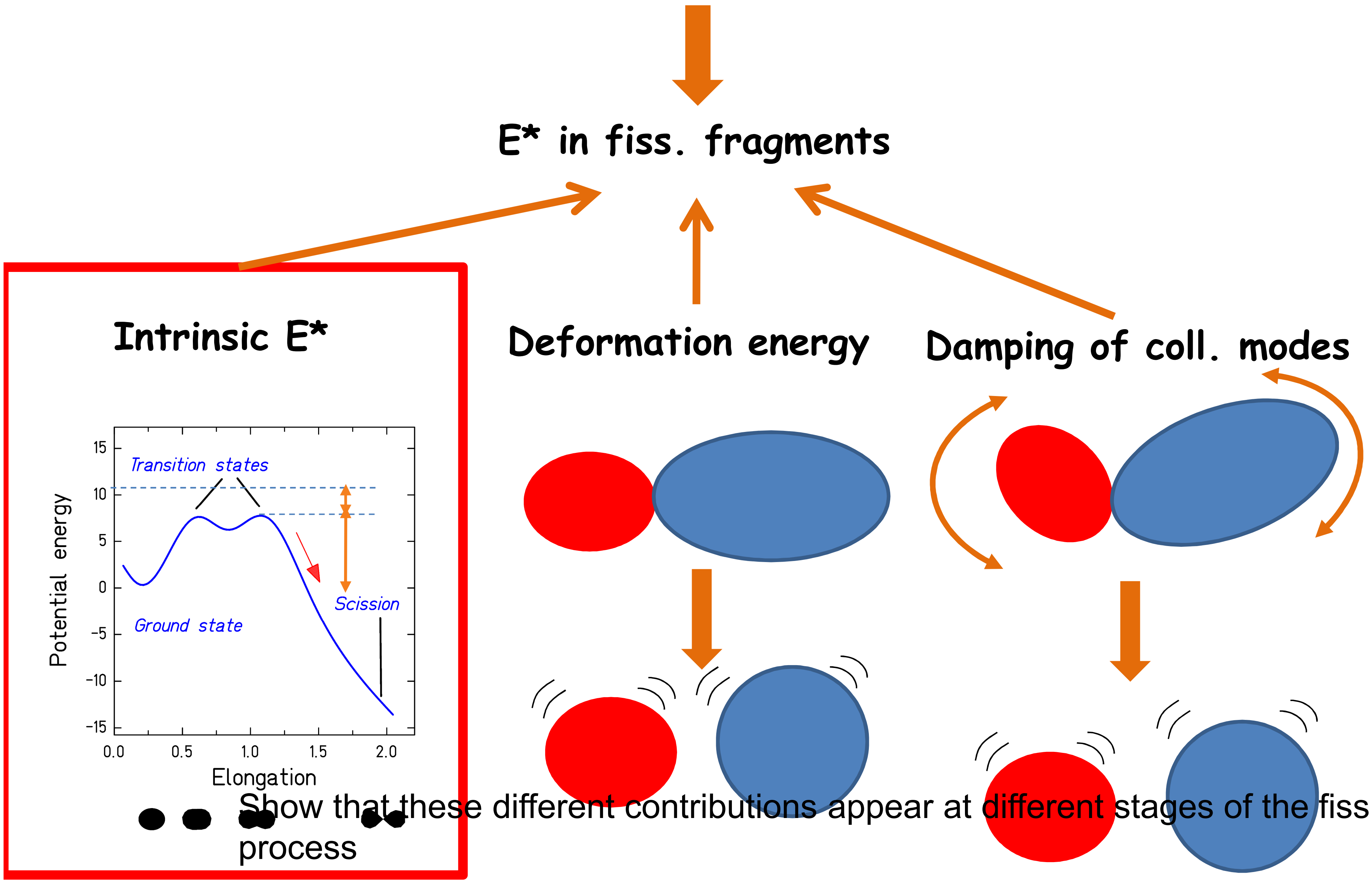
E* flow

E* keeps flowing from the hot (light) to the cold (heavy) nucleus until the E* of the hot nucleus is completely exhausted!!!

-> Process of excitation energy sorting

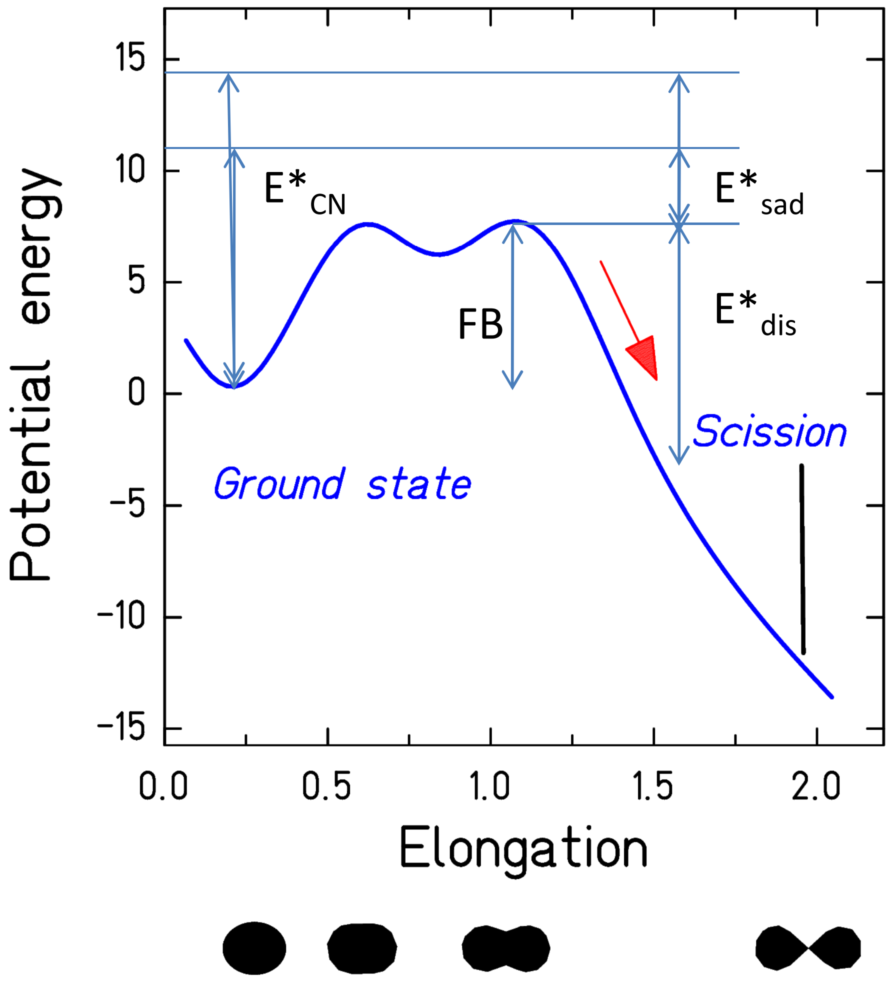
Unique! All other objects in nature reach thermal equilibrium (T1=T2) before the hot object has exhausted all its heat

Neutrons emitted by the de-excitation of fission fragments



Show that these different contributions appear at different stages of the fiss process

The fission process



Energy considerations:

•Energy at saddle $E^*_{sad}=E^*_{CN}-FB$

Intrinsic E*, “heat”

(We consider E^*_{CN} < 15 MeV)

