Extended RPA method for the study of Nuclear Giant Resonances

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Contents of the talk

1. Density Functional Theory in relativistic static phenomena
2. Method to describe nuclear collective phenomena (RPA)
3. Exact treatment of the continuum
4. Results in spherical nuclei and comparison with experiment
5. Conclusions

Density Functional Theory

$$E[\hat{\rho}] = \langle \Psi | H | \Psi \rangle$$ exact!

density matrix $\rho(\mathbf{r}, \mathbf{r}') = \sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{A}} |\phi_{\mathbf{i}}(\mathbf{r})\rangle\langle\phi_{\mathbf{i}}(\mathbf{r}')|$

Mean Field

$$\hat{h} = \frac{\delta E}{\delta \hat{\rho}}$$

Eigenfunctions:

$$\hat{h}|\phi_i\rangle = \varepsilon_i|\phi_i\rangle$$

Interaction:

$$V = \frac{\delta^2 E}{\delta \hat{\rho}^2}$$

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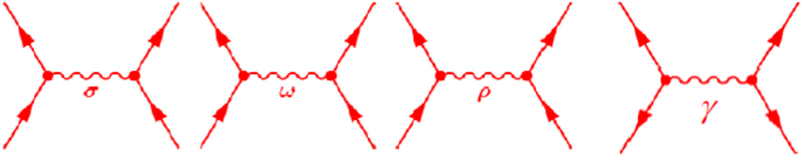
- Extensions: Pairing correlation (BCS)

Relativistic DFT

Nucleons are coupled by exchange of mesons



$(J^\pi,T)=(0^+,0)$ $(J^\pi,T)=(1^-,0)$ $(J^\pi,T)=(1^-,1)$

$$S(\mathbf{r}) = g_\sigma \sigma(\mathbf{r}) \qquad V(\mathbf{r}) = g_\omega \omega(\mathbf{r}) + g_\rho \vec{\tau}\vec{\rho}(\mathbf{r})\omega(\mathbf{r}) + \mathbf{eA}(\mathbf{r})$$

Sigma-meson:
Attractive scalar field

Omega-meson:
short range
repulsive

Rho-meson:
isovector field

Relativistic DFT

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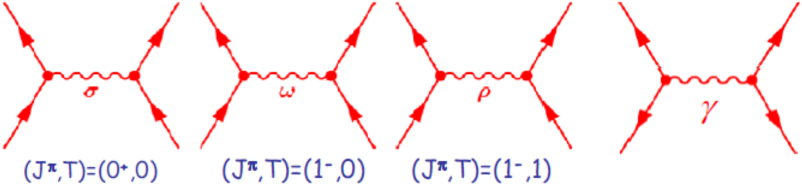

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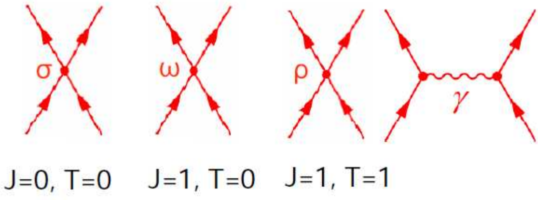
Dirac equation: $[\alpha \cdot p + \beta(m + S(\mathbf{r})) + \mathbf{V}(\mathbf{r})]\, \psi_i(\mathbf{r}) = \varepsilon_\mathbf{i} \psi_\mathbf{i}(\mathbf{r})$

Relativistic Point Coupling

P. Manakos and T. Mannel Z. Phys. A 330, (1989)



Linear terms $\alpha_S, \alpha_V, \alpha_{TV}$

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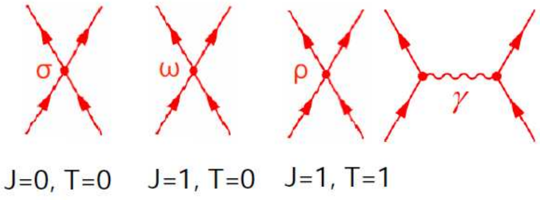

Linear terms $\alpha_S, \alpha_V, \alpha_{TV}$
+ density dependent couplings $\beta_S, \gamma_S, \gamma_V$

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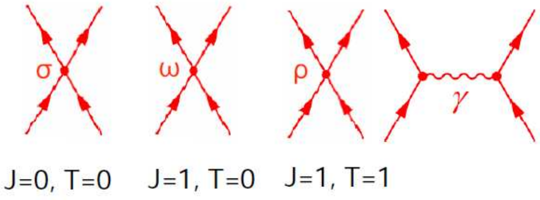

Linear terms	$\alpha_S, \alpha_V, \alpha_{TV}$
+ density dependent couplings	$\beta_S, \gamma_S, \gamma_V$
+ gradient terms	$\delta_S, \delta_V, \delta_{TV}$

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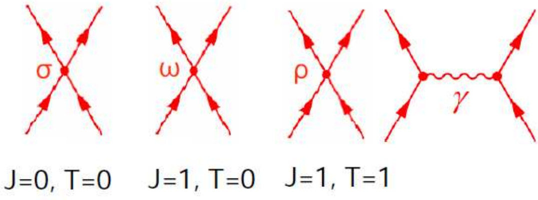

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$$E[\hat{\rho}]$$

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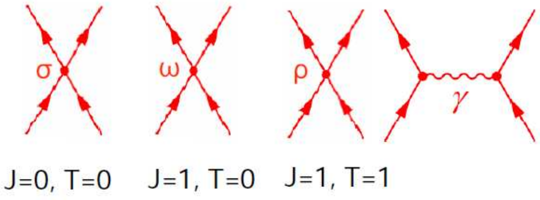

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$$E\left[\hat{\rho}\right]$$

PCF1:Buervenich et al. Phys. Rev. C65, 044308 (2002).
DDPC1:Niksic et al. Phys. Rev. C78,034318 (2008).

Beyond DFT

$\surd$ Ground state properties
(binding energies, nuclear radii, etc.)

X Collective excitations
(surface oscillations, Giant Multipole Resonances)

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How can we explain reactions that lead to collective excitations from individual motion?

$$\sigma = (2L+1)\pi^2(\hbar c)^2 E^{2L-1} S(E)$$

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How can we explain reactions that lead to collective excitations from individual motion?

$$\sigma = (2L+1)\pi^2(\hbar\, c)^2 E^{2L-1} S(E)$$

Random Phase Approximation

$$S(E) = -\frac{1}{\pi} Im\left[F_{ext} R(E) F_{ext}\right] \qquad F_{ext} = \text{external field}$$

Random Phase Approximation

Bethe-Salpeter equation:

$$R(E) = R^0(E) + R^0(E)V_{ph}R(E)$$

Random Phase Approximation

Bethe-Salpeter equation:

$$R(E) = R^0(E) + R^0(E)\, V_{ph}\, R(E)$$

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$$V_{ph} = \frac{\delta^2 E}{\delta \hat{\rho}^2}$$

Random Phase Approximation

Bethe-Salpeter equation:

$$R(E) = R^0(E) + R^0(E)\, V_{ph}\, R(E)$$

Free response function:

$$R^0(E) = ?$$

Interaction :

$$V_{ph} = \frac{\delta^2 E}{\delta \hat{\rho}^2}$$

Free response function

$$R^0(E) = \sum_h \langle h | \frac{1}{E - \hat{h} + \varepsilon_h} | h \rangle + b.g. = \sum_h \langle h | G(E + \varepsilon_h) | h \rangle + b.g.$$

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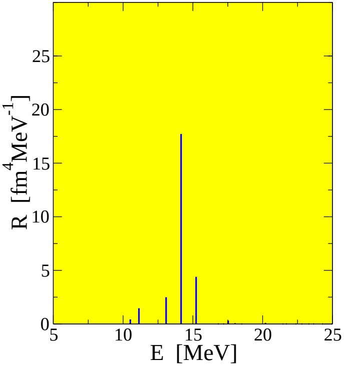
- Spectral representation (Discrete RPA)



$$G(\mathbf{r}, \mathbf{r}'; \mathbf{E}) = \sum_p^{< E_{max}} \frac{|p\rangle_r \langle p|_{r'}}{E - (\varepsilon_p - \varepsilon_h) + i\eta} + \sum_a^{< E^a_{max}} \frac{|a\rangle_r \langle a|_{r'}}{E - (\varepsilon_a - \varepsilon_h) + i\eta}$$

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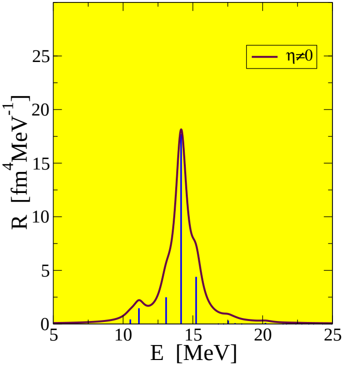

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- Smearing parameter η is needed to produce a width.

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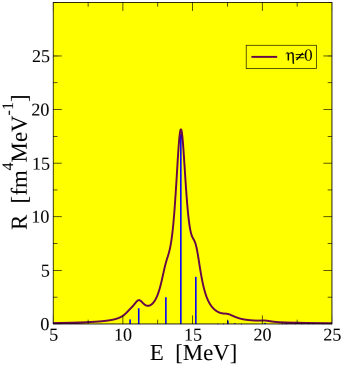

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