Reentrance in nuclei

Competitive phenomena

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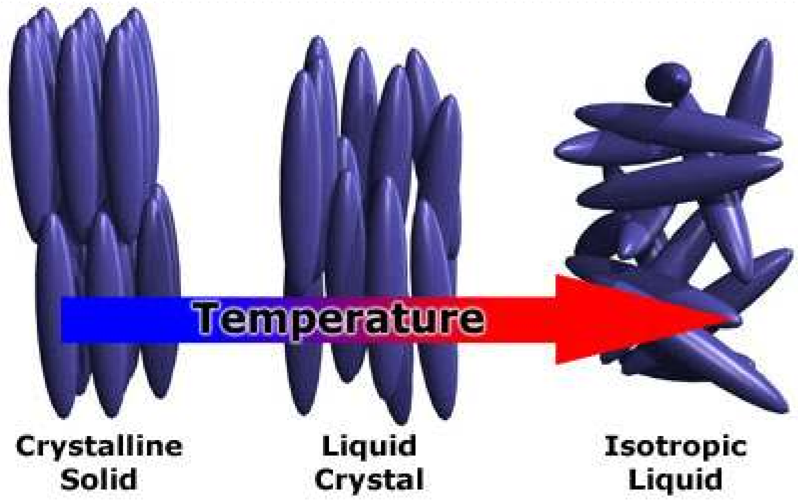
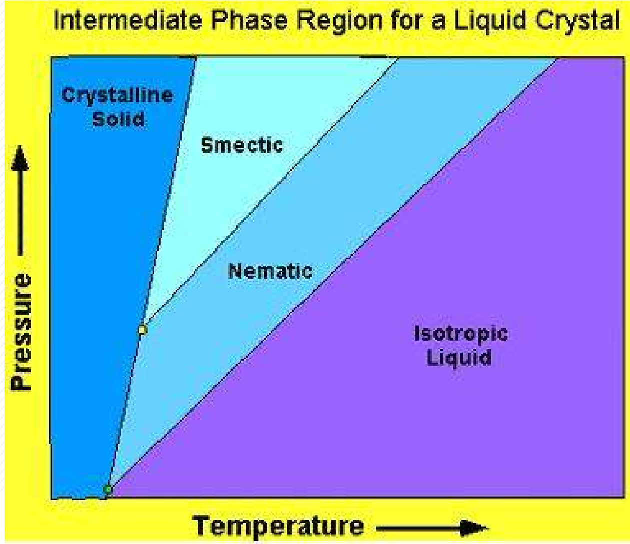
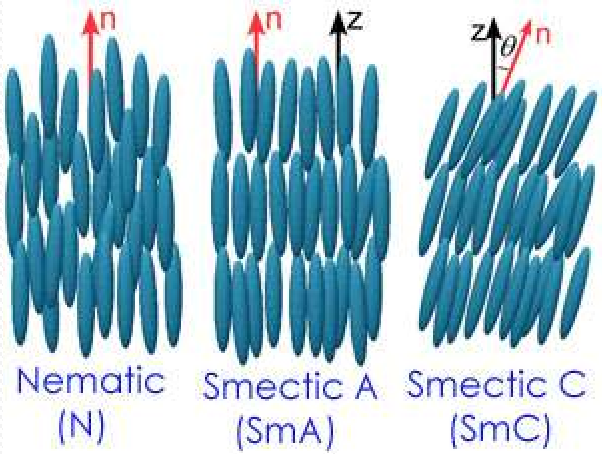
Outline

- A brief survey of *reentrance*
- Nuclear “phases” (this is not Italy)
- The competition between deformation and temperature
- The competition between rotation, deformation and temperature
- A few words on simulations and computing mixed in

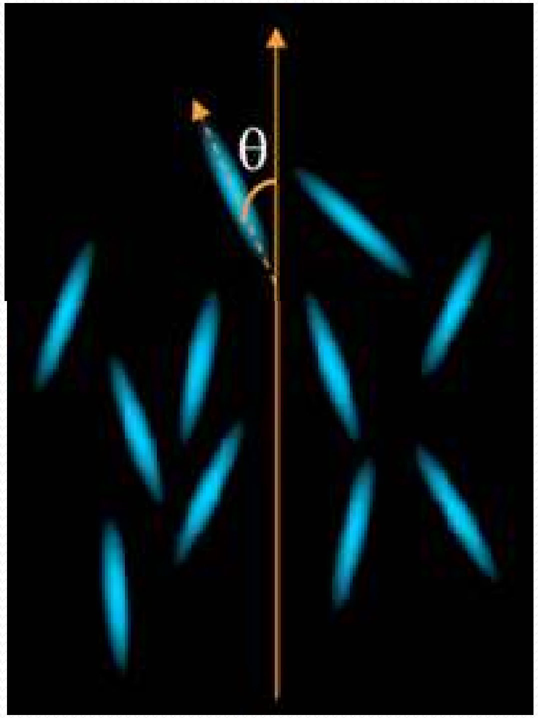
Liquid Crystal Phases

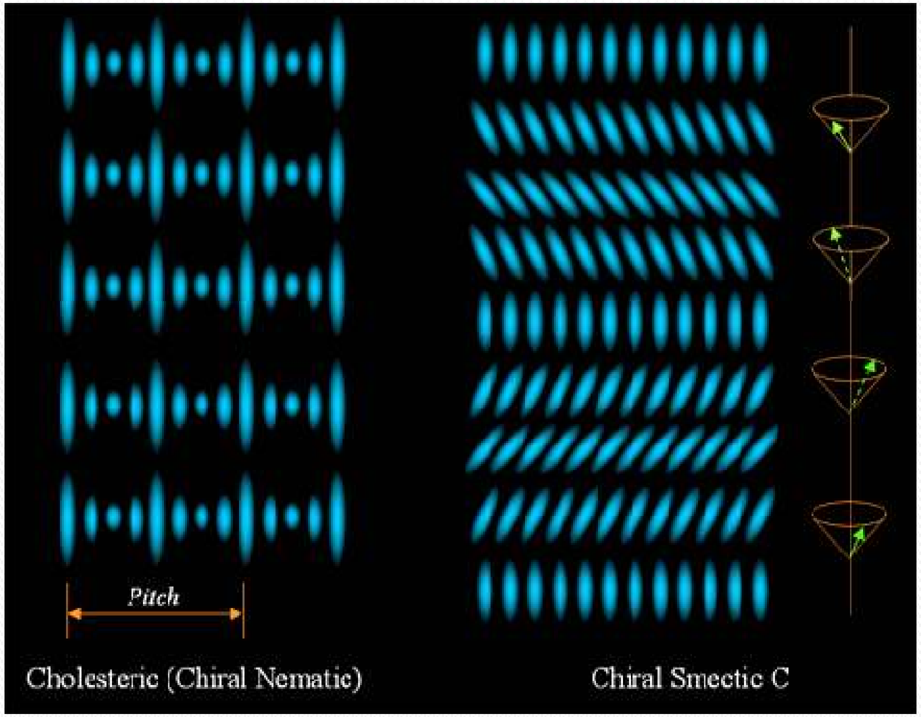






Liquid crystal alignment





$$S = \langle P_2(\cos\theta)\rangle = \left\langle \frac{3\cos^2\theta - 1}{2} \right\rangle$$

"Re-entrance" discovered for liquid crystals

New Liquid-Crystal Phase Diagram

P. E. Cladis
Bell Laboratories, Murray Hill, New Jersey
(Received 7 April 1975)

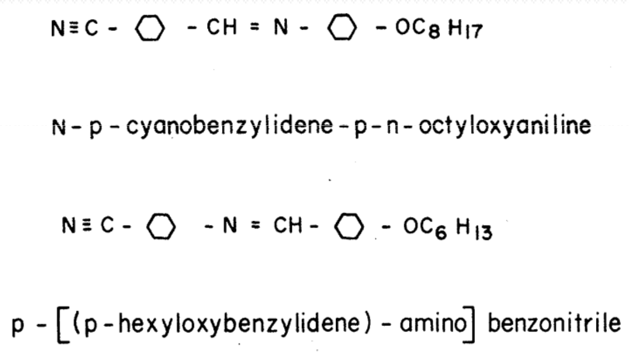
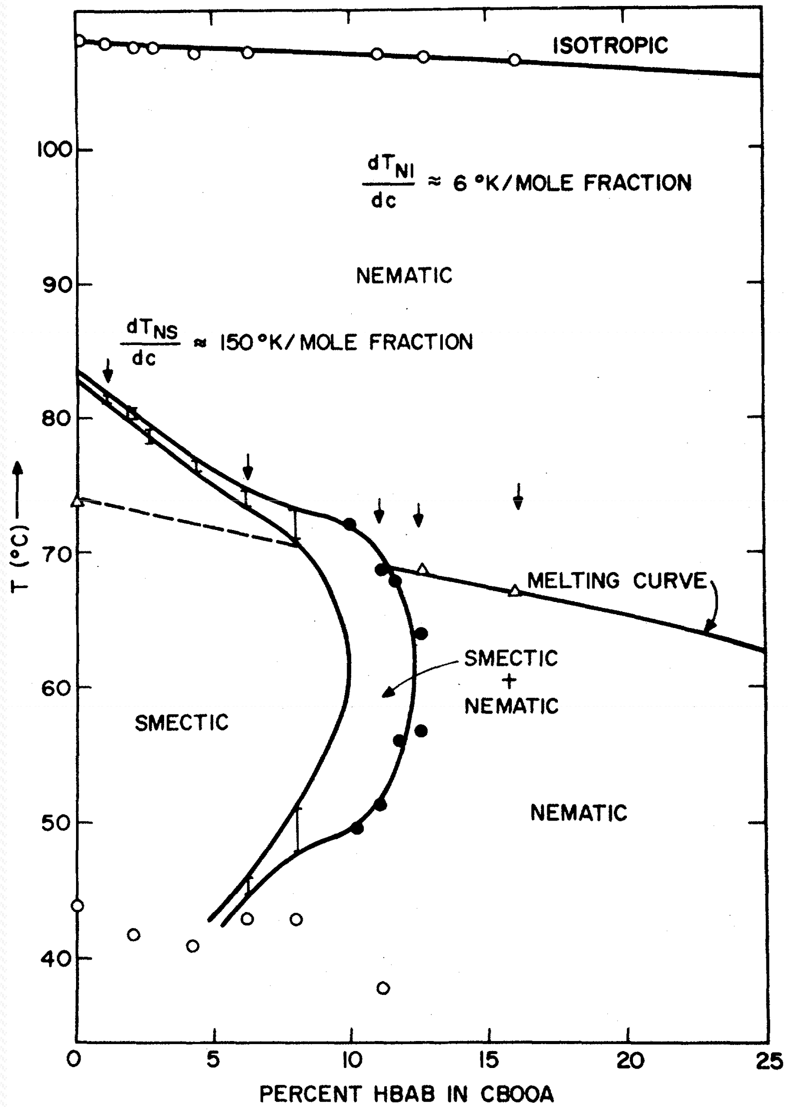