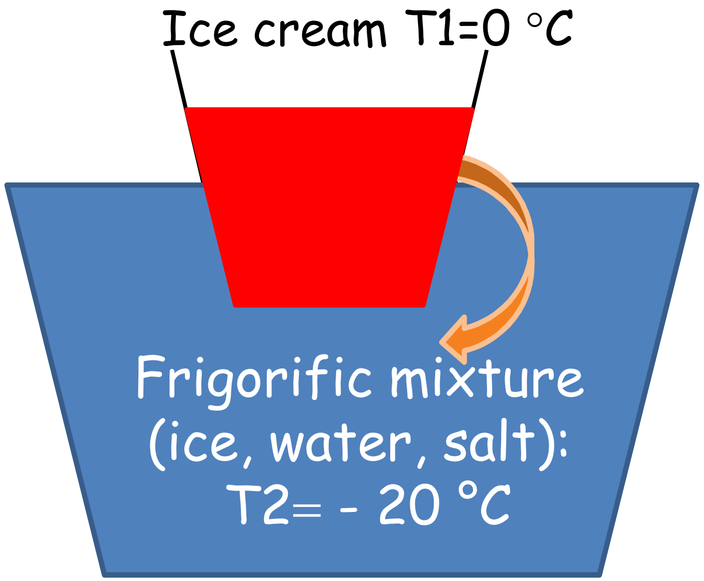
•E release from saddle to scission

-kinetic energy of fission fragments

-excitation of collective modes

-Dissipation->Intrinsic E* (heat)

Technical analogon:
The classical ice machine



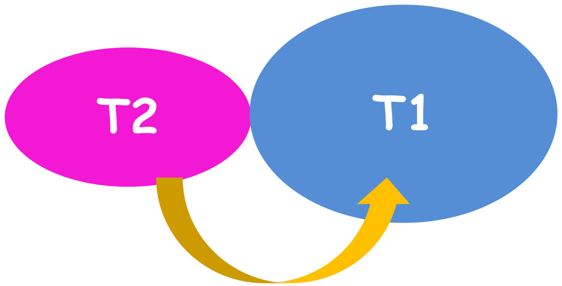
When ice cream has become ice → T1 is not cte. and T1=T2 before T1≈ 0°K!

What about entropy?

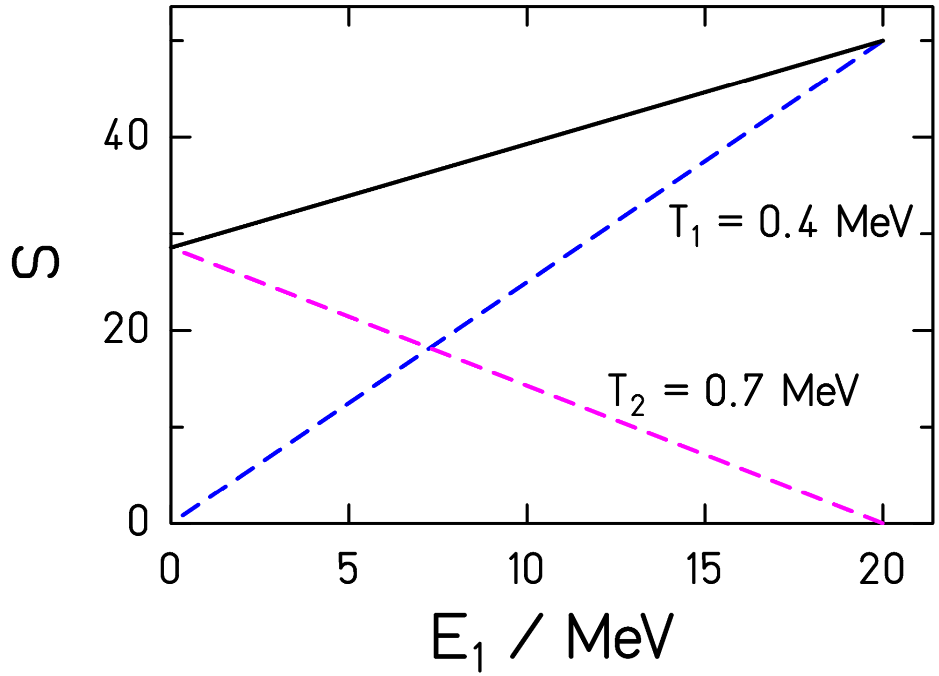
$$S = \ln \rho$$

$$S_{tot} = S_1 + S_2 = \frac{E_1}{T_1} + \frac{E_2}{T_2} = \frac{(T_2 - T_1)E_1}{T_1 T_2} + \frac{E_{tot}}{T_2}$$

T2 > T1



E* flow

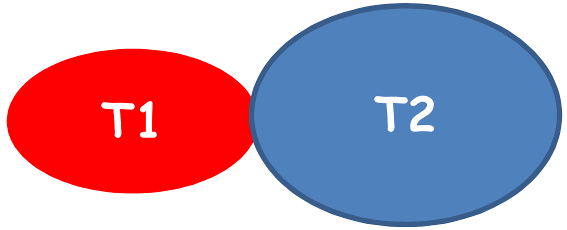


Maximum entropy obtained when all E* is found in the cold nucleus.

Energy sorting is entropy driven!

Last step of energy sorting: nucleon transfer

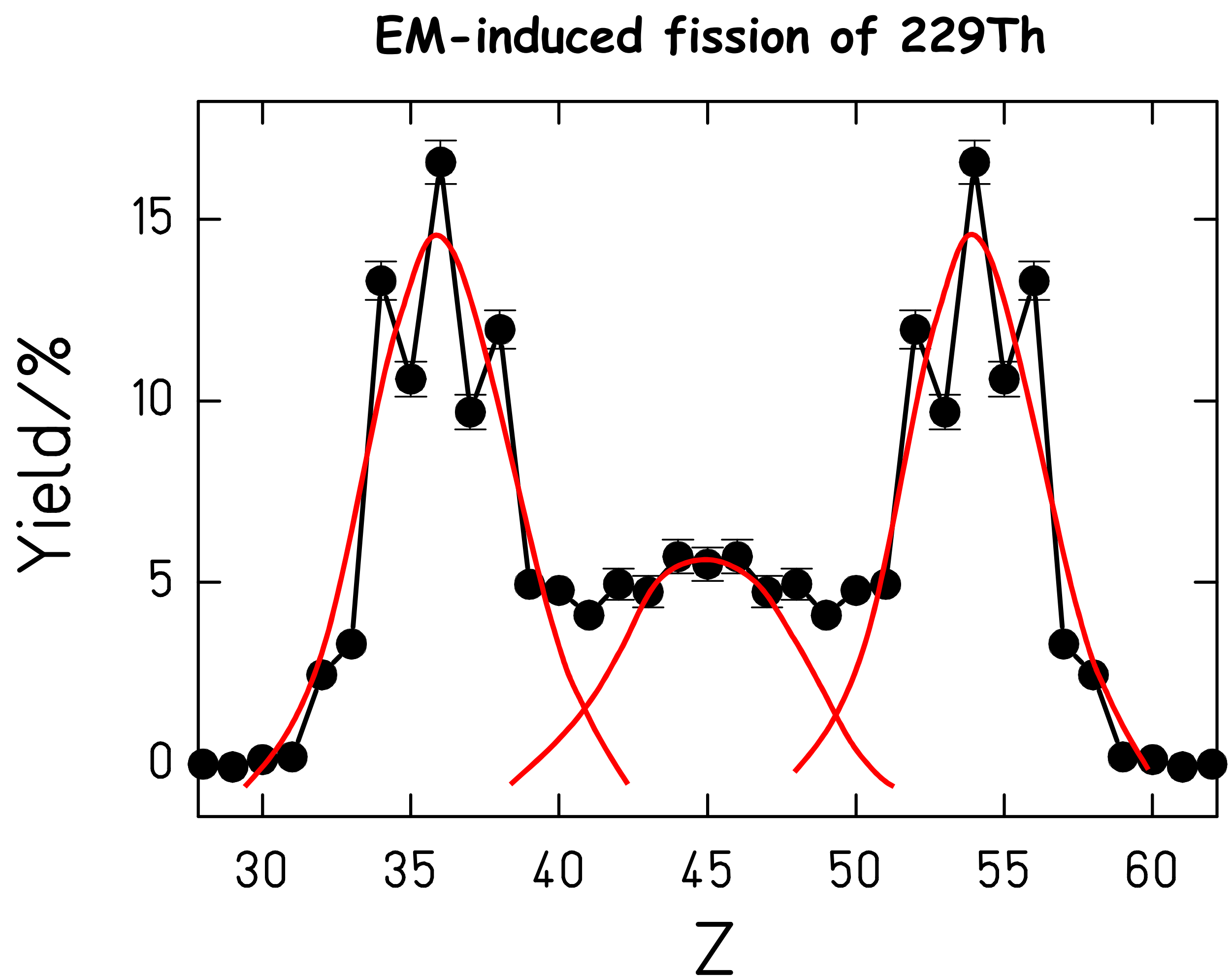
Assume the light fragment has arrived at $E^*_1 = 0$, but Z and/or N are odd.



