Quantum chaos in nuclei and hadrons

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Contents

J.M.G. Gómez, K. Kar, V.K.B. Kota, R.A. Molina, A. Relaño, J. Retamosa, *Phys. Rep.* **499**, 103 (2011)

Quantum chaos and spectral fluctuations

The concept of chaos in Classical Mechanics can not be easily carried to Quantum Mechanics

- → A quantum system is said to be regular when its classical analogue is integrable and it is said to be chaotic when its classical analogue is chaotic
- → Berry and Tabor, *Proc. R. Soc. London* **A356**, 375 (1977)

  The spectral fluctuations of a quantum system whose classical analogue is fully integrable are well described by Poisson statistics, i.e. the successive energy levels are not correlated.

- → Bohigas, Giannoni, and Schmit, *Phys. Rev. Lett.* **52**, 1 (1984)

  CONJECTURE: Spectra of time-reversal invariant systems whose classical analogs are K systems show the same fluctuation properties as predicted by GOE.

  An analytical proof in the semiclassical framework has been obtained by Heusler *et al.* Rev. Lett. 98 (2007) 044103.

- → A. Relaño, J. M. G. Gómez, R. A. Molina, J. Retamosa, and E. Faleiro, *Phys. Rev. Lett.* **89**, 244102 (2002)

  CONJECTURE: The energy spectra of chaotic quantum systems are characterized by $1/f$ noise.

We consider three spectral statistics

→ The nearest neighbor level-spacing distribution $P(s)$

→ The spectral rigidity $\Delta_3(L)$ of Dyson and Mehta

→ The power spectrum P_k^δ of the δ_n statistic

The nearest level spacing s_i is defined by

$$s_i = \varepsilon_{i+1} - \varepsilon_i$$

and the average values are

$$\langle s \rangle = 1$$

$$\langle \varepsilon_n \rangle = n$$

There is an analogy between a discrete time series and a quantum energy spectrum, if time t is replaced by the energy E of the quantum states.

In time series analysis, fluctuations are usually studied by means of the power spectrum of the signal.

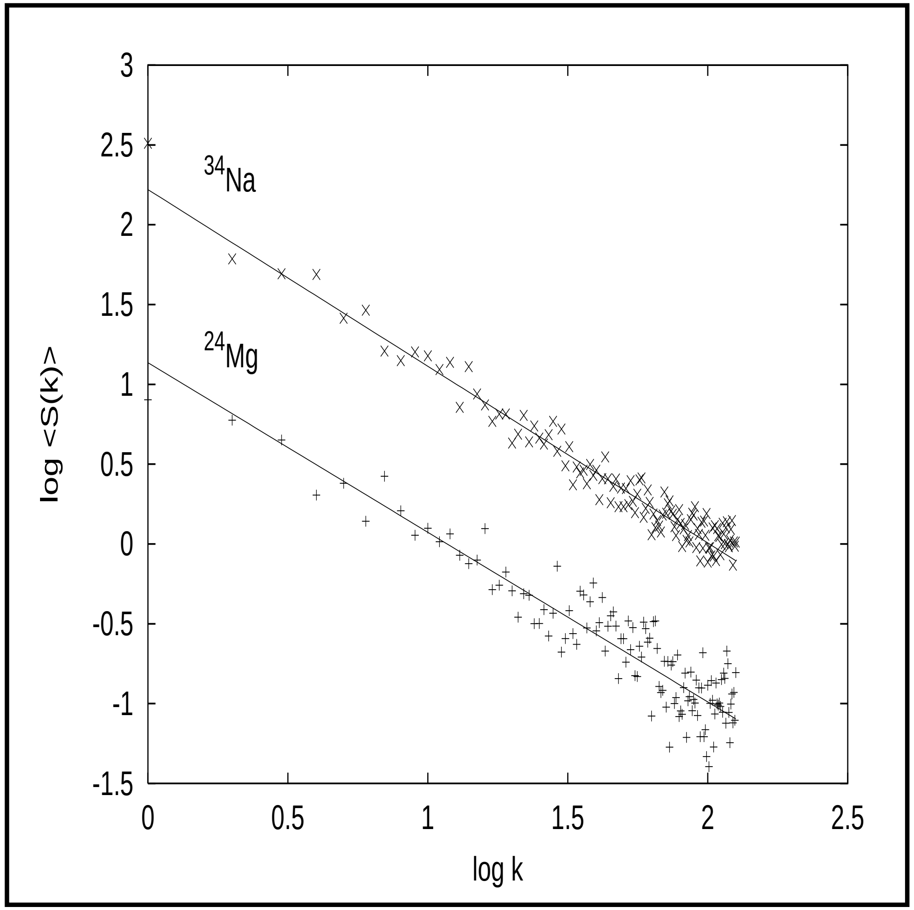
We define the statistic δ_n as a signal,

$$\delta_n = \sum_{i=1}^{n} (s_i - \langle s \rangle) = \varepsilon_{n+1} - \varepsilon_1 - n$$

and the discrete power spectrum is

$$P_k^\delta = \left| \widehat{\delta_k} \right|^2, \quad \widehat{\delta_k} = \frac{1}{\sqrt{N}} \sum_n \delta_n \exp\left(\frac{-2\pi i k n}{N} \right)$$

where $\widehat{\delta_k}$ is the Fourier transform of δ_n, and N is the size of the series.

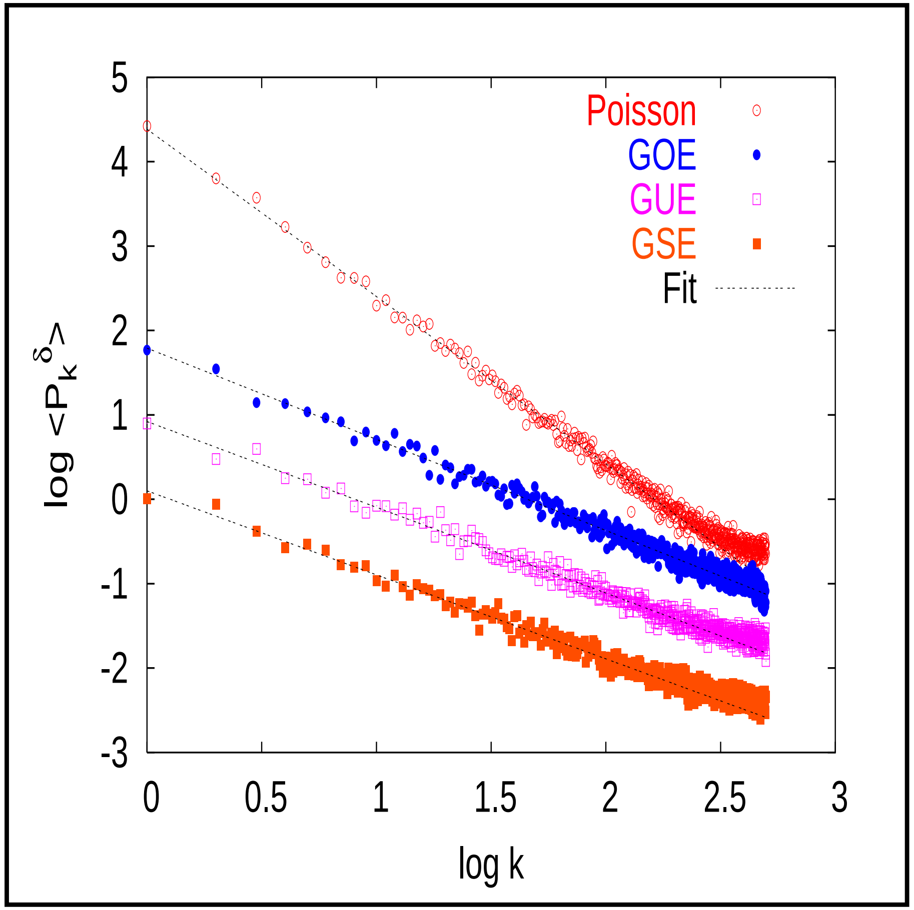


$$\langle P_k^\delta \rangle \propto \frac{1}{k^\alpha}$$

^{34}Na: $\alpha = 1.11$

^{24}Mg: $\alpha = 1.06$

A. Relaño, J. M. G. Gómez, R. A. Molina, J. Retamosa, and E. Faleiro, *Phys. Rev. Lett.* **89**, 244102 (2002)



$$\langle P_k^\delta \rangle \propto \frac{1}{k^\alpha}$$

Poisson: $\alpha = 2.00$

GOE: $\alpha = 1.08$

GUE: $\alpha = 1.02$

GSE: $\alpha = 1.00$

RMT formula for P_k^δ

$$\langle P_k^\delta \rangle_\beta = \frac{N^2}{4\pi^2}\left[\frac{K_\beta\left(\frac{k}{N}\right)-1}{k^2}+\frac{K_\beta\left(1-\frac{k}{N}\right)-1}{(N-k)^2}\right]+\frac{1}{4\sin^2\left(\frac{\pi k}{N}\right)}+\Delta, \quad N\gg 1$$

$$\beta = \begin{cases} 0 & \text{Poisson} \\ 1 & \text{GOE} \\ 2 & \text{GUE} \\ 4 & \text{GSE} \end{cases} \qquad \Delta = \begin{cases} -\dfrac{1}{12}, & \text{for RMT} \\ 0, & \text{for Poisson} \end{cases}$$