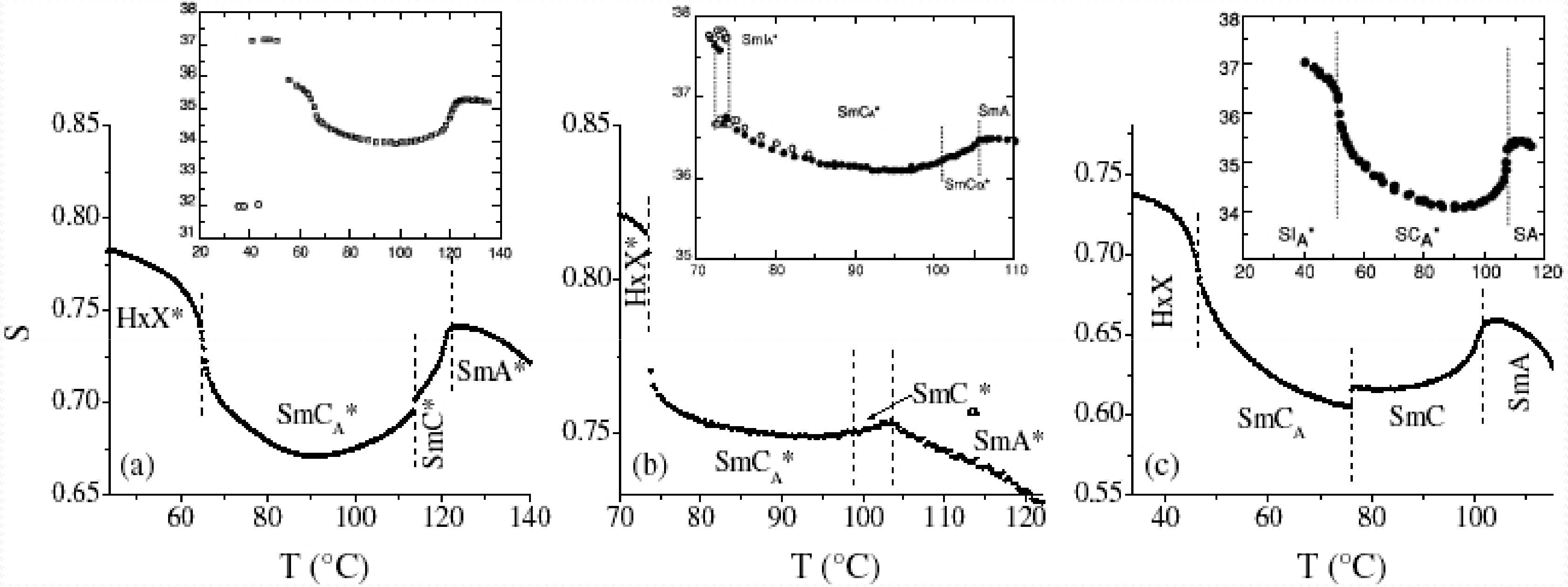
FIG. 1. CBOOA (top) and HBAB.

"By mixing HBAB in CBOOA, I have found that a smectic phase may be formed which reverts to the nematic phase at still lower temperatures. As far as I can ascertain, this is the first time such an effect has been observed."

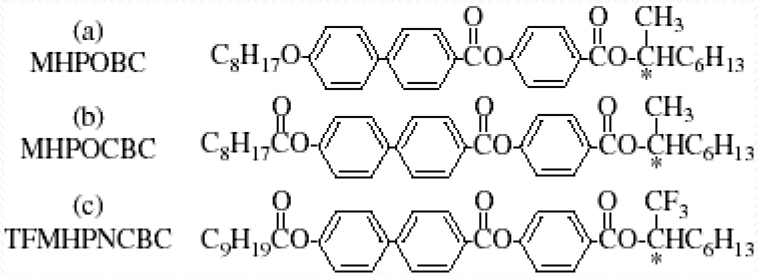


PRL 35, 48 (1975)

Liquid crystal phases



R. Korlacki et al., 2007 *EPL* **77** 36004



Quantum phase reentrance

Ferromagnetic Insulator

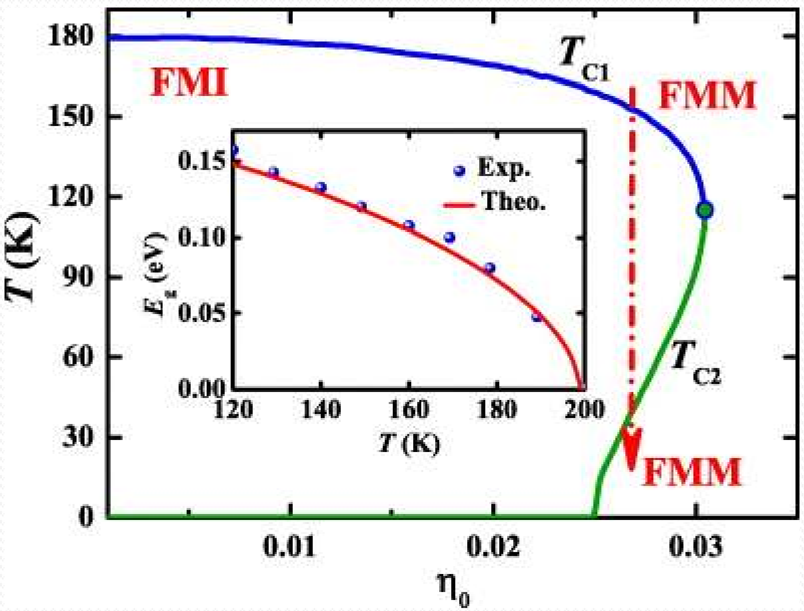


Figure 2. Two critical temperatures of metal–insulator transitions (T_{c1} and T_{c2}) as a function of interchain coupling strength η_0. The large dots are the coincident points of T_{c1} and T_{c2}. The re-entrance of the ferromagnetic metal state is shown by the big arrow.

Wei et al., N. J. Phys. 12, 053201 (2010)