The system can still gain considerable entropy by transferring 1 proton and /or 1 neutron to form an even-even light fragment!
Entropy gain up to $\Delta S = 2 \cdot \Delta / T2$!

The "hotter" (generally lighter) fragment tends to be even-even!!!!!

Even-Odd effect in fission-fragment yields



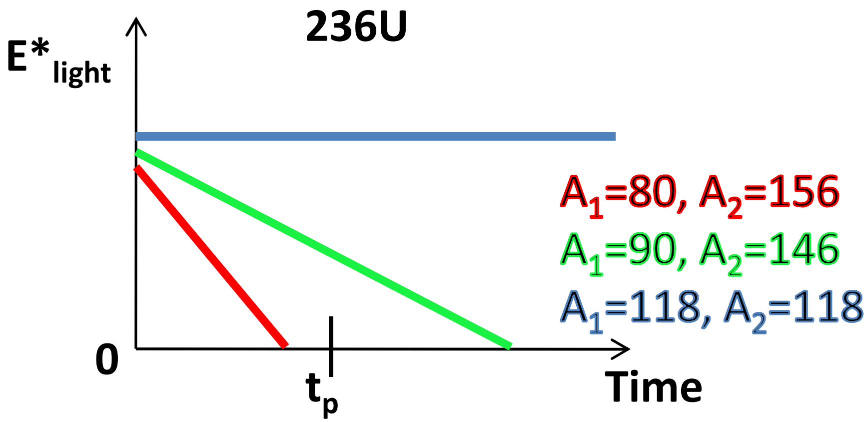
Even-odd quantified by: δ_p (local deviation with respect to gaussian)

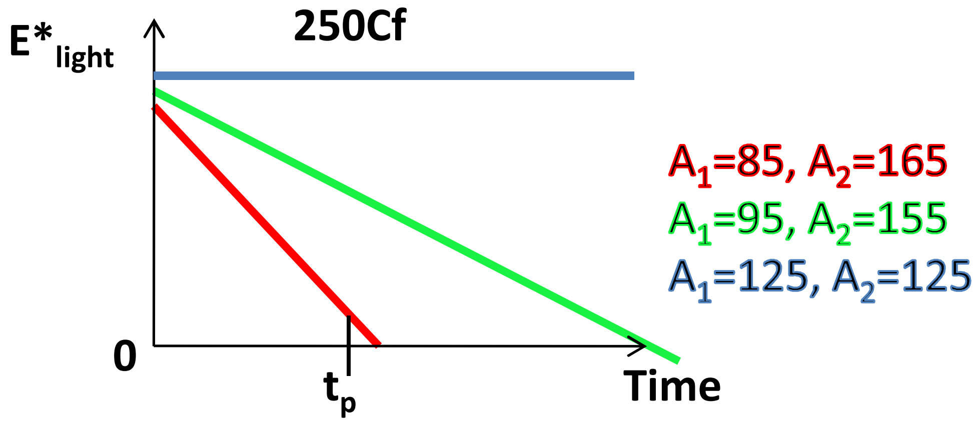
Energy sorting and even-odd (e-o) effect in fission

t→ time for E* transfer + time to transfer a protons

Amount of E* to be transferred
$E^*_{light} \propto E^*_{total} = E^*_{fb} + E^*_{sad\text{-}sci}$

Temperature difference T1-T2





$t \sim E^*_{total}/(T1-T2)$ {
T1-T2 ↑ with mass asymmetry
E^*_{total} ↑ A of fiss. nucleus and beam energy

t_p→time at which the exchange of protons through the neck is very much hindered

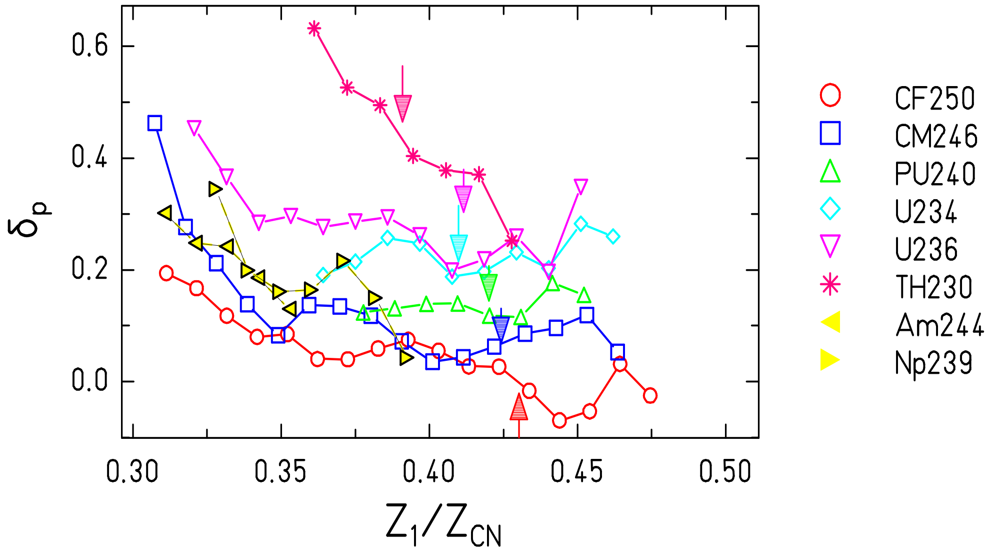
If $t>t_p$, no even-odd effect is possible !!!

- ✓e-o effect sets in at a certain asymmetry {
  which ↑ with A of fiss. nucleus
  which ↑ with E* of fiss. nucleus
- ✓Equal description for even-Z and odd-Z fissioning nuclei

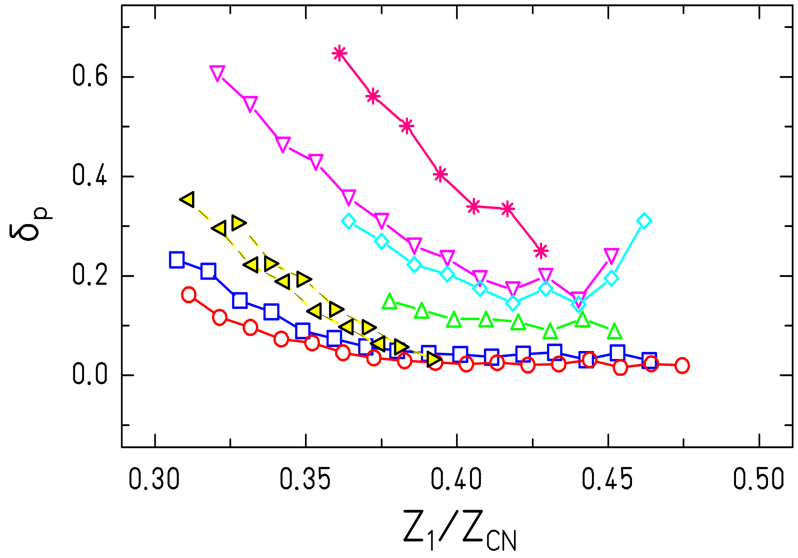
Comparison with experimental data

Experiment

Local even-odd effect



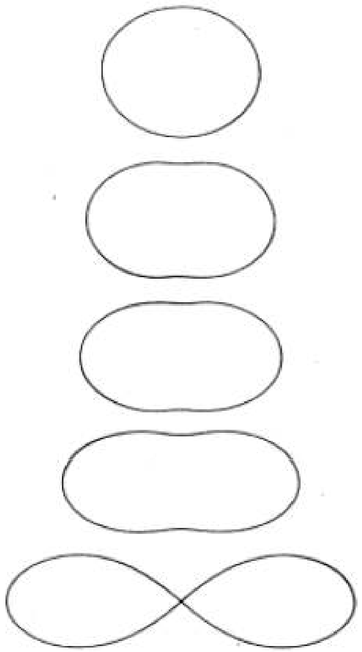
GEF calculation



(Thermal neutron-induced fission, Lohengrin)