E. Faleiro, J. M. G. Gómez, R. A. Molina, L. Muñoz, A. Relaño, and J. Retamosa, *Phys. Rev. Lett.* **93**, 244101 (2004)

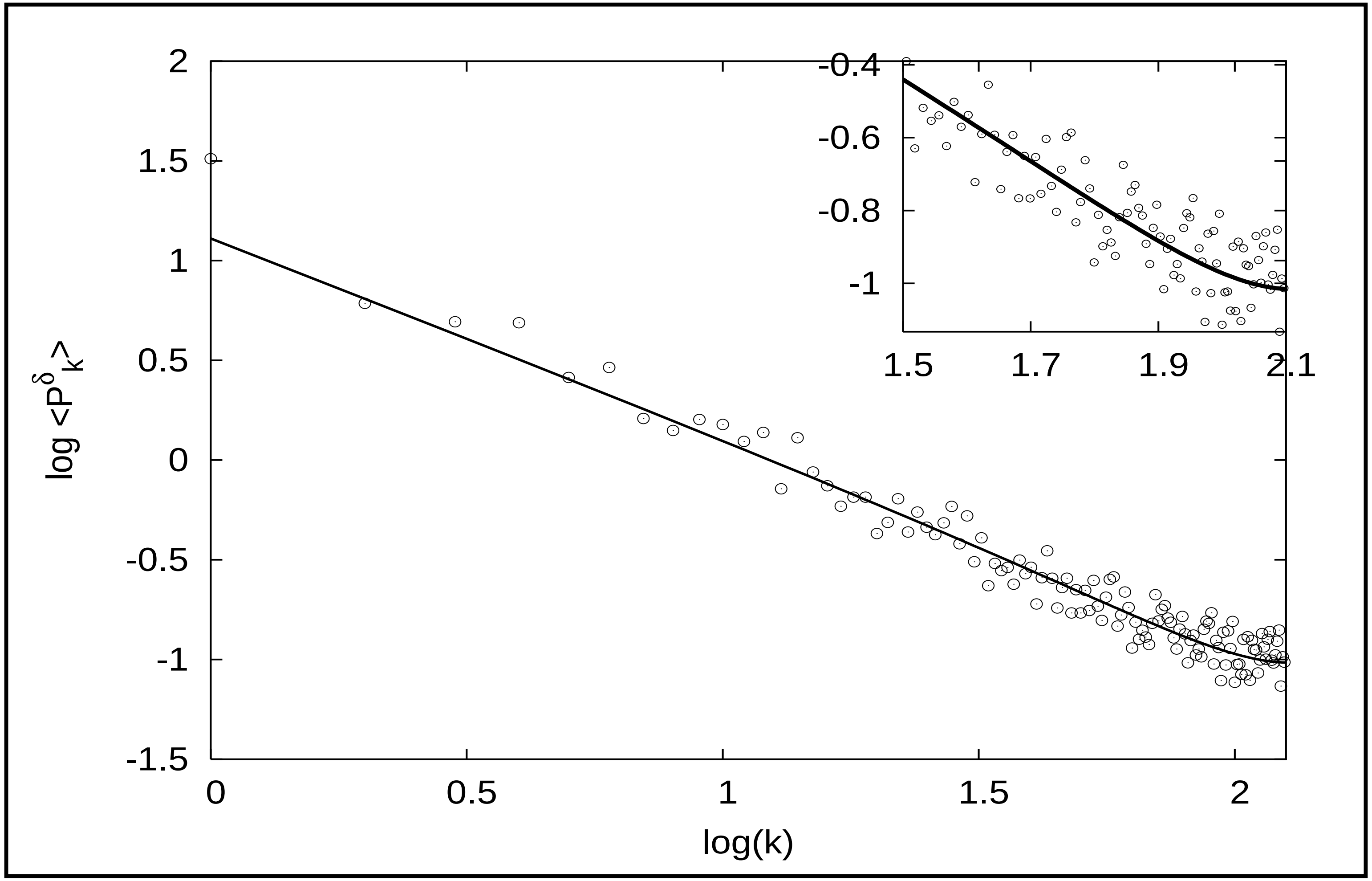
$$K(\tau) = \left\langle \lim_{L\to\infty} \frac{1}{2L} \left| \int_{-L}^{L} d\epsilon \widetilde{\rho}(\epsilon) e^{-2\pi i \epsilon \tau} \right|^2 \right\rangle$$

RMT formula for small frequencies

$$K_\beta(\tau) \simeq \frac{2\tau}{\beta}, \qquad \tau \ll 1.$$

$$\left\langle P_k^\delta \right\rangle_\beta = \begin{cases} \dfrac{N}{2\beta\pi^2 k}, & \text{for chaotic systems} \quad \Rightarrow 1/f \;\; \text{noise} \\ \dfrac{N^2}{4\pi^2 k^2}, & \text{for integrable systems} \Rightarrow 1/f^2 \;\; \text{Brown noise} \end{cases}$$

Theoretical vs numerical P_k^δ values for ^{34}Na



E. Faleiro *et al.* Phys. Rev. Lett. **93** (2004) 244101.

Features of the $1/f$ conjecture

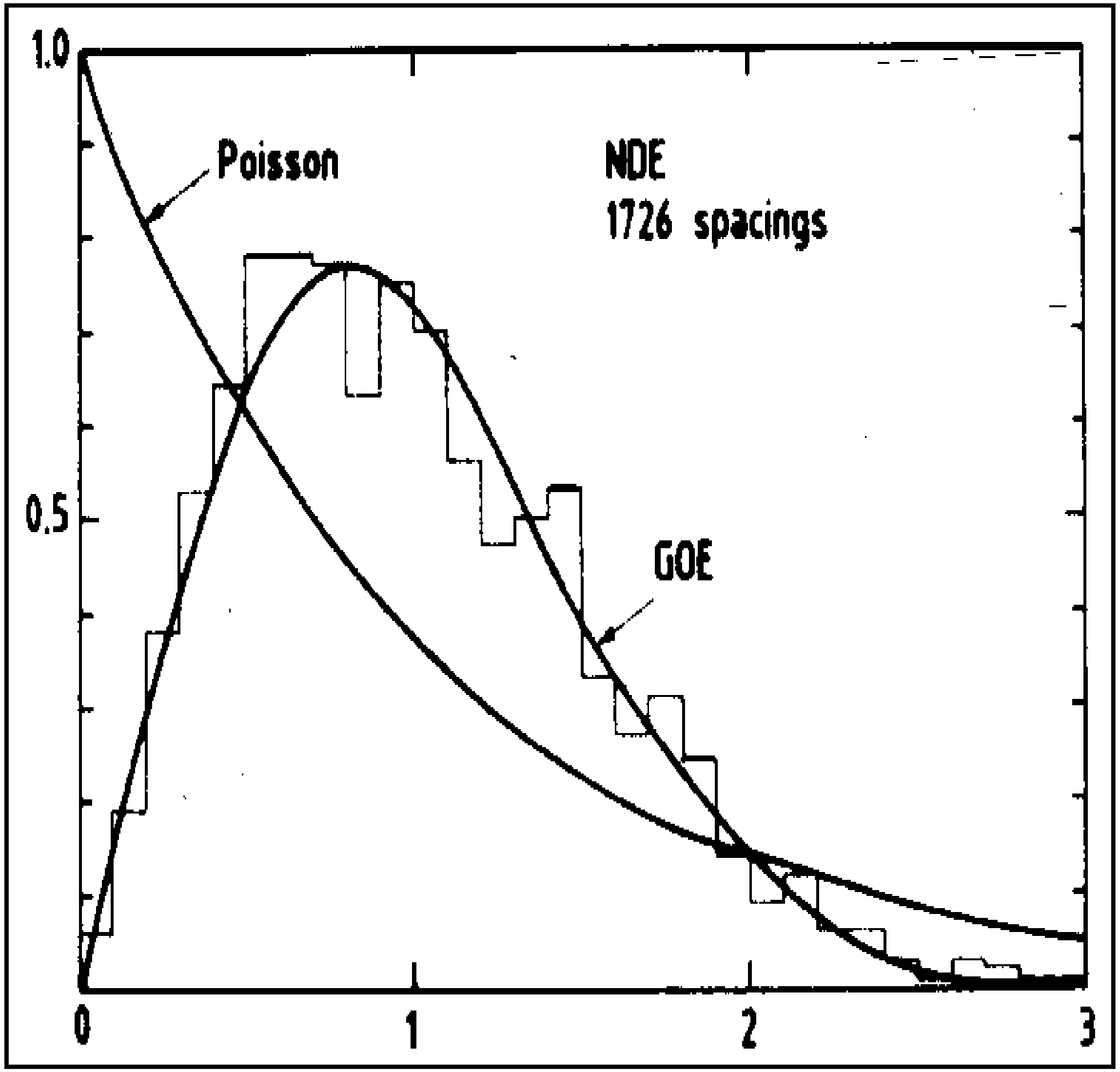
→ The $1/f$ noise is an intrinsic property characterizing the chaotic spectrum by itself, without any reference to the properties of other systems such as GOE.

→ The $1/f$ feature is universal, i.e. this behavior is the same for all kinds of chaotic systems, independently of their symmetries: either time-reversal invariant or not, either of integer or half-integer spin.

Quantum chaos in nuclei: experimental results

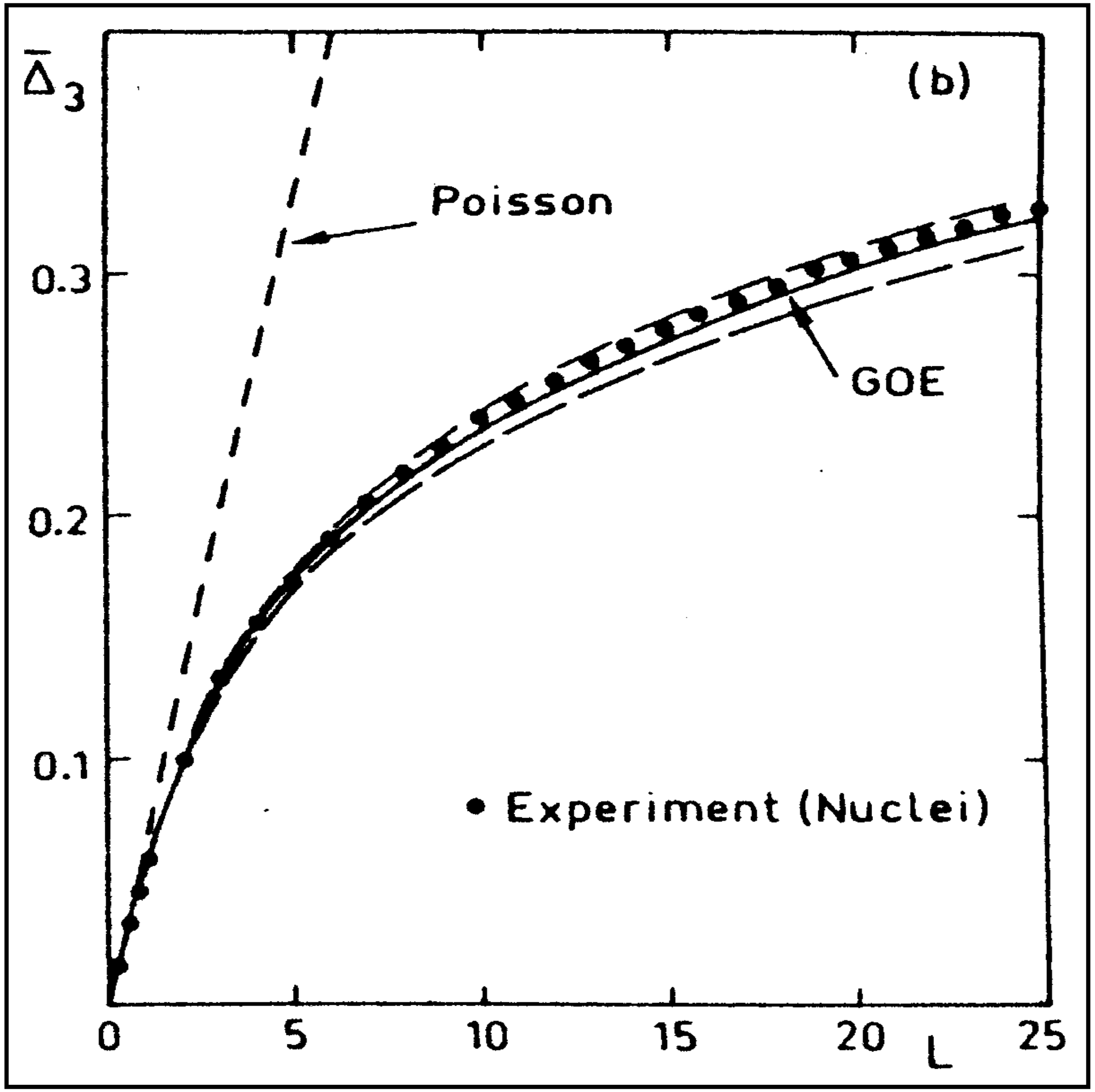
- → The analysis of spectral fluctuations in nuclei requires energy level sequences that are pure, complete and as long as possible. Pure means that all the states have the same quantum numbers (J, π, $T, \ldots$). Complete means that there are no missing levels.
- → In practice, there are strong difficulties to analyze the spectral fluctuations of experimental bound states. It is usually done with very short sequences because experimental data are plagued with uncertain or unknown quantum numbers, and there are also possible missing levels.
- → In the region of slow neutron and proton resonances the known level sequences with $J^\pi = 1/2^+$ are quite large, and thus the statistical analysis is easier and more reliable.

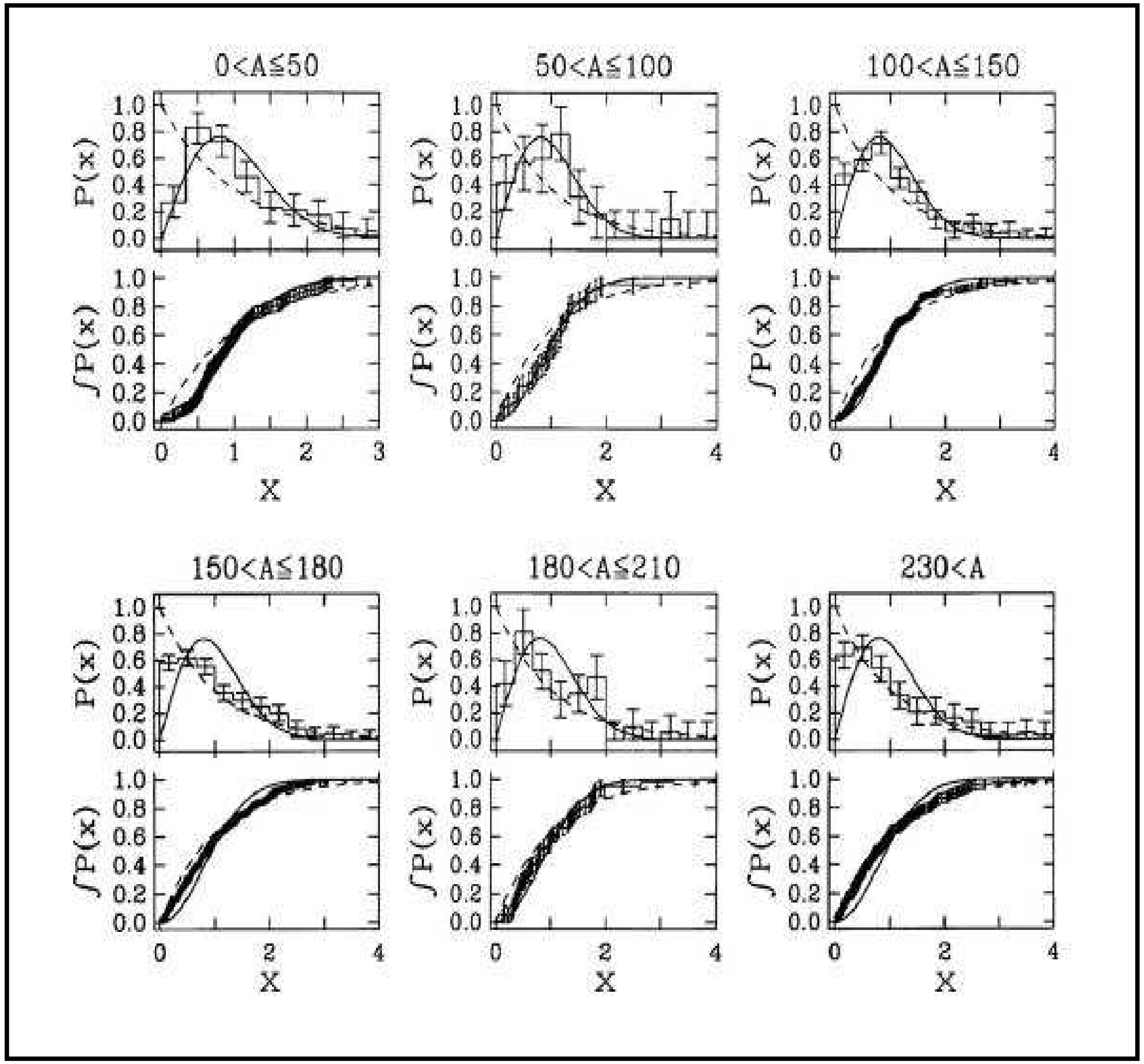
$P(s)$ distribution for the nuclear data ensemble (NDE)



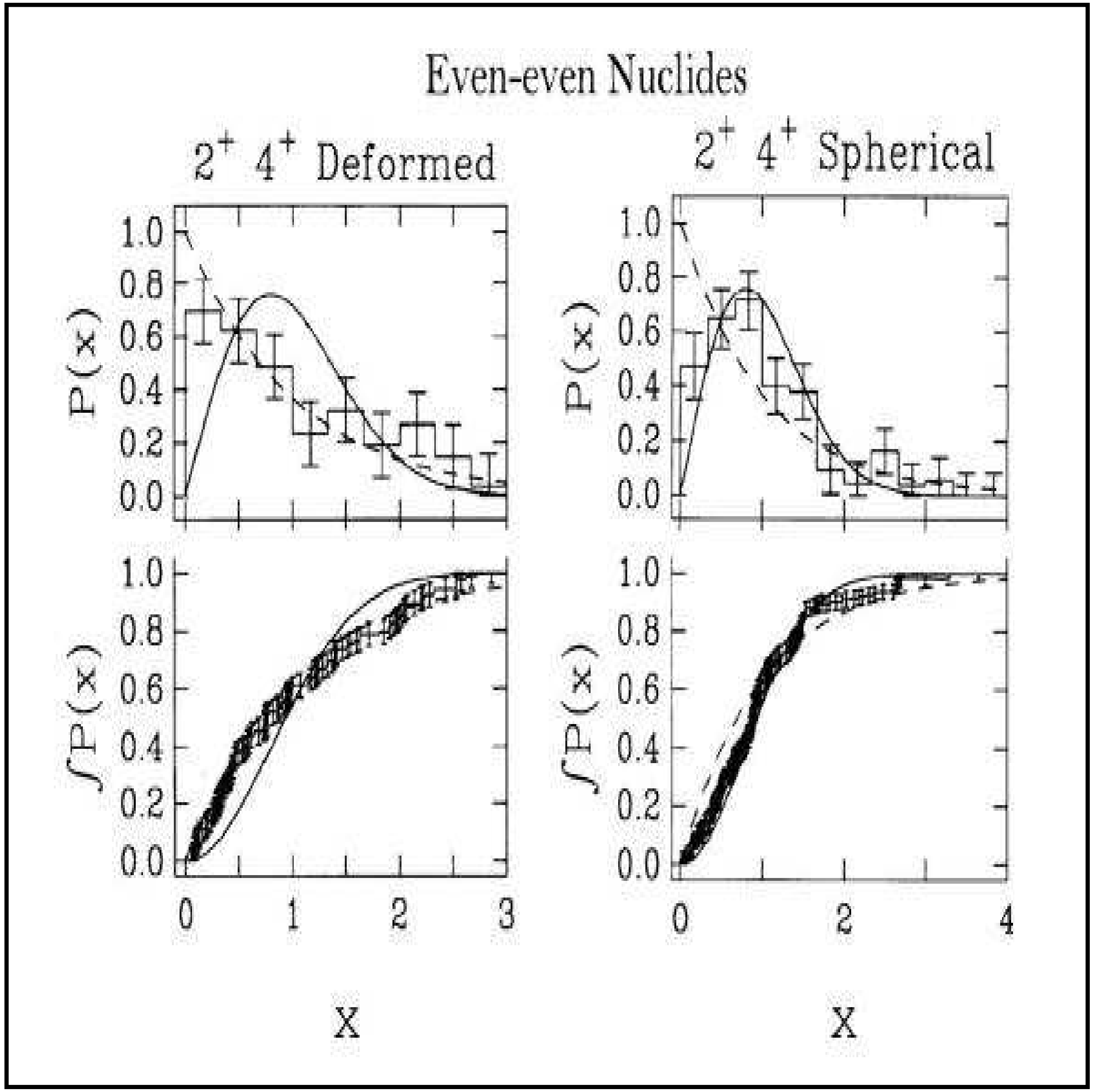
Bohigas, Haq and Pandey, *Nuclear Data for Science and Technology* (1983)

The $\langle \Delta_3(L) \rangle$ statistic for NDE





J.F. Shriner *et al.*, Z. Phys. A 338, 309 (1991)


Even-even Nuclides
2+ 4+ Deformed
2+ 4+ Spherical
P(x)
∫P(x)
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
X

Quantum chaos in nuclei: shell-model results

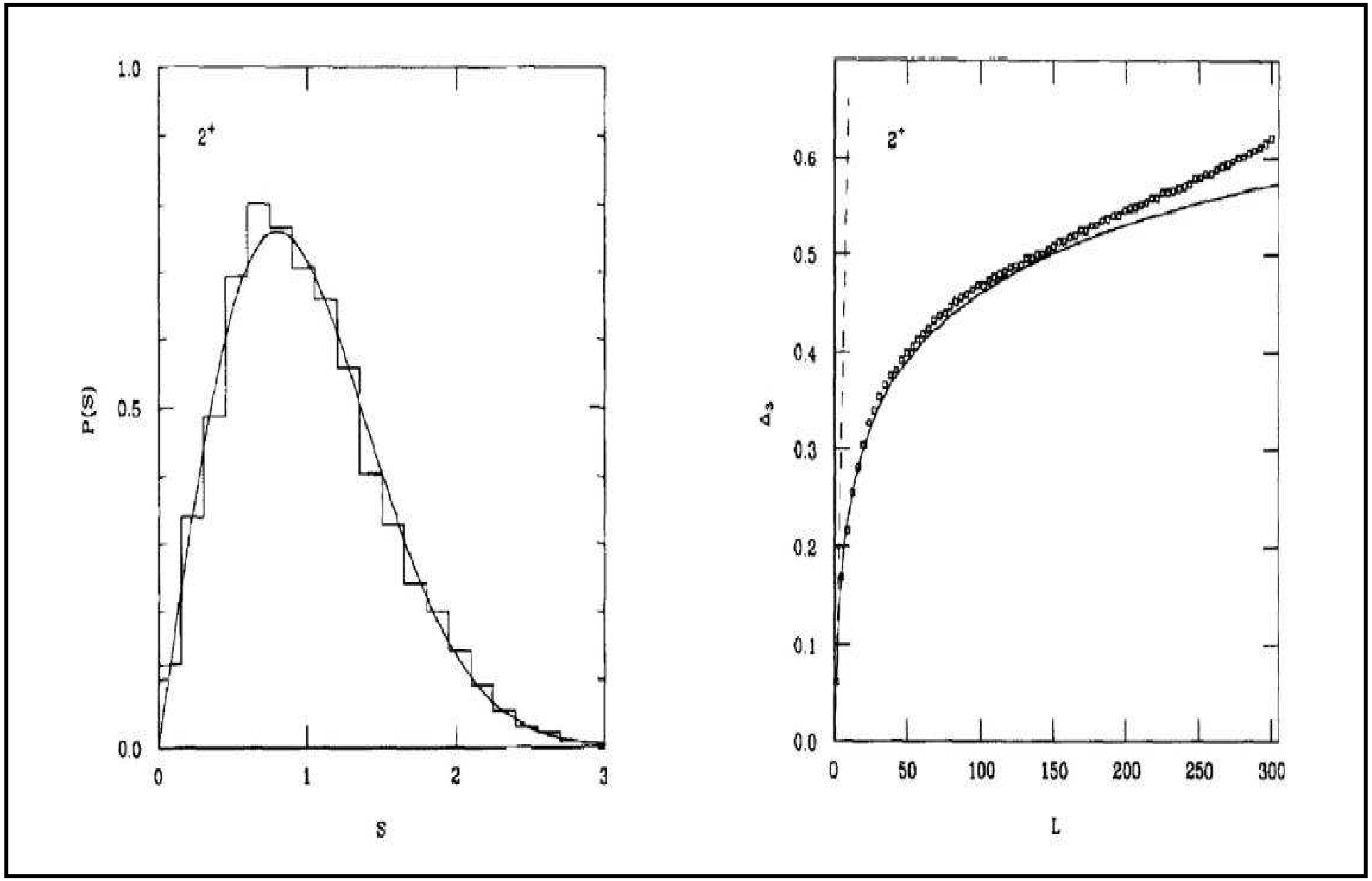
→ Large-scale shell-model calculations provide very long pure and complete sequences of energy levels.

→ This makes possible the study of long-range spectral correlations and in general the statistical analysis is quite reliable.

→ For large valence spaces, like the pf shell, one can also study the dependence of the statistical properties on the excitation energy, angular momentum, isospin, etc.

→ On the other hand, detailed agreement with the experimental levels is good only for the lower energy region.

Shell-model $P(s)$ and $\langle \Delta_3(L) \rangle$ values for the $J^\pi = 2^+, T = 0$ states of ^{28}Si



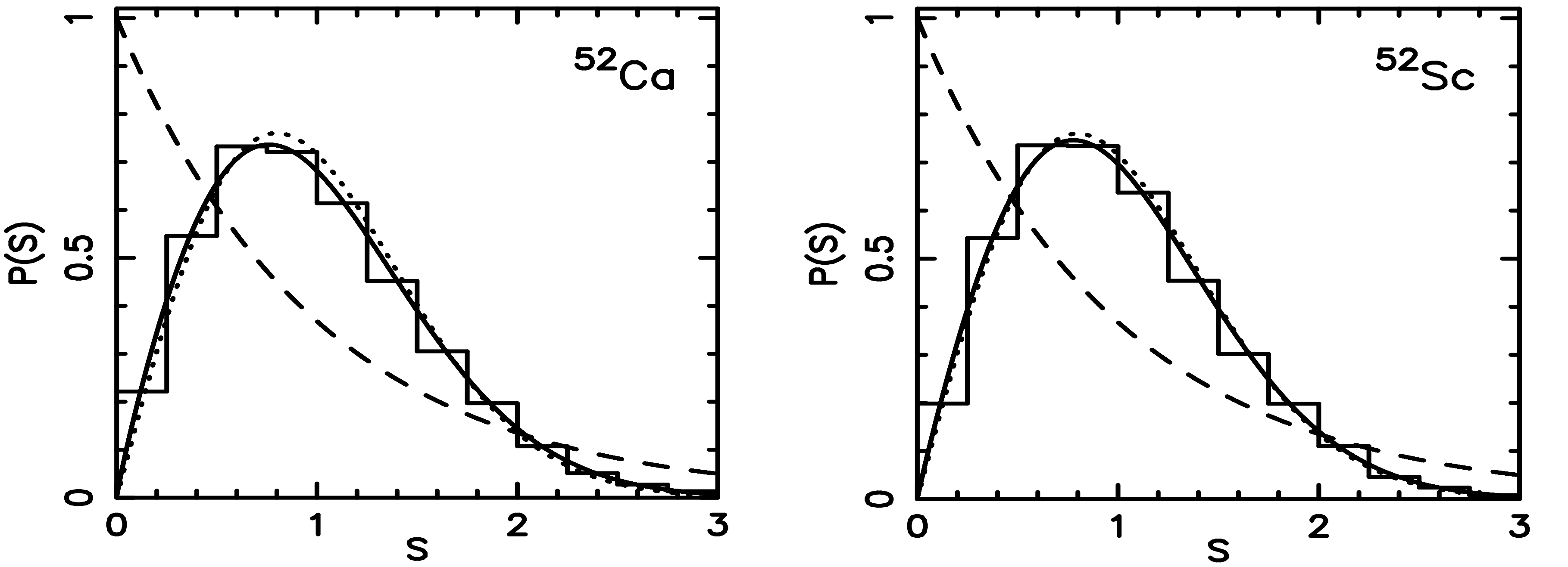
V. Zelevinsky *et al.*, Phys. Rep. 276, 85 (1996)

Shell-model $P(s)$ values for ^{52}Ca

$J^{\pi} = 0^{+} - 12^{+}, T = 6$ states

Shell-model $P(s)$ values for ^{52}Sc

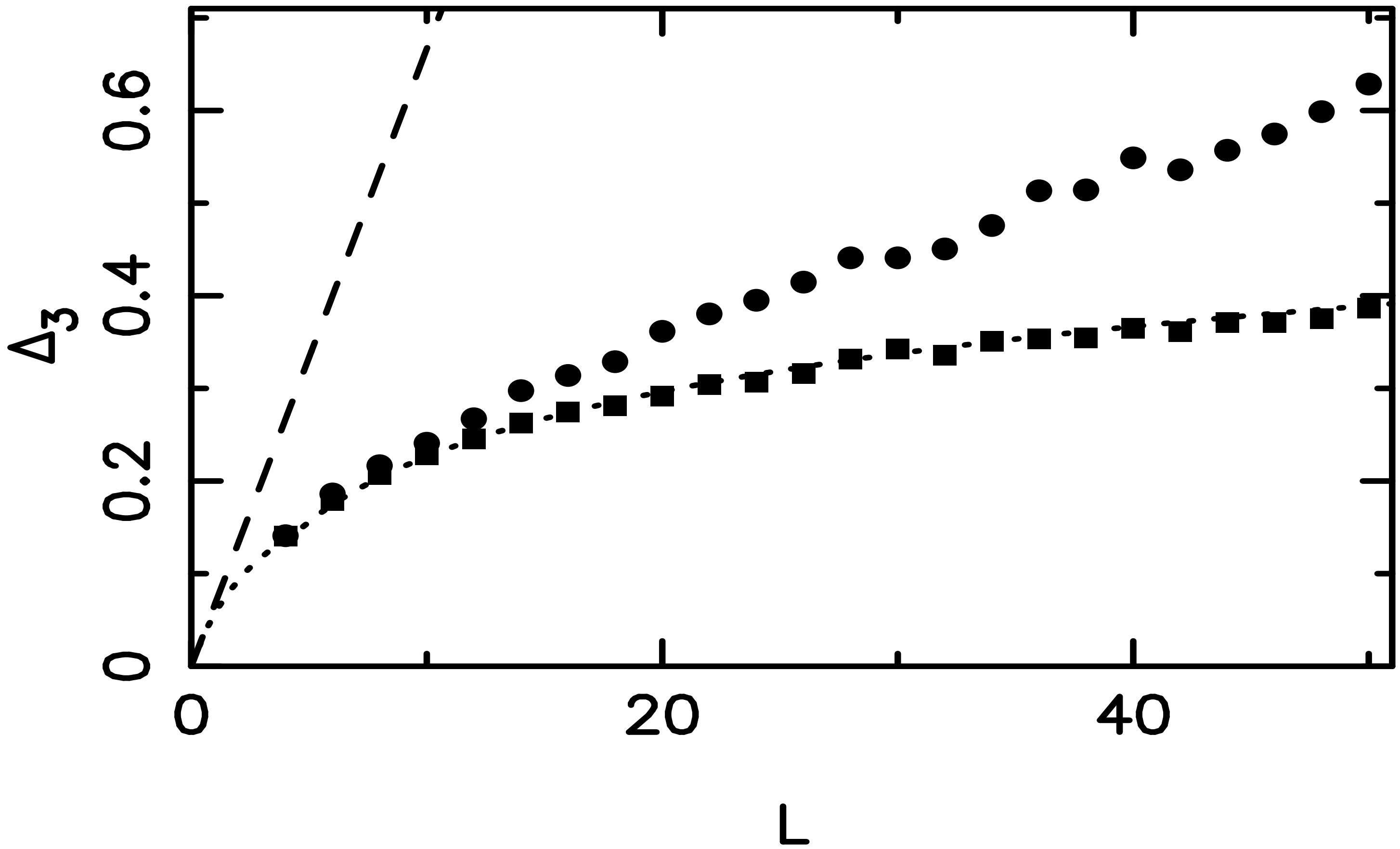
$J^{\pi} = 0^{+}, 11^{+}, 12^{+}, T = 4$



R. A. Molina *et al.*, Phys. Rev. C 63, 014311 (2000)

Shell-model $\langle \Delta_3(L) \rangle$ values for ^{52}Ca (dots) and ^{52}Sc (squares)

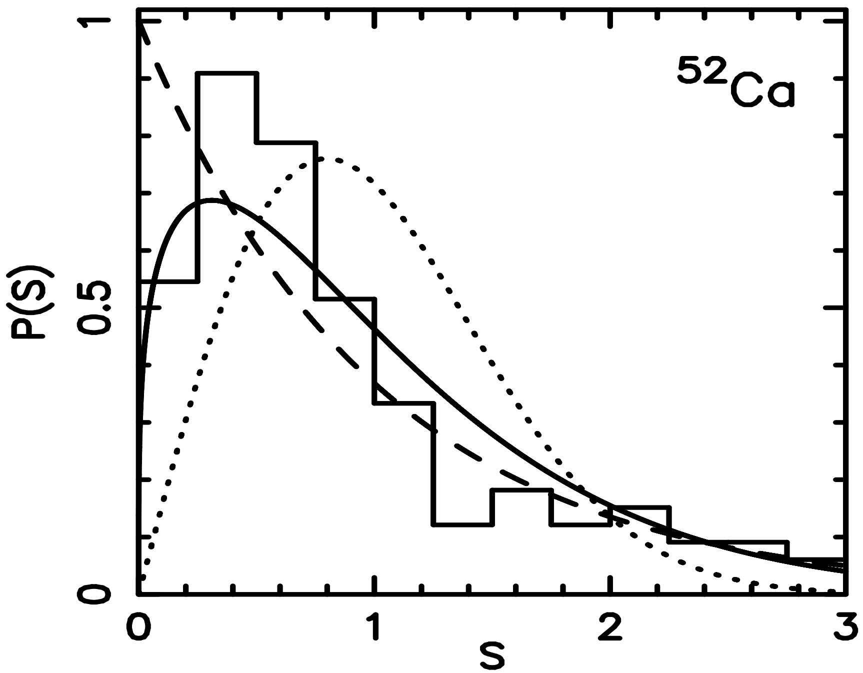
$J^\pi = 0^+$, $T = T_z$ states

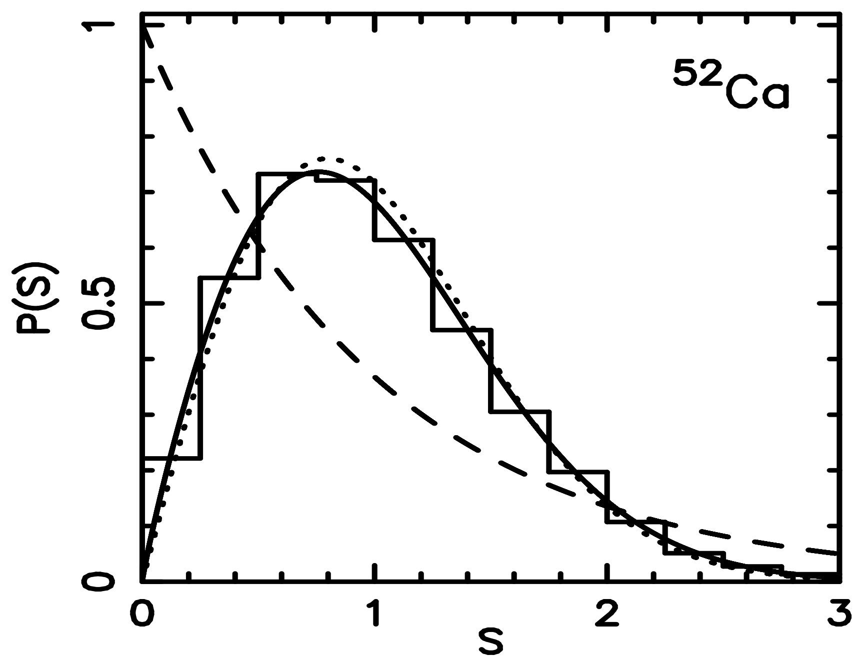


Shell-model $P(s)$ values for ^{52}Ca, $J^\pi = 0^+ - 12^+$, $T = 6$ states

$E_x \leq 5$ MeV

Whole spectrum

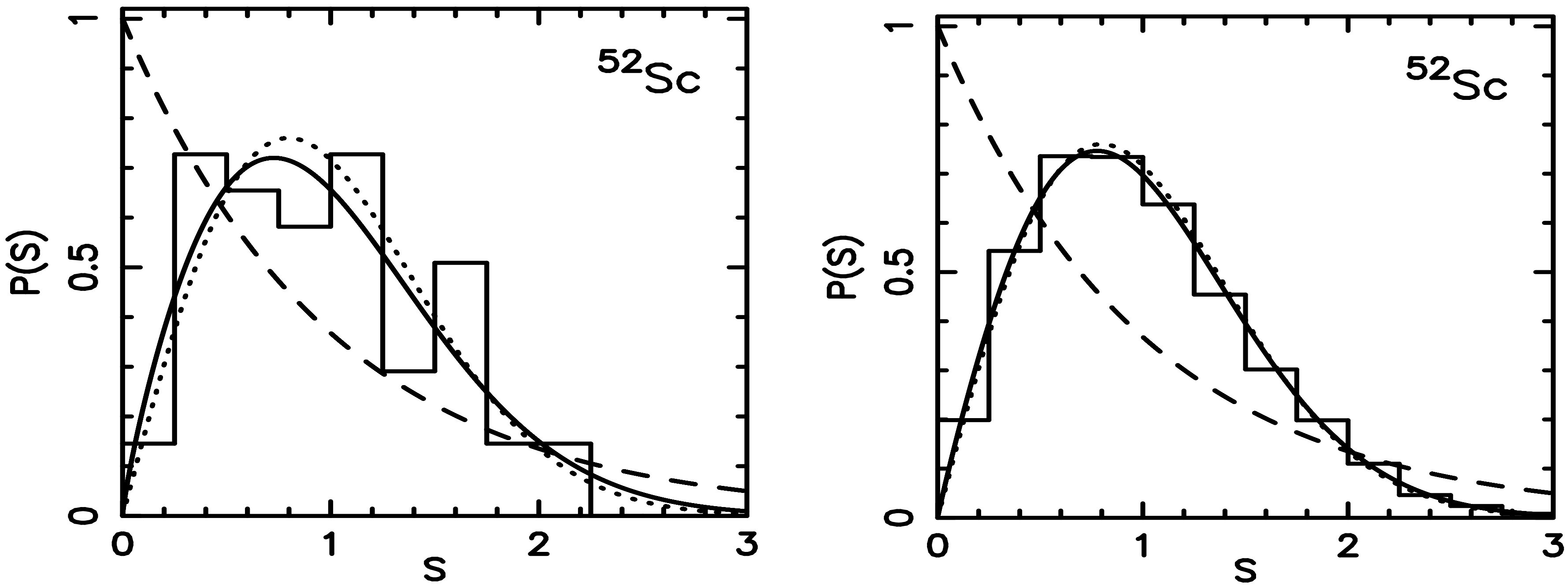




Shell-model $P(s)$ values for ^{52}Sc, $J^\pi = 0^+, 11^+, 12^+, T = 4$ states

$E_x \leq 5$ MeV

Whole spectrum



→ The study of wave functions provides deeper insight in the properties of quantum chaotic systems.

→ The degree of the complexity of $|E\rangle = \sum_{k=1}^{d} c_k(E)\,|k\rangle$, can be measured by the information entropy,

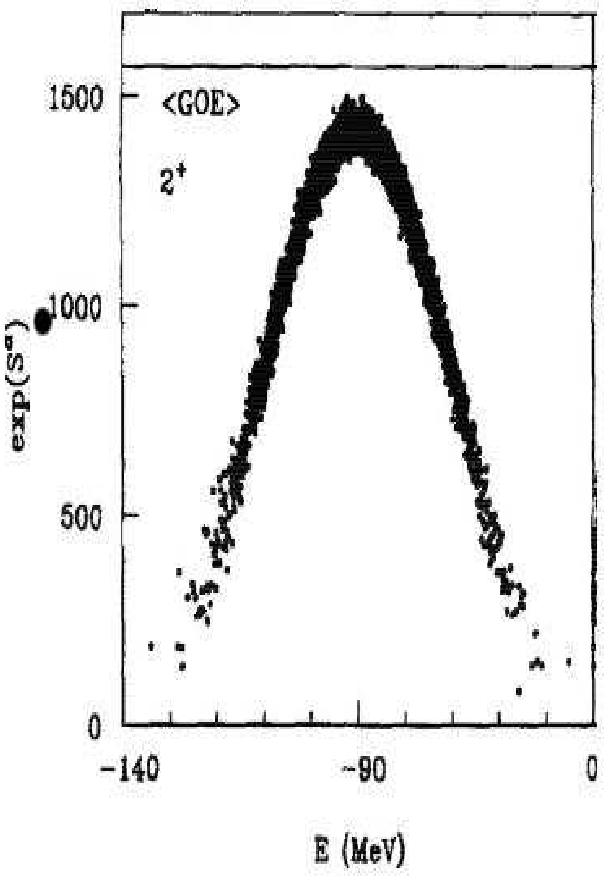
$$S^{inf}(E) = -\sum_{k=1}^{d} |c_k(E)|^2 \,\log\, |c_k(E)|^2 \,.$$

→ In order to compare states with different dimension, the localization length is defined as

$$l_{loc}(E) = \frac{\exp\left(S^{inf}(E)\right)}{\exp\left(S^{inf}_{\mathrm{GOE}}\right)} \simeq \frac{\exp\left(S^{inf}(E)\right)}{0.48d}.$$

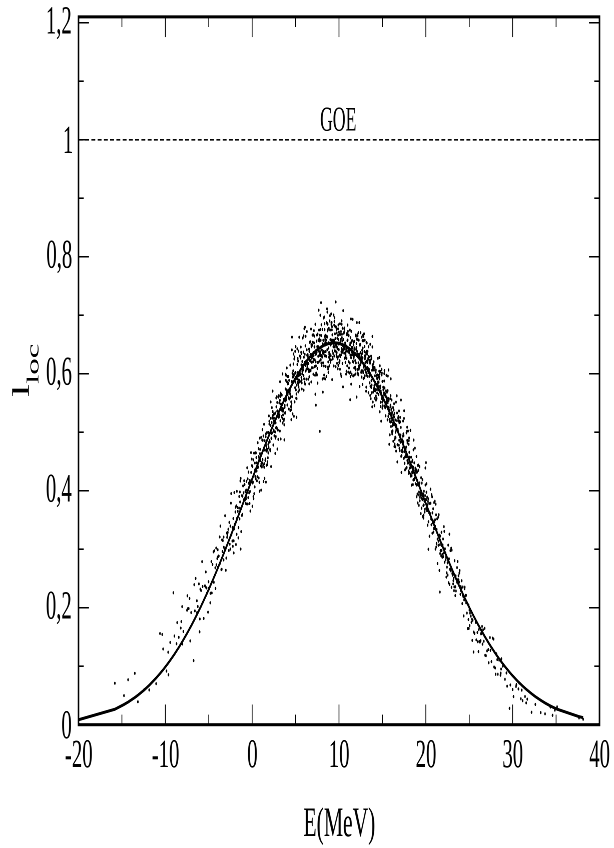
$\exp\left(S^{inf}\right)$ values for ^{28}Si

$J^{\pi} = 2^{+}, T = 0$ (Zelevinsky 1996)



l_{loc} values for ^{46}Ti

$J^{\pi} = 2^{+}, T = 1$ (Gómez 2011)

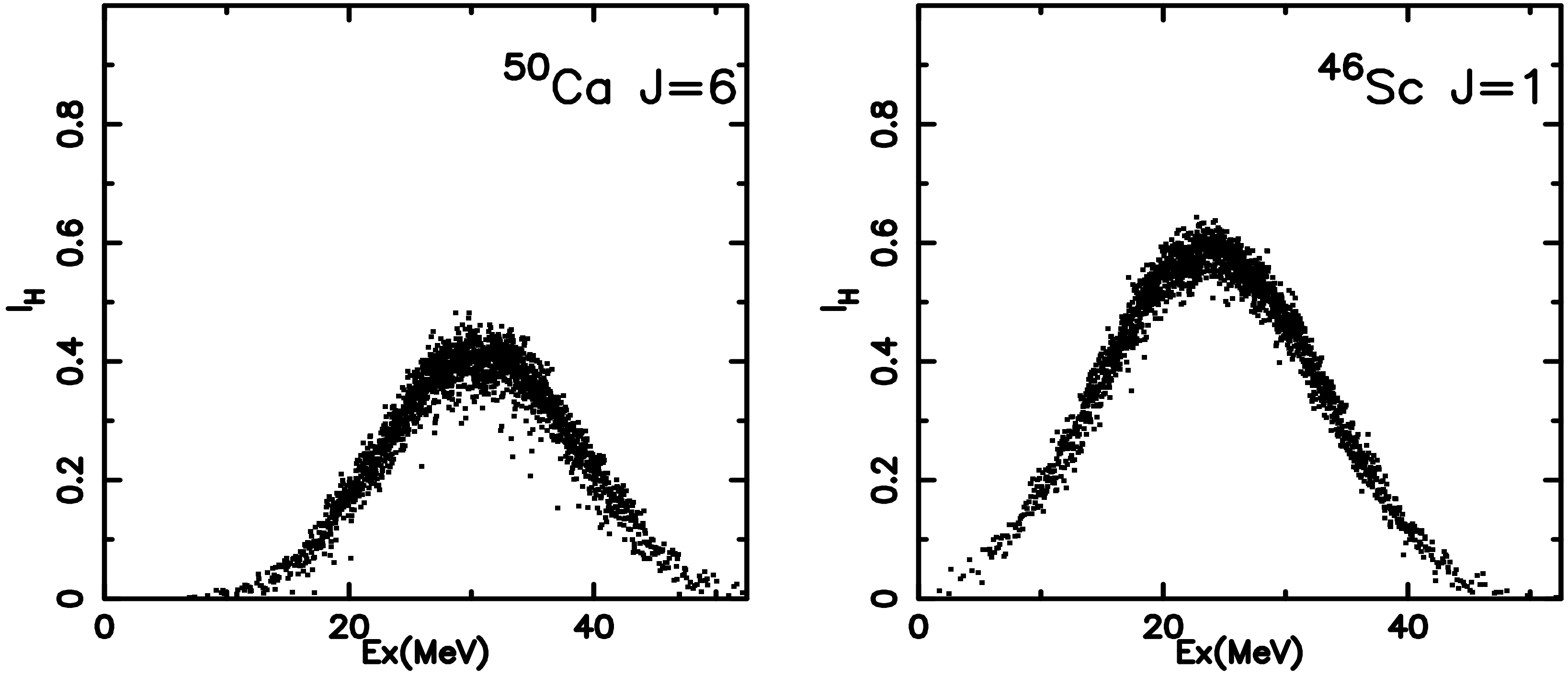


l_{loc} values for ^{50}Ca

$J^\pi = 6^+, T = 5$

l_{loc} values for ^{46}Sc

$J^\pi = 1^+, T = 2$



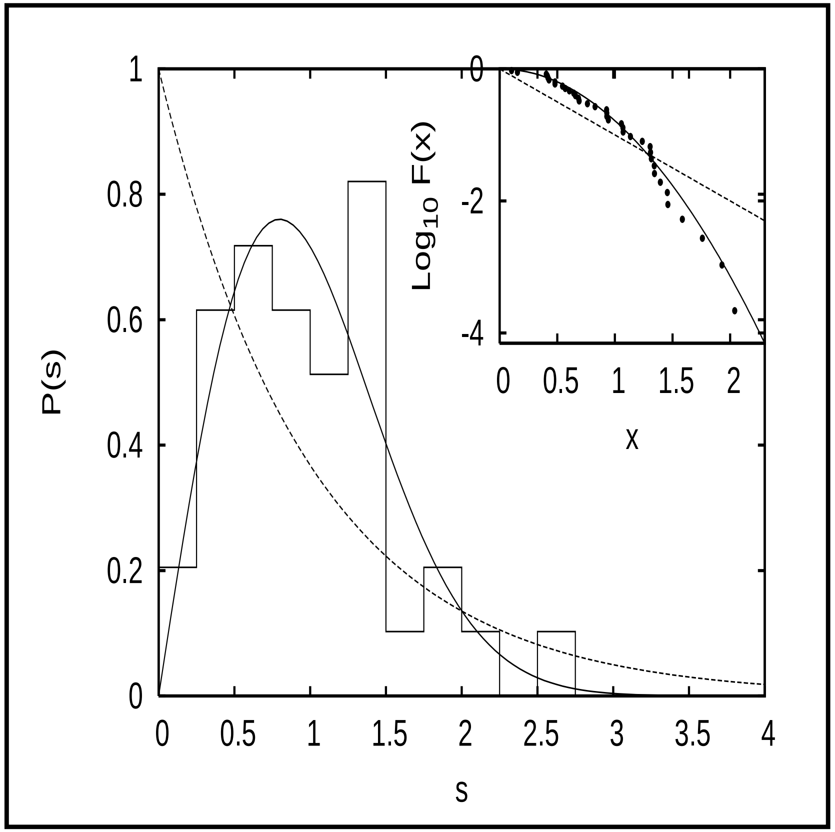
R. A. Molina *et al.*, Phys. Rev. C 63, 014311 (2000)

Baryon spectra

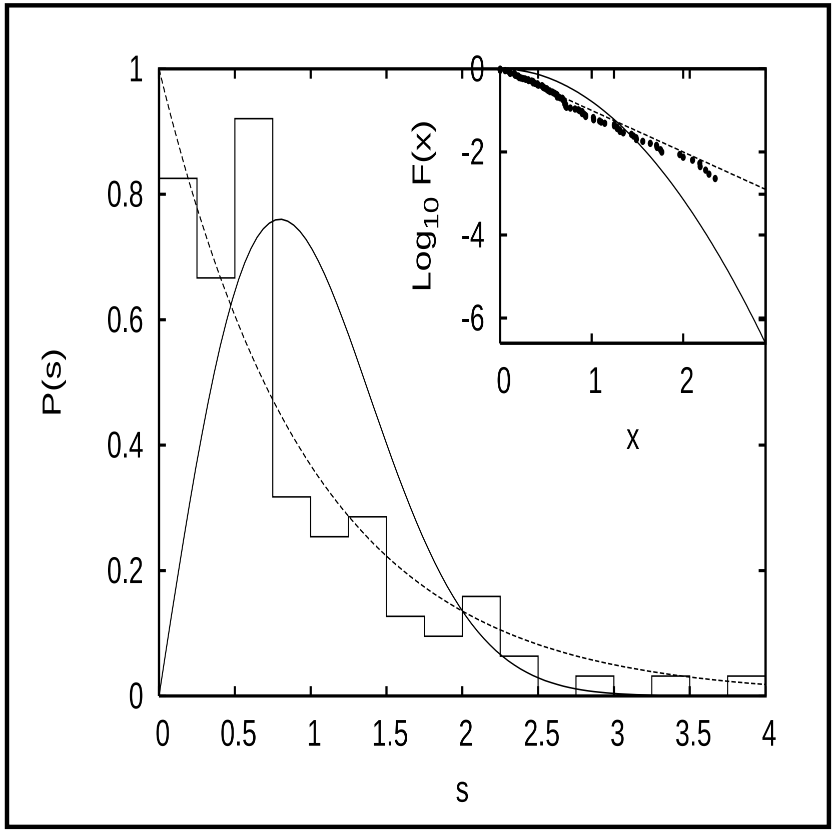
Problem: The number of baryons predicted by constituent quark models is larger than what is observed experimentally.

- → A spectral fluctuation analysis has been performed.
- → The experimental $P(s)$ distribution is close to GOE. Quark model results are close to the Poisson distribution.

Expt. Baryon Data



Quark Model (Capstick and Isgur)



$$F(x) = 1 - \int_0^x P(s) ds$$

C. Fernández-Ramírez and A. Relaño, Phys. Rev. Lett. **98**, 062001 (2007)

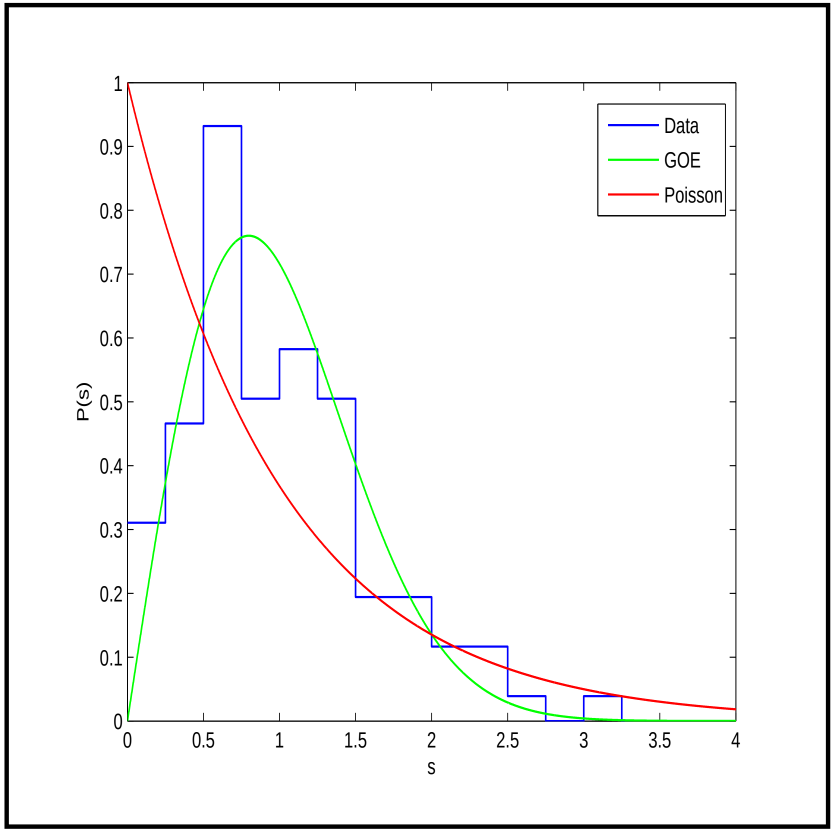
→ If the observed experimental spectra is incomplete, the experimental $P(s)$ distribution should be much closer to Poisson than the theoretical one. The situation is just the opposite.

→ Present quark models are not able to reproduce the statistical properties of the experimental baryon spectrum.

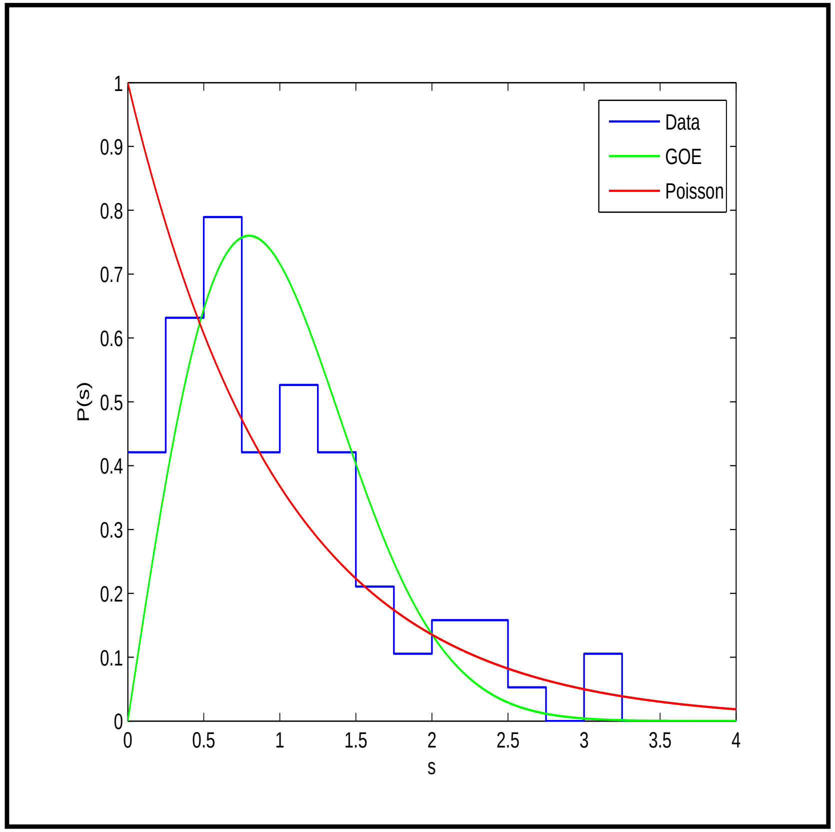
Meson spectra

- The number of mesons predicted by constituent quark models is in reasonable agreement with experiment.
- A preliminary spectral fluctuation analysis has been performed.
- The experimental $P(s)$ distribution is close to GOE. Some quark models give results close to GOE.

Expt. Meson Data



Quark Model (Vijande *et al.*)



Concluding remarks

→ There is a formal analogy between a discrete time series and the energy level spectrum of a quantum system, and the spectral fluctuations can be characterized by the power spectrum P_k^δ of the statistic δ_n.

→ P_k^δ is easy to calculate and easily interpreted. It characterizes the fully chaotic or regular behavior of a quantum system by a single quantity, the exponent α of the $1/f^\alpha$ noise. This result is valid for all quantum systems, independently of their symmetries (time-reversal invariance or not, integer or half-integer spin, etc.).

- → The analysis of shell-model spectra in pf-shell nuclei shows that for fixed A chaos depends strongly on the isospin T_z. This behavior can be explained in terms of the strong pn residual interaction in comparison with the nn residual interaction.
- → The analysis of shell-model wave functions shows that l_{loc} is clearly smaller than in GOE ($l_{loc}^{GOE} = 1$) and for fixed A exhibits a strong isospin dependence in good agreement with the behavior of spectral fluctuations.

- → A spectral fluctuation analysis of the low-energy baryon spectrum shows that the experimental $P(s)$ distribution is close to GOE, while quark model results are close to the Poisson distribution. Therefore, present quark models are not able to reproduce the statistical properties of the experimental baryon spectrum.

- → A preliminary spectral fluctuation analysis of the low-energy meson spectrum shows that the experimental $P(s)$ distribution is close to GOE. There are constituent quark models that give results in agreement with this behavior.