FMM=Ferromagnetic metal state

FMI=Ferromagnetic insulator state

Hidden Order States –
orbital antiferromagnetism

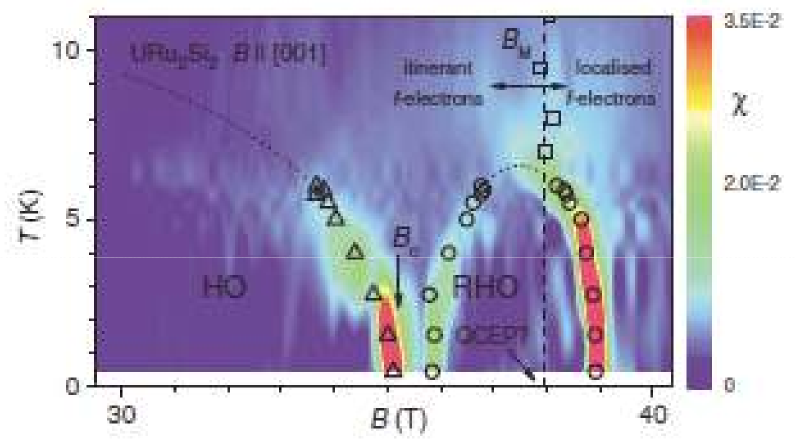
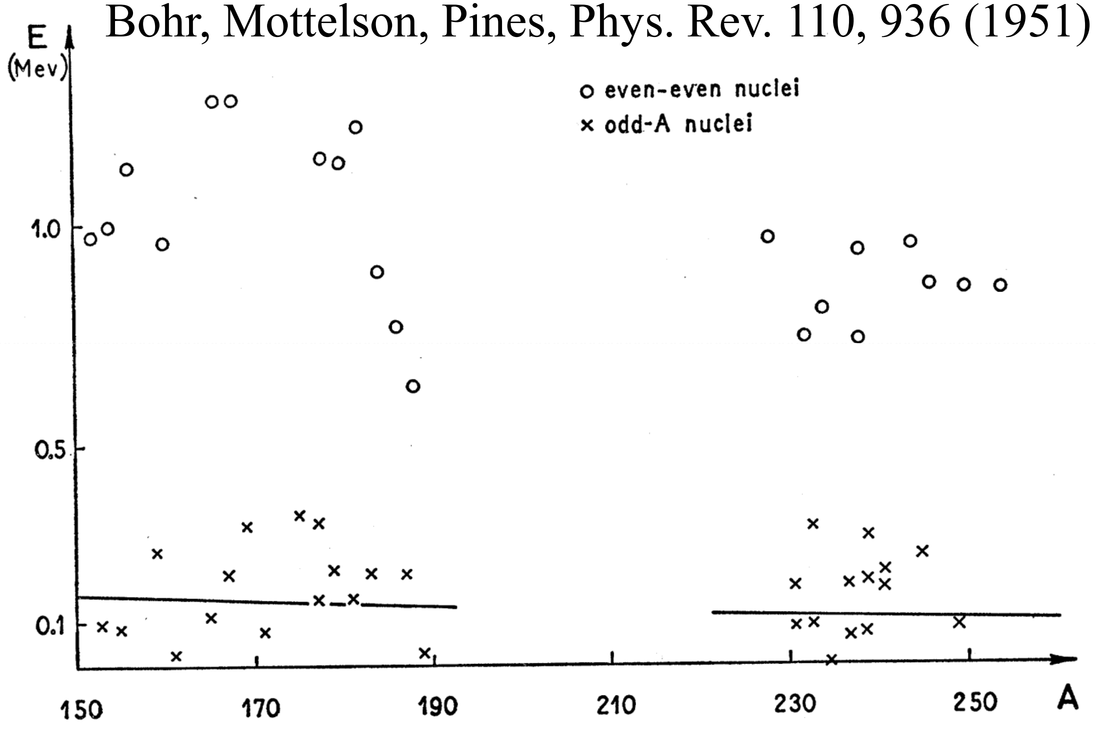


FIG. 1 (color). The $B > 30$ T versus T phase diagram of URu_2Si_2 combined with a color intensity plot of χ measured at many different temperatures. Square, triangle, and circle symbols mark B_M and transitions into and out of the HO and RHO hidden order (RHO) phases, respectively. The curved dotted lines depict the continuation of the phase boundaries revealed by specific heat and transport studies [4].

Harrison et al., PRL 90 96402 (2003)

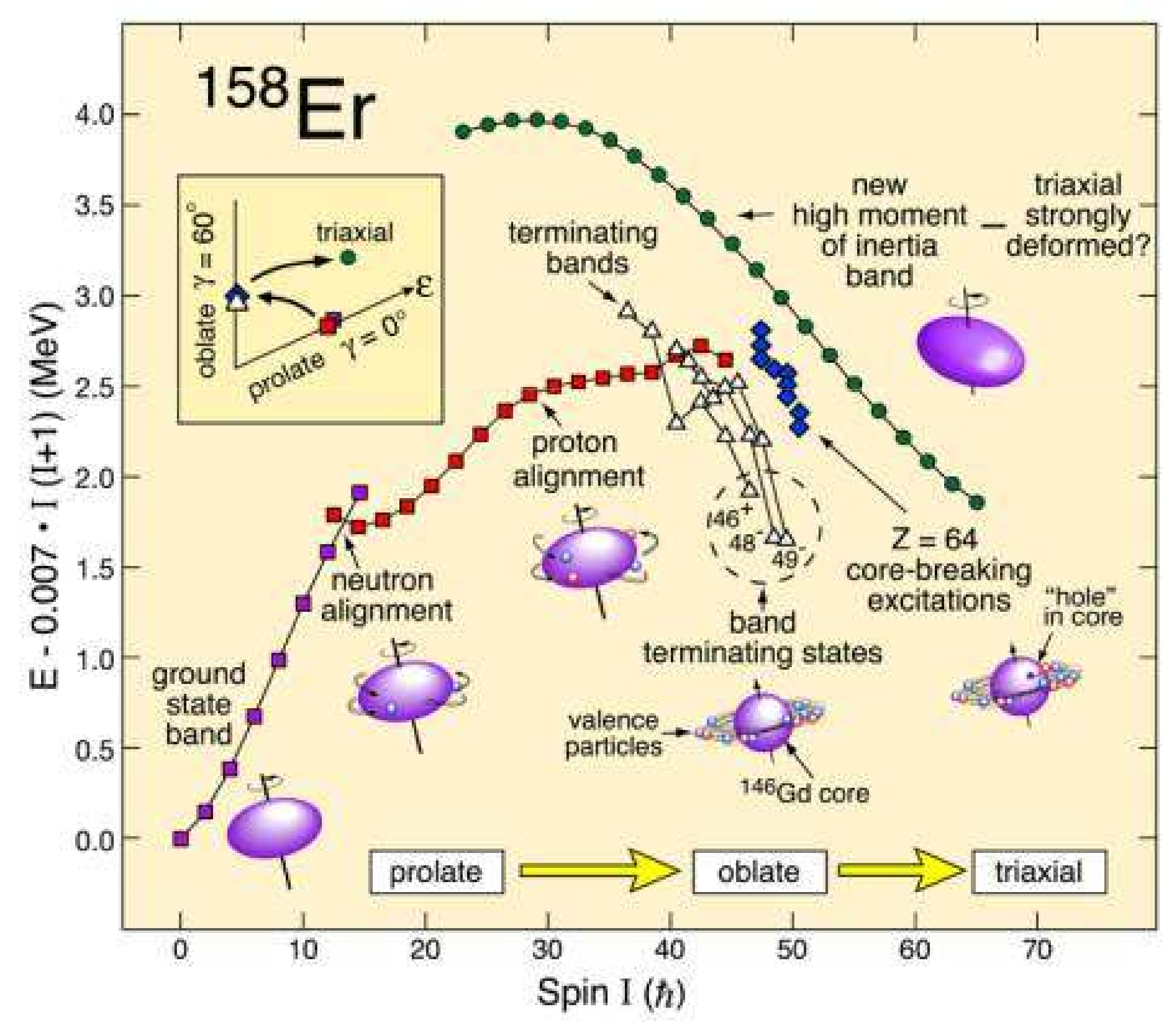
Nuclear Pairing: one phase of nuclei

Bohr, Mottelson, Pines, Phys. Rev. 110, 936 (1951)



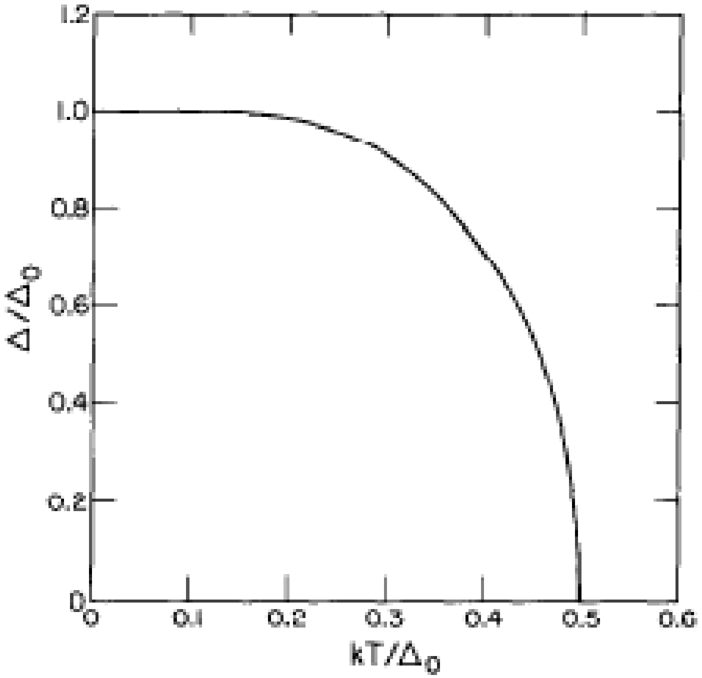
- Pairing effects cause the gap
- Collective behavior and emergent phenomena
- Points to superconductivity

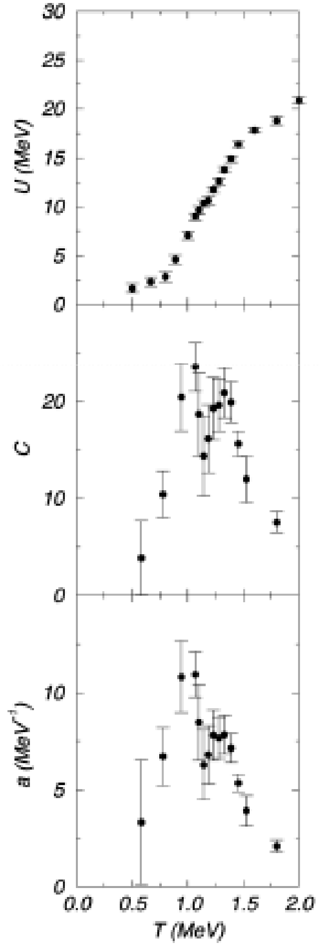
Nuclear Deformation: collective behavior phases



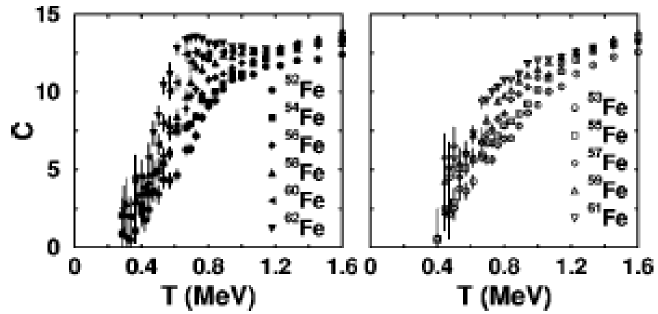
Pairs can be thermally broken

Goodman, NPA 352, 30 (1981)
FT-HFB

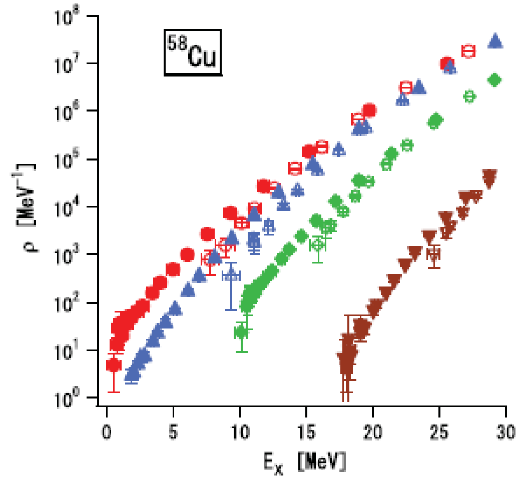




Dean et al., PRL 1994



Liu, Alhassid, PRL 2001



Nakada, Alhassid, PRC, 2008

Thermal transition to unpaired state occurs around 0.5-1.0 MeV

Early work on thermally assisted pairing

Tamura, Prog. Theor. Phys. 31, 595 (1964)

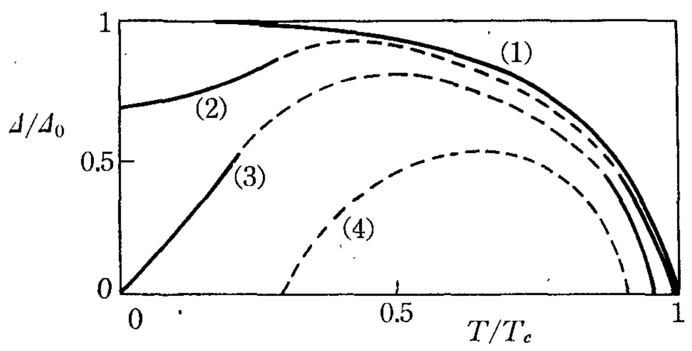


Fig. 2. Ratio of the energy gap to the gap at $T=0$ vs temperature for various values of spin projection M: (1) $M=0$, (2) $M=m$, (3) $M=2m$, (4) schematic plot for some higher M.

$$H = \sum_{s,\rho} (e_s - \lambda)\, a^*_{s\rho}\, a_{s\rho} - G \sum_{s,s'} a^*_{s-}\, a^*_{s+}\, a_{s'+}\, a_{s'-} - \omega J_z$$

Exact model shows a pairing-deformation relation

Sheikh et al, PRC 72, 041301 (2005)

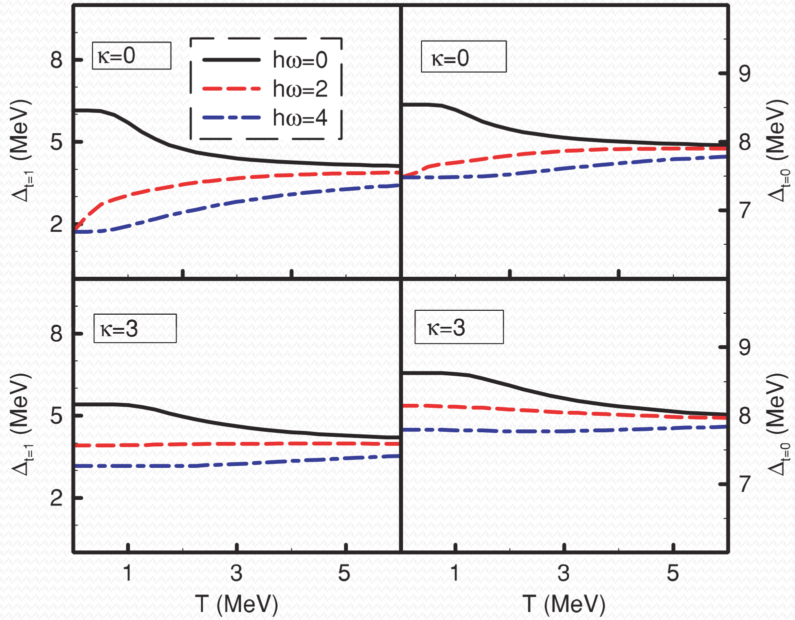


FIG. 2. (Color online) Results of the total isovector ($\Delta_{t=1}$) and isoscalar ($\Delta_{t=0}$) pair gaps are plotted as a function of temperature for three different rotational frequencies of $\hbar\omega = 0, 2,$ and 4 MeV. The upper panel shown the results for $\kappa = 0$ and the lower panel depicts the results for $\kappa = 3$ MeV.

Hamiltonian for this work

- Pairing+Quadrupole Hamiltonian

$$H = \sum_{jmt_z} e(j) a^{+}_{jmt_z} a_{jmt_z} - \frac{G}{4} \sum_{\alpha\alpha' t_z} P^{+}_{JT=01,t_z}(\alpha) P_{JT=01,t_z}(\alpha') - \chi \sum_{\mu} (-1)^{\mu} Q_{2\mu} Q_{2-\mu}$$

$0g_{7/2}$-1d-2s

0f-1p-$0g_{9/2}$

- Solve using Auxiliary Field Monte Carlo techniques

- Parameters:

One-body from W-S for ^{56}Ni

$$e_{0f_{7/2}} = 0.000 \quad e_{0f_{5/2}} = 6.42$$
$$e_{1p_{3/2}} = 4.350 \quad e_{1p_{1/2}} = 6.54$$
$$e_{0g_{9/2}} = 8.980 \quad e_{0g_{7/2}} = 17.59$$
$$e_{2d_{5/2}} = 12.95 \quad e_{2d_{3/2}} = 15.99$$
$$e_{2s_{1/2}} = 14.64$$

$$x = 0.0104 \text{ MeV}^{-1}$$
$$G = 0.106 \text{ MeV}$$

Reproduces collective Spectrum in ^{64}Ni and ^{64}Ge

For rotations:

$$H^{\omega} = H - \omega J_z$$

Langanke, Dean, Nazarewicz, NPA757, 360 (2005);
Dean, Langanke, Nam, Nazarewicz, PRL105, 212504 (2010)

Shell Model Monte Carlo Essentials

$$\hat{H} = \varepsilon\hat{\Omega} + \frac{V}{2}\hat{\Omega}^2$$

$$Z = \mathrm{Tr}\left[\exp\left(-\beta\hat{H}\right)\right] \quad \rightarrow \quad \left\langle \hat{H} \right\rangle = \frac{\mathrm{Tr}\left[\exp\left(-\beta\hat{H}\right)\hat{H}\right]}{Z}$$

$$\exp\left(-\beta\hat{H}\right) = \sqrt{\frac{\beta|V|}{2\pi}} \int_{-\infty}^{\infty} d\sigma \exp\left(-\beta|V|\sigma^2/2\right)\exp\left(-\beta\hat{h}\right)$$

$$\hat{h} = \varepsilon\,\hat{\Omega} + sV\sigma\hat{\Omega}$$

$$s = 1 \quad for \quad V < 0$$
$$s = i \quad for \quad V > 0$$

Koonin et al., Phys. Repts. 287, 1 (1997)

SMMC for a general interaction

$$\hat{H} = \sum_{\alpha} \left(\varepsilon_{\alpha} \hat{\Omega}_{\alpha} + \frac{V_{\alpha}}{2} \hat{\Omega}_{\alpha}^{2} \right)$$

$$Z = \mathrm{Tr}\left[\exp(-\beta \hat{H})\right] \quad \rightarrow \quad \mathrm{Tr}\left[\exp\left(-\Delta\beta \hat{H}\right)\right]^{N_t} \quad \rightarrow$$

$$\int D[\sigma] G(\sigma) \mathrm{Tr}\left\{ \prod_{n=1}^{N_t} \exp\left[(-\Delta\beta \hat{h}(\sigma_n))\right] \right\} = \int D[\sigma] W(\sigma) \Phi(\sigma)$$

$$W(\sigma) = G(\sigma) \left|\mathrm{Tr}[\;\;]\right| \qquad \Phi(\sigma) = \frac{\mathrm{Tr}[\;\;]}{\left|\mathrm{Tr}[\;\;]\right|}$$

Particle number projection

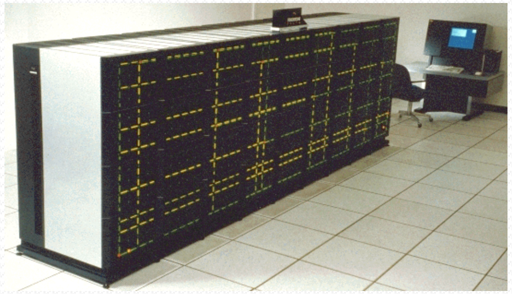
$$\mathrm{Tr}\left[\hat{\Omega}\right] \equiv \mathrm{Tr}_N\left[\hat{\Omega}\right] = \sum_i \langle i | \hat{P}_N \hat{\Omega} | i \rangle$$

$$\hat{P}_N = \delta\left(\hat{N} - N\right) = \int_0^{2\pi} \frac{d\varphi}{2\pi} \exp\left\{ i\varphi\left(\hat{N} - N\right)\right\}$$

Thermal operator expectation

$$\left\langle \hat{\Lambda} \right\rangle = \frac{Tr\left[\exp\left(-\beta \hat{H}\right) \hat{\Lambda}\right]}{Tr\left[\exp\left(-\beta \hat{H}\right)\right]}$$

Simulations in science (Nortur2011 talk)



Intel Touchstone Delta
512 nodes
10 Gflops peak; 8 Gbytes total memory

1992-1995

ORNL Jaguar, Cray XT5
224,500 cores
2.3 Pflops peak; 289 Tbytes total memory

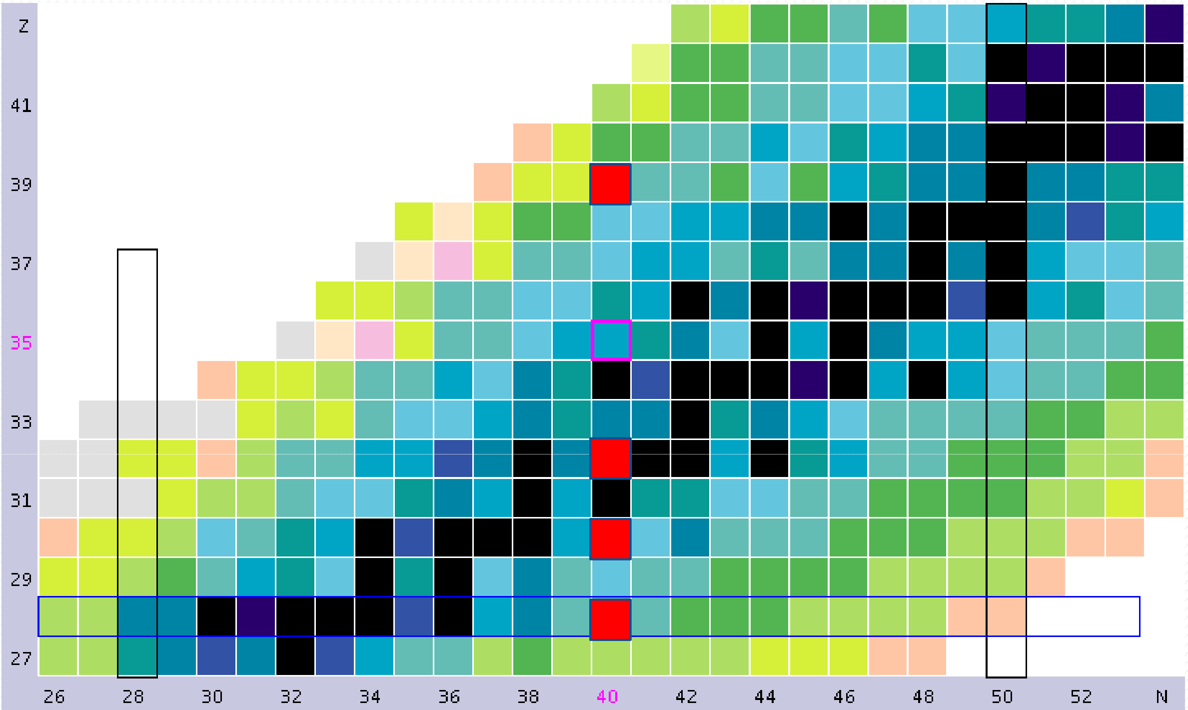
2011

- Flops are free
- Memory onto the chip (3-d)
- More cores/chip
- Exascale science by end of the decade

ELECTRICITY	Today	Tomorrow (exa)
Electricity Cost	$0.1/kW-hr	$0.1/kW-hr
Requirement	7MW	21MW
Cost/hour	$700/hour	$2100/hour
Cost/year	$5.6M	$16.8M

Power is a new constraint

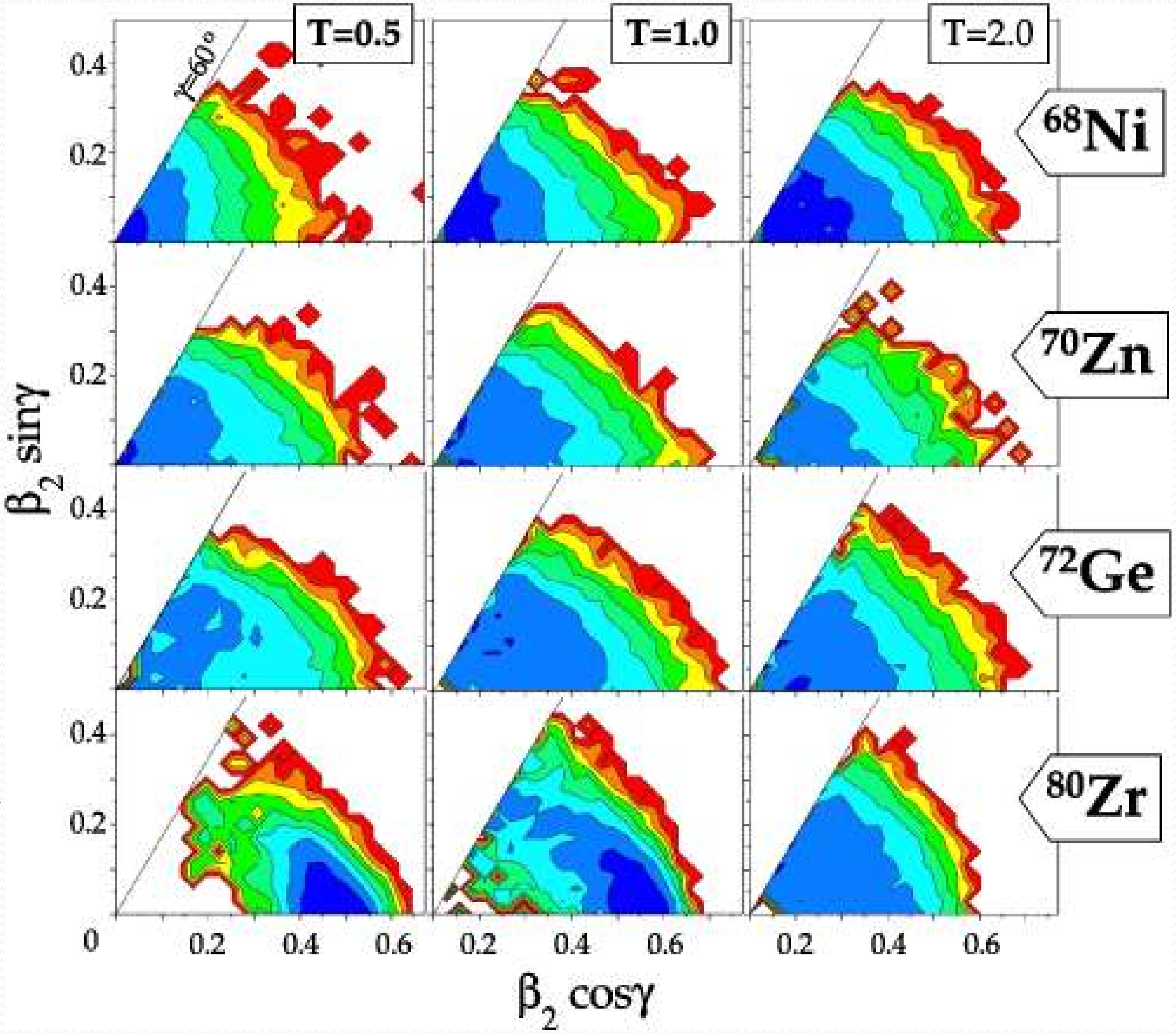
Nuclei in the fp-gds region



- ^{68}Ni Spherical ground state; weak N=40 shell closure
- ^{70}Zn Stronger proton pairing correlations;
  Some quadrupole collectivity; erosion of N=40 shell gap
- ^{72}Ge Shape coexistence phenomena; static proton and neutron pairing
- ^{80}Zr Very deformed; large N=40 shell effects, weakened pairing