- ✓The e-o effect increases with asymmetry
- ✓The lighter the nucleus the smaller the threshold asymmetry
- ✓Similar behavior of even-Z and odd-Z fissioning nuclei
- ✓General trends nicely reproduced with GEF code
- ✓No data to test variation of threshold asymmetry with E*

Two-centre shell-model calculations (Mosel and Schmitt, 1976)



^{236}U

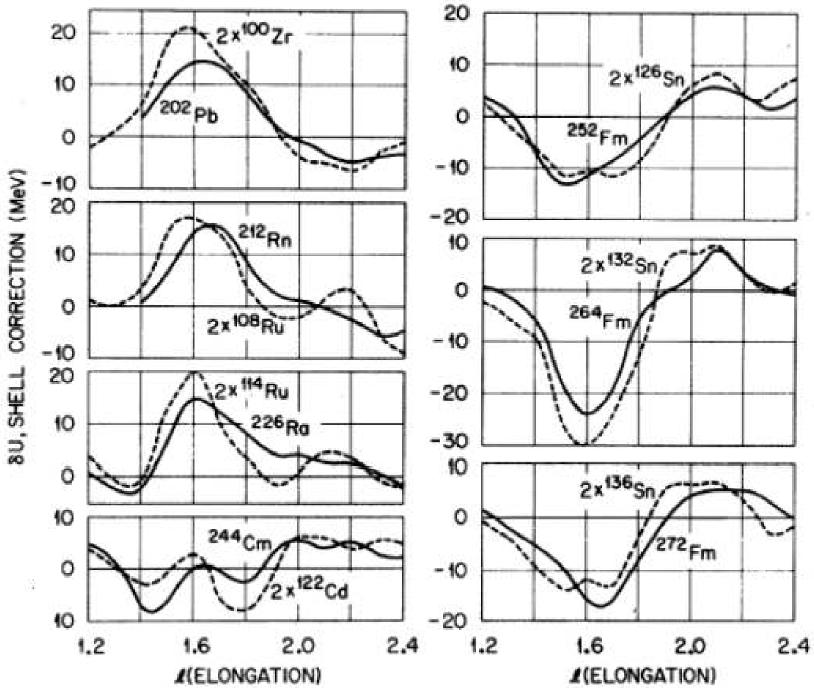


FIG. 14. Shell corrections in the realistic model at $d=0.5$ as a function of elongation (solid curves) are compared with the total shell corrections for the two independent-fragment nuclei indicated in the figure (dashed curves).

Individual structural properties of nascent fragments appear well before scission!

Non-consistent properties

	Mono-nucleus	Fragments
Pairing	$12/\mathrm{Sqrt}(A_{CN})$	$12/\mathrm{Sqrt}(A_{fragment})$
Congruence energy	(N-Z)/A	(N-Z)/A
Shells	One object	Separate objects

Theoretical studies on the gradual transition:
H. J. Krappe, S. Fadeev, Nucl. Phys. A 690 (2002) 431.
W. D. Myers, W. J. Swiatecki, Nucl. Phys. A 612 (1997) 249.
U. Mosel, H. Schmitt, Phys. Rev. C 4 (1971) 2185.

Fragments acquire their individual properties well before scission!

Energy sorting in fission:
General phenomenon without a convincing explanation.

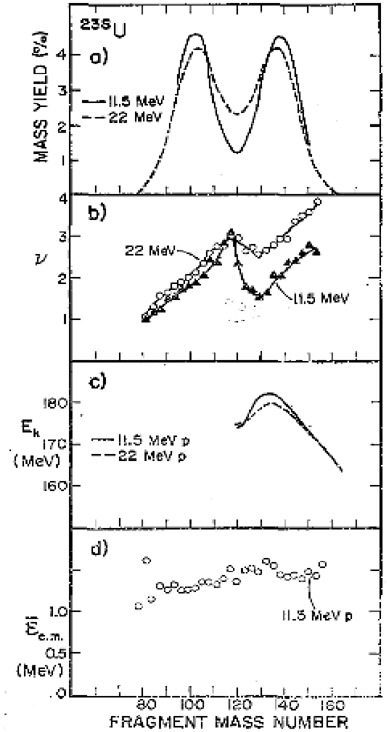


Fig. 3. The fission mass yield (a) neutron yield (b) total fragment kinetic energy (c) and average neutron c.m. energy (d) are illustrated as a function of the initial fragment mass for proton-induced fission of ^{238}U.

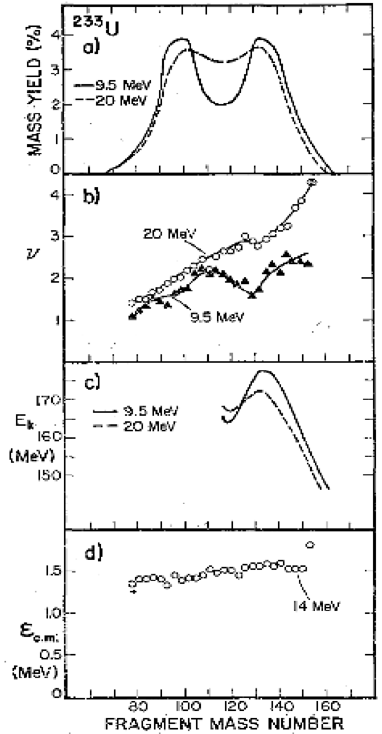


Fig. 4. The fission mass yield (a) neutron yield (b) total fragment kinetic energy (c) and average neutron c.m. energy (d) are illustrated as a function of the initial fragment mass for proton-induced fission of ^{233}U.

(From Bishop et al., 1970)

Is this Maxwell's demon on the nuclear level?

Contributions to neutron yields

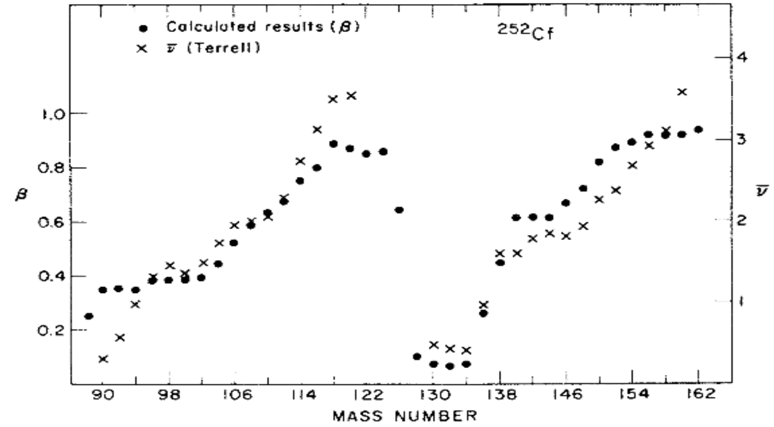
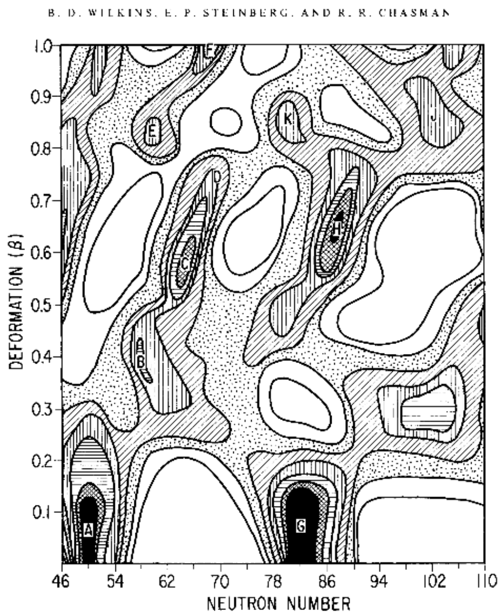


FIG. 9. The average deformation β of the fragments (symbol ●) calculated for the fissioning system ^{252}Cf compared with the results of Terrell (Ref. 32) for $\bar{\nu}(A)$ in ^{252}Cf(sf) (symbol ×).



Sawtooth shape reflects deformation at scission due to shell effects. (Wilkins et al. 1976)☑

Variation of E*initial : TKE = const.
-> Contribution of deformation to neutron yield stays constant!

Energy removed by gamma emission is small and stays constant.
-> We may neglect it.

Other contributions to neutron yields:

- o Collective excitations (normal modes)☑
- o Intrinsic excitations

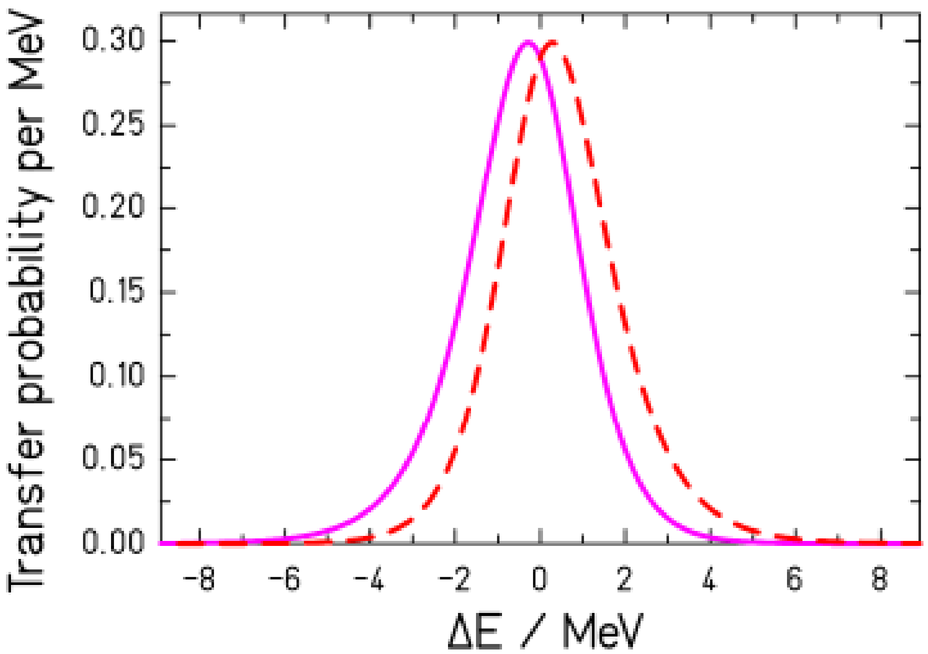
As an example, a constant single-particle level density was assumed with an occupation probability as a function of single-particle energy ε given by the Fermi-Dirac distribution:

$$f(E) = \frac{1}{\exp(\frac{\varepsilon}{T}) + 1}$$

$$f_{12} = f_{p1} \times f_{h2}$$

$$f_{p1}(E) = \frac{1}{\exp(\frac{\varepsilon}{T_1}) + 1}$$

$$f_{h2}(E) = 1 - \frac{1}{\exp(\frac{\varepsilon}{T_2}) + 1}$$



Probability function for energy transfer ΔE between two nuclei in thermal contact by thetransfer of one nucleon.. Full line: Change of excitationenergy ΔE1 of the first nucleus by the transfer of one nucleon from the first to the second nucleus.Dashed line: Change of excitation energy ΔE2 of the second nucleus by the transfer of one nucleonfrom the second to the first nucleus.

Hindrance for proton exchange through the neck

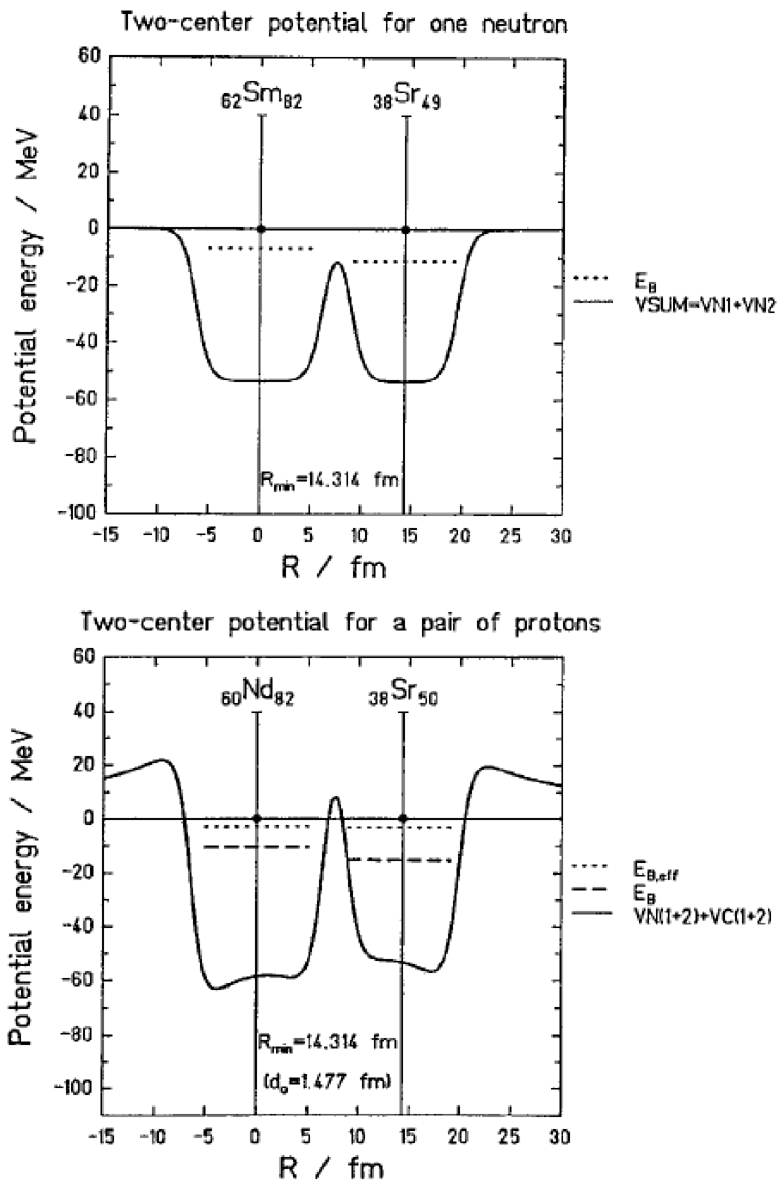


Figure 16. Two-centre wells for neutrons and protons for the system $^{88}Sr + ^{144}Sm$ for the same internuclear distances (given by d_0 values) corresponding to different heights of the internal barrier. The binding energies (and E_{eff} for the protons) are indicated.

Nuclear level density:
Independent-particle model (approximative)

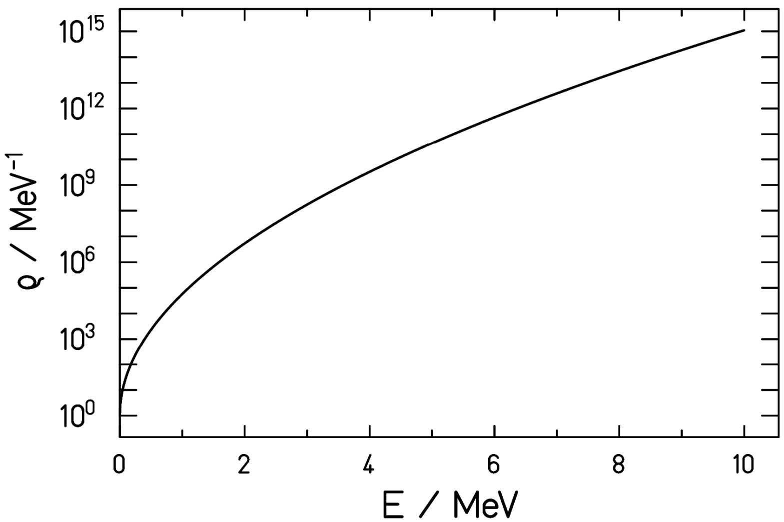
Combinatorics of different particle-hole configurations

numerical (e.g. Hilaire et al.)

analytical (Bethe, "Fermi gas", better “equidistant model”)

$$\rho \propto \exp\left(2\sqrt{(aE)}\right)$$

$$T = \left(d\,\frac{(ln\rho)}{dE}\right)^{-1}$$



$E = a\,T^2$

$a = A/10$

$E = T^2 A/10$

This leads to:

$E_1/E_2 = A_1/A_2$

Generally used!

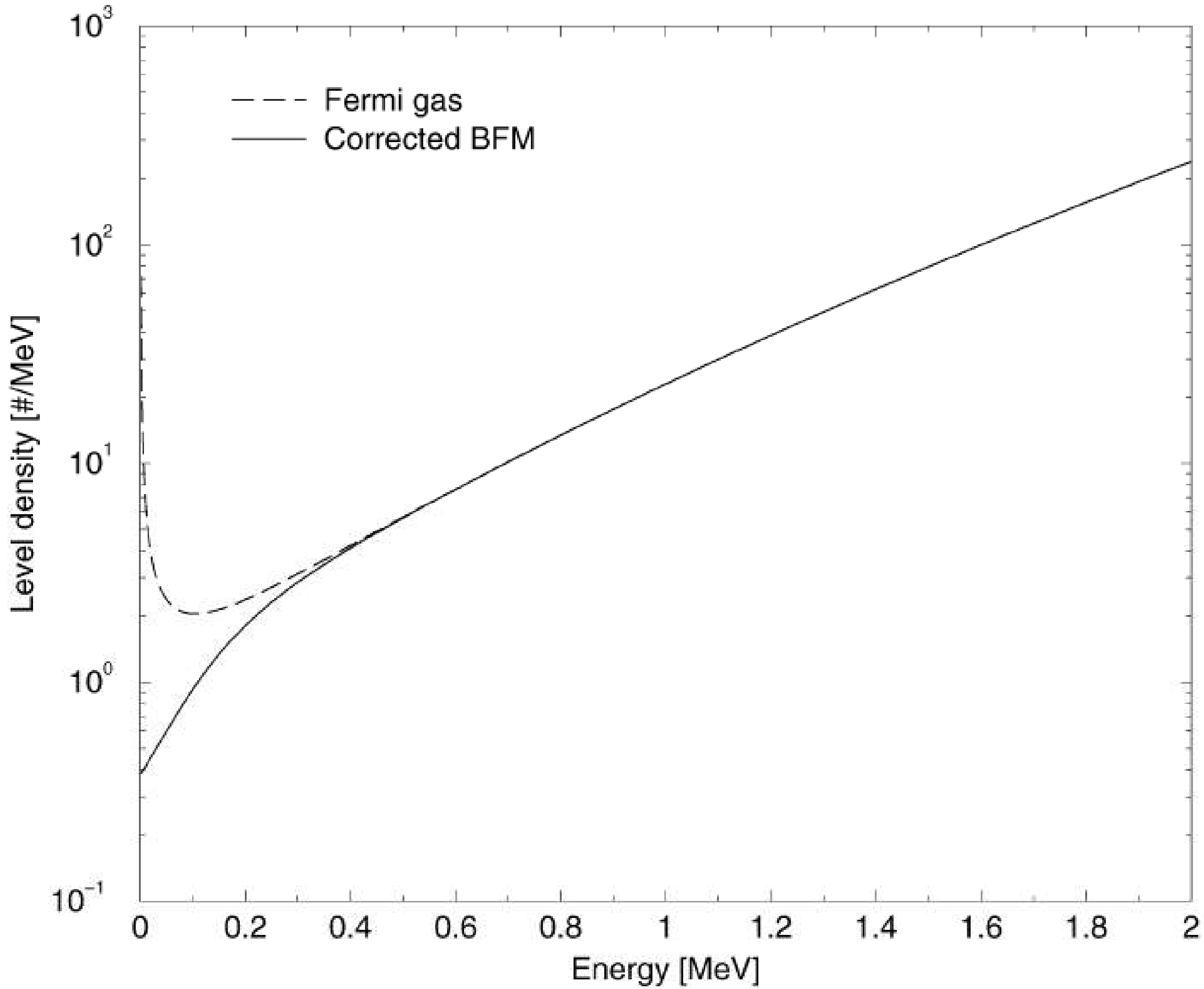
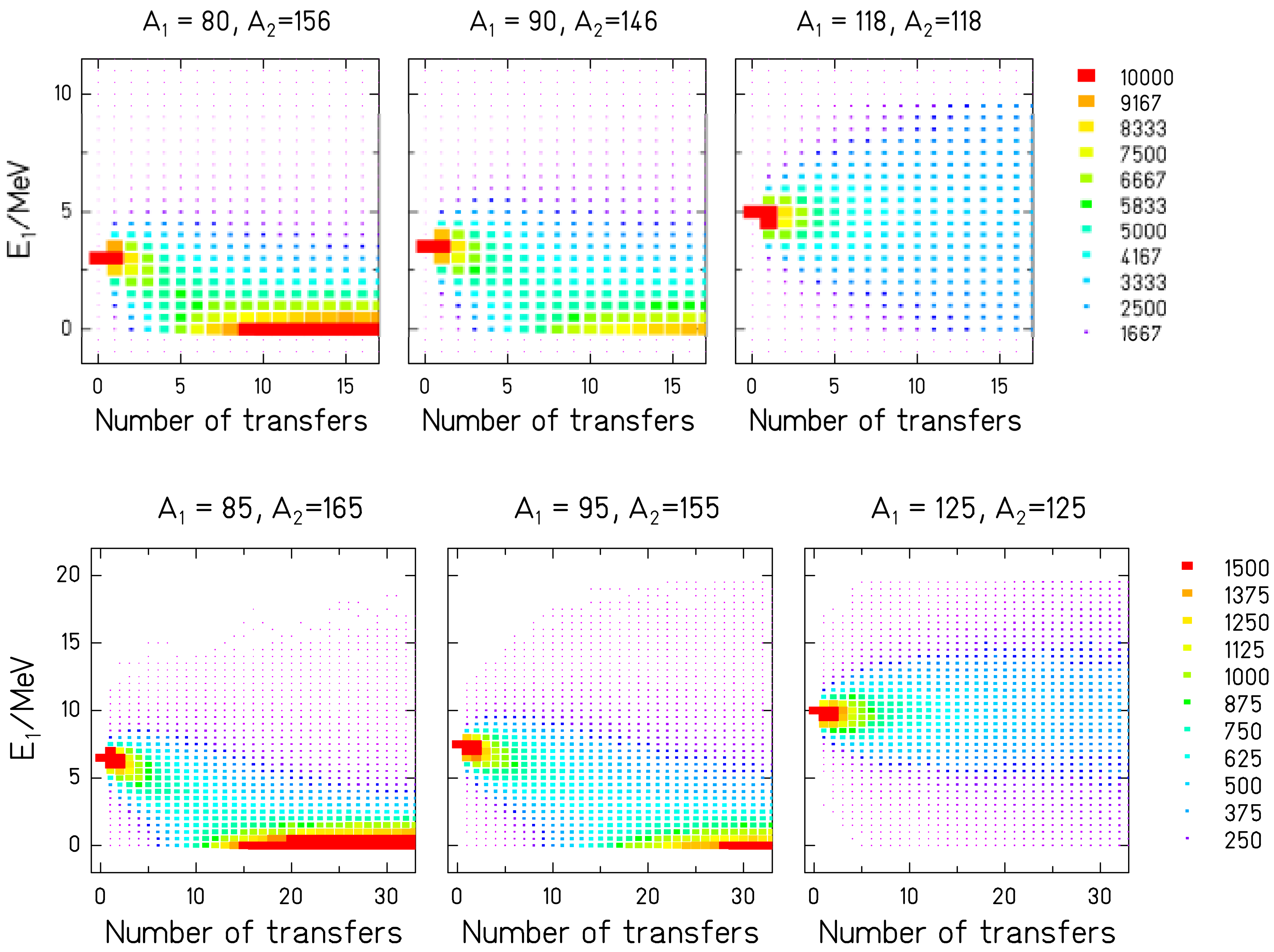


Fig. 1. Grossjean–Feldmeier correction of the Fermi gas formula at low energies for a medium mass nuclide with $a = 15\ \mathrm{MeV}^{-1}$.



A1 = 80, A2=156
A1 = 90, A2=146
A1 = 118, A2=118
E1/MeV
Number of transfers
10000
9167
8333
7500
6667
5833
5000
4167
3333
2500
1667
A1 = 85, A2=165
A1 = 95, A2=155
A1 = 125, A2=125
1500
1375
1250
1125
1000
875
750
625
500
375
250

E* sharing and level density

Fermi gas

$T \propto \mathrm{sqrt}(E^*/A)$

$T1=T2$

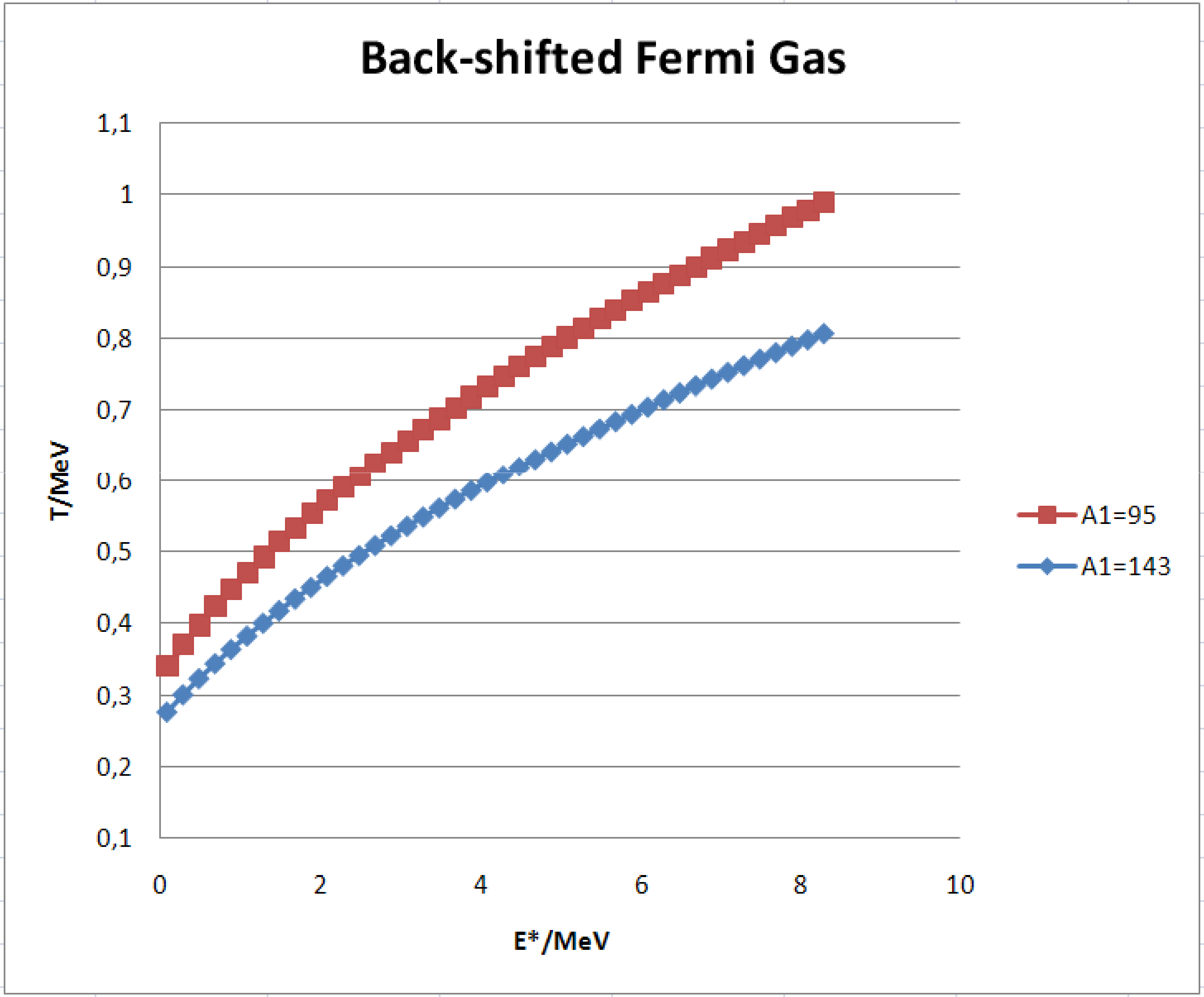
$E^*1/E^*2=A1/A2$

Constant temperature

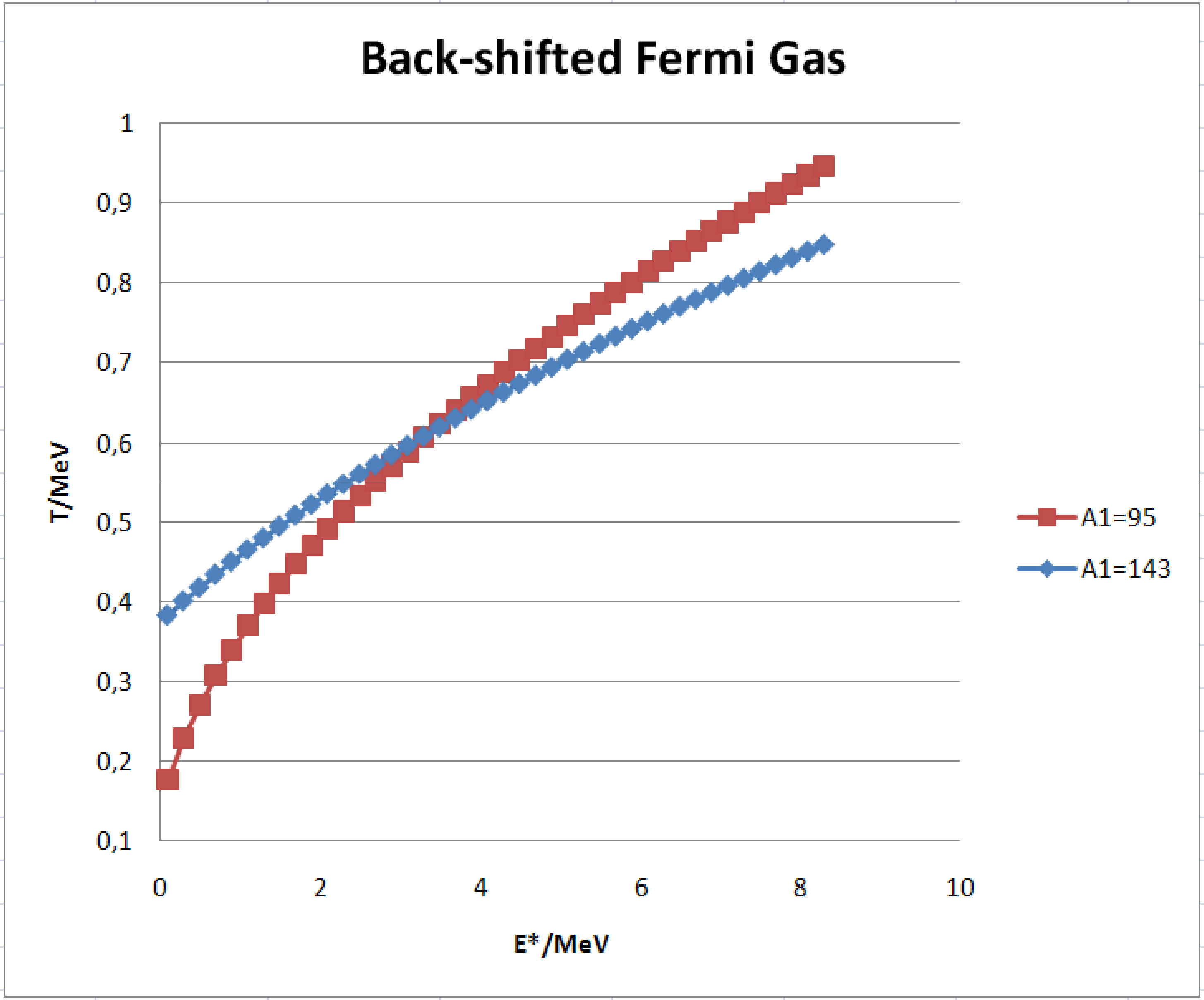
$T1 \neq T2$

Back-shifted Fermi Gas

$T=\mathrm{sqrt}[(E^*-EbF)/A]$



Back-shifted Fermi Gas
1,1
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
T/MeV
0
2
4
6
8
10
E*/MeV
A1=95
A1=143


Back-shifted Fermi Gas
T/MeV
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
2
4
6
8
10
E*/MeV
A1=95
A1=143

Division of E^*_{intr} at scission: Assumption of thermal equilibrium

Assumption:
Collective motion in fission direction is slow enough that thermal equilibrium of intrinsic excitations is maintained until scission.

Individual properties of fragments establish well before scission:

Shell effects (Maruhn, Mosel, Greiner)

Congruence energy (Myers, Swiatecki)

Pairing strength (Myers, Swiatecki)

Partition of energy governed by level density

$$T_1 = T_2$$

$$\beta = 1/T = d(\ln\rho)/dE^*$$

Thermodynamical interpretation

Fundamental meaning of *T*:

❍ean energy per (effective) degree of freedom

Ideal gas: T E

Constant number of degrees of freedom: n = const.

(Translational degrees of freedom of point-like mass objects.)

Fermi gas: T sqrt(E)

Number of degrees of freedom n_{eff} sqrt(E).

(Great part of nucleons are frozen by Pauli blocking.)

Constant temperature: T = const.

Number of degrees of freedom n_{eff} E

(Energy is used for melting pairs.)

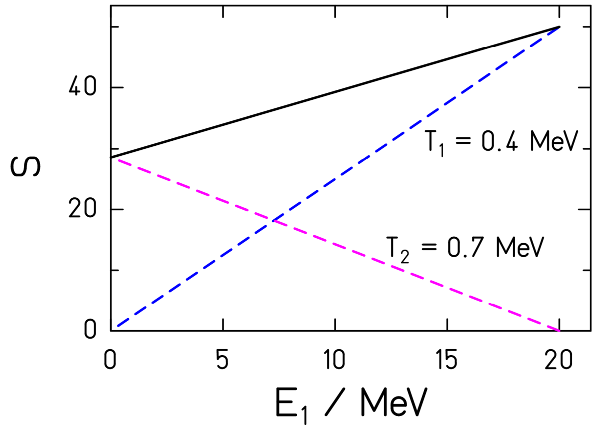
Entropy considerations

dS/dt is [illegible]
collective dissipative motion.
(General feature of Langevin
equations.)

[illegible]

$d(S_1 + S_2)/dE_1$ [illegible] is [illegible]
zero.

Energy sorting is entropy



It is more efficient (in terms of entropy) to invest all energy in the colder (heavier) fragment than to share and portion of energy with the hotter (lighter) fragment.

Hindrance for proton exchange through the neck

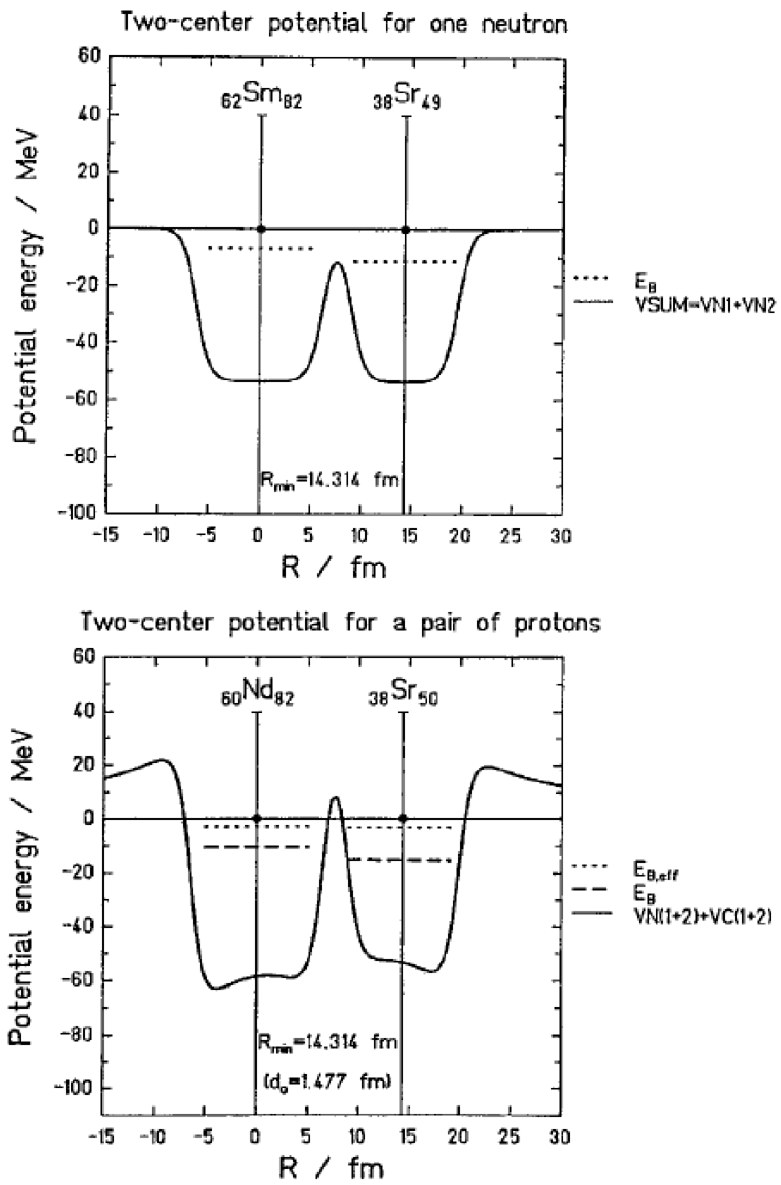


Figure 16. Two-centre wells for neutrons and protons for the system $^{88}Sr + ^{144}Sm$ for the same internuclear distances (given by d_0 values) corresponding to different heights of the internal barrier. The binding energies (and E_{eff} for the protons) are indicated.

Zu Deiner Frage, wie der Asymmetrie-assoziierte Gerade-Ungerade-Effekt in GEF ausgerechnet
wird: Bei gerade-Z spaltenden Kernen gibt es zunächst einmal einen Gerade-Ungerade-Effekt, der konstant ist über den ganzen Z-Bereich.
Der restliche Bereich (bis zu 100%) wird mit einem skalierten Gauß-Integral parametrisiert.
Sie setzt sich auf dem allgemeinen Gerade-Ungerade-Effekt auf und wächst bis 100%.
(Die Error-Funktion ist eine etwas andere Funktion, die von dem Gauß-Integral abgeleitet ist.)
Der Schwellenwert (der Wert, an dem das Gauß-Integral den Wert 1/2 hat) ist gegeben durch die Bedingung, dass die Energie-Sortierungs-Zeit
(proportional zu E*/(T2-T1)) gleich dem "dynamischen Zeitfenster" ist. Dieses dynamische Zeitfenster ist durch die kollektive Spaltdynamik gegeben.
Sie beginnt, wenn die beiden Kerne ihre individuellen Temperaturen ausbilden und sie endet, wenn der Widerstand für Protonentransport durch den Hals
zu groß wird. Der Wert ist für alle der gleiche. Er ist an die Daten angepasst. Die Breite des Gaußintegrals wächst proportional zu |T2-T1|. (Ich hatte mir
überlegt, dass diese Proportionalität physikalisch sinnvoll ist. Es ist einfach eine Skalierung.) Der Proportionalitätsfaktor ist wieder an die Daten angepasst.