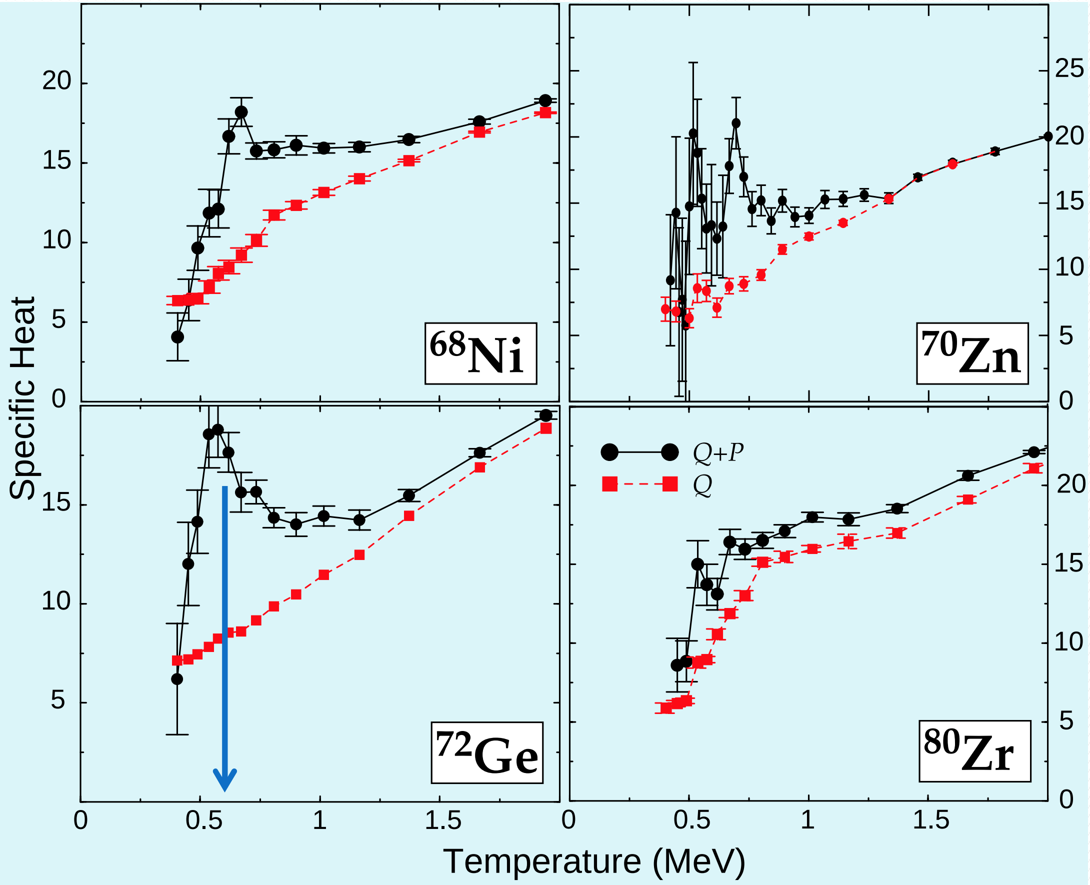
Calculated free energy surfaces

Spherical



Deformed

Pairing, deformation and the specific heat



$$E(\beta) = \frac{Tr\left[\exp(-\beta H)H\right]}{Tr\left[\exp(-\beta H)\right]}$$

$$C_v = -\beta^2 \frac{dE}{d\beta}$$

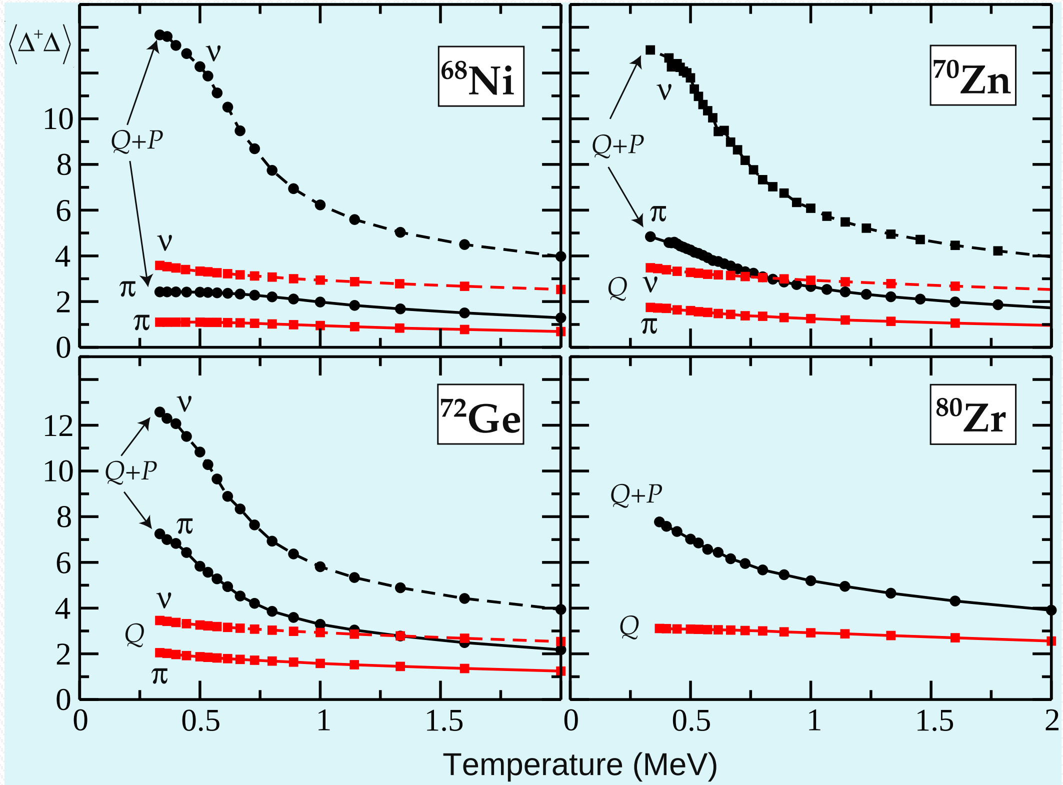
$T_c \sim 0.6$ MeV

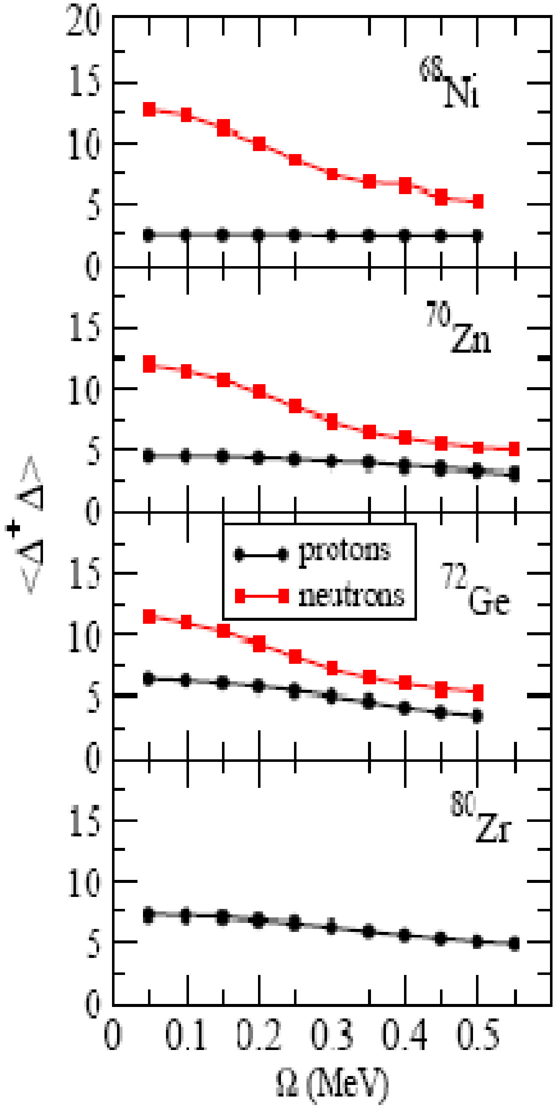
Red: quadrupole only

Black: pairing+quadrupole

How does rotation affect pairing?

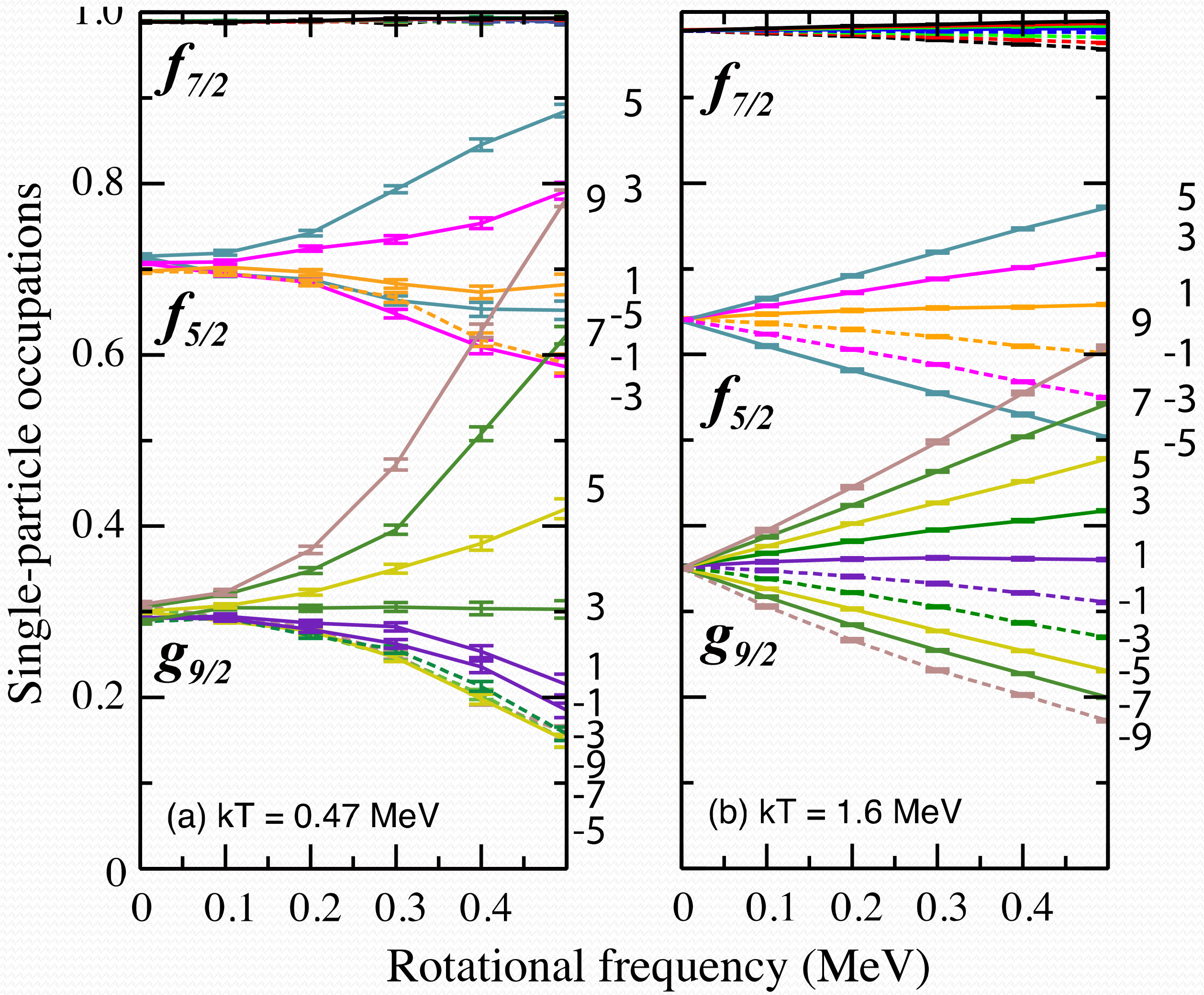
No rotation – thermal effect only





Rotation low-temp

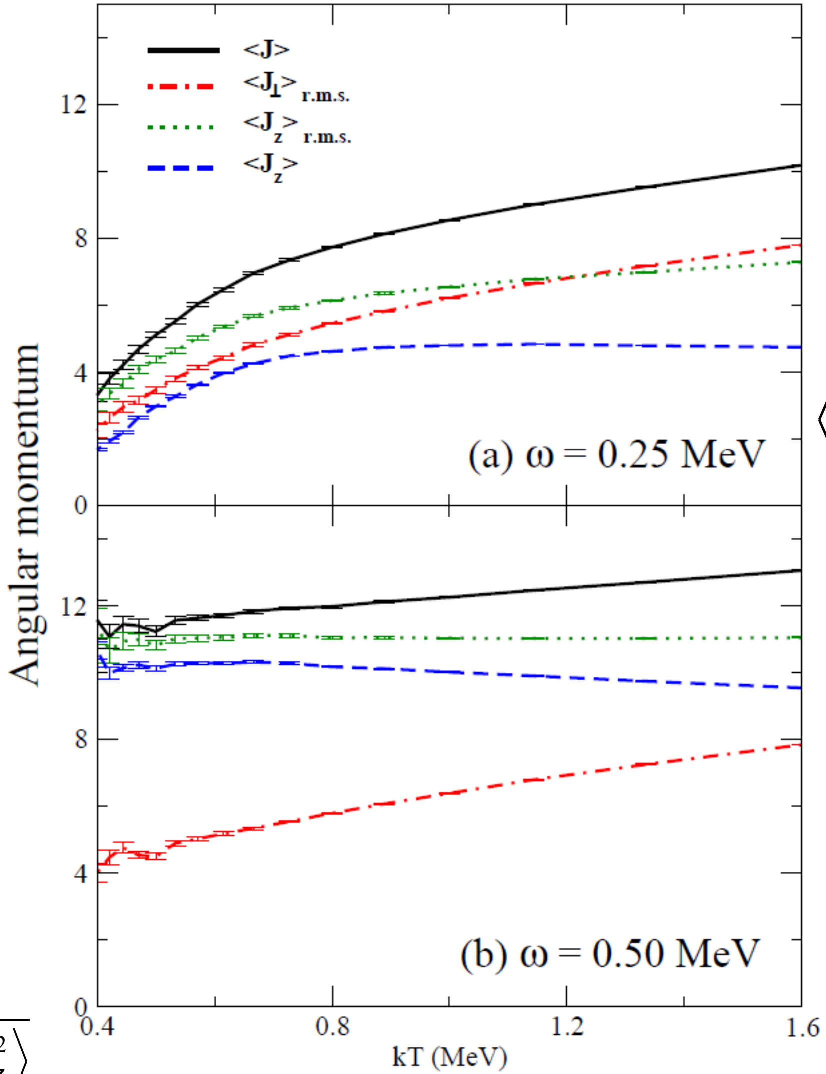
^{72}Ge occupations with rotation



Below T_c

Above T_c

Temperature and rotation



At low ω, <J> increase due to increased thermal fluctuations both parallel and perpendicular to rotation

$\langle J_z \rangle \approx 4.5\hbar$

$\langle J_z \rangle = 2.5\hbar$

At high ω, <J> increase due to increased quantum fluctuations perpendicular to rotation

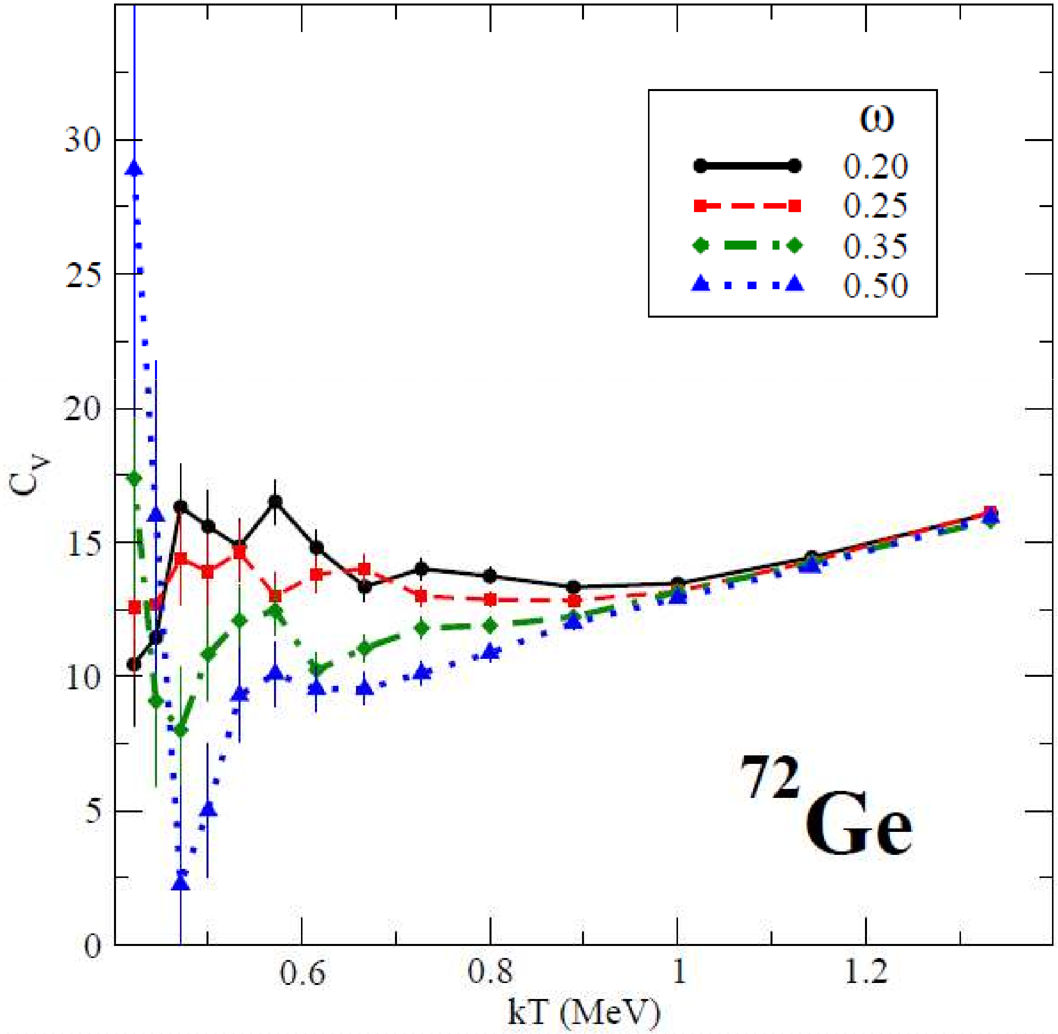
$\langle J_z \rangle \sim 10.5\hbar$

$$\langle \hat{J}^2 \rangle = \langle J \rangle (\langle J \rangle + 1)$$

$$\langle J_z \rangle_{r.m.s.} = \langle \hat{J}_z^2 \rangle$$

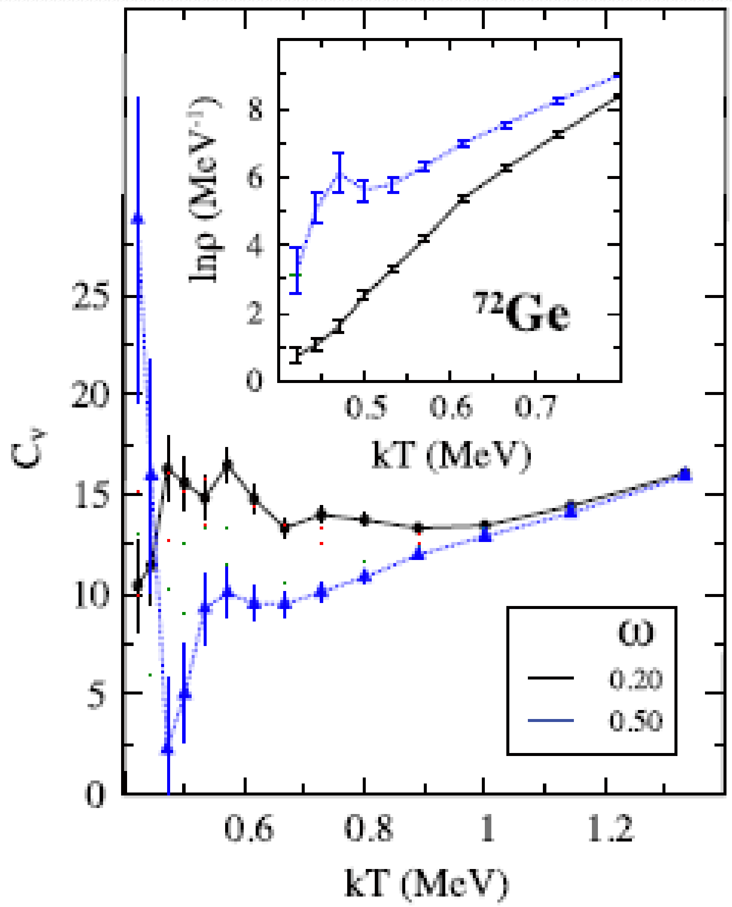
$$\langle J_\perp \rangle_{r.m.s.} = \sqrt{\langle \hat{J}^2 - \hat{J}_z^2 \rangle}$$

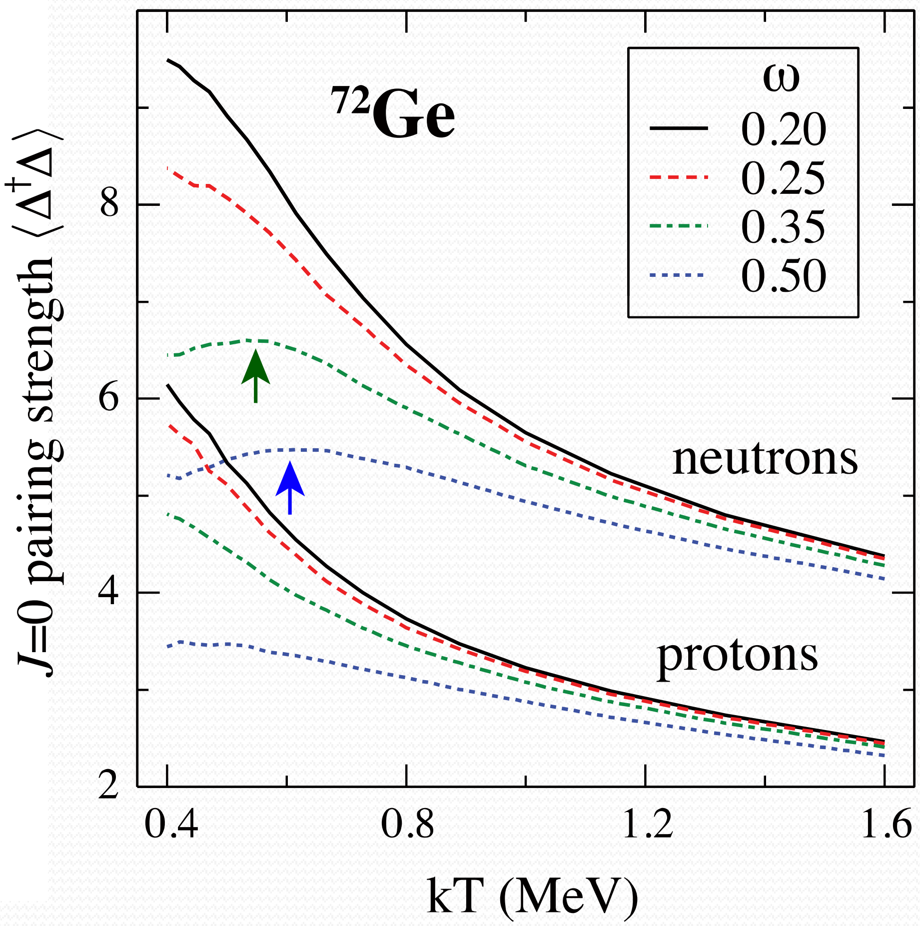
Nuclear Specific heat



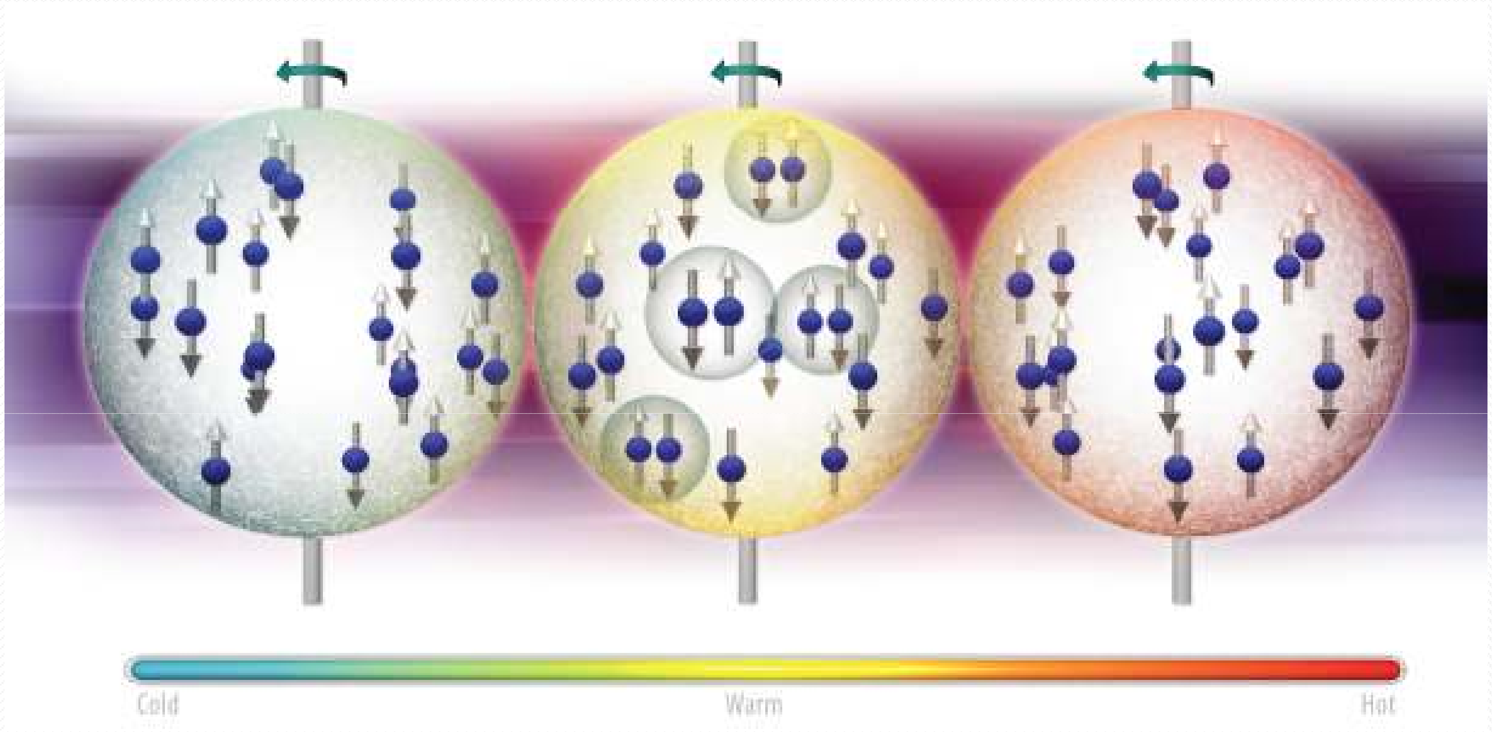
- Gradual entrance of the dip with increasing ω
- Statistical errors are large at large ω
- Dip definitely influences level density
- Due to pairing reentrance

Specific heat and pairing: reentrance





Conclusions



- In many electron systems magnetic fields produce reentrance effects
- In nuclei, rotation acts as the ‘magnetic’ field
- Reentrance should be visible in level density data at a given value of rotation
- HPC enables physics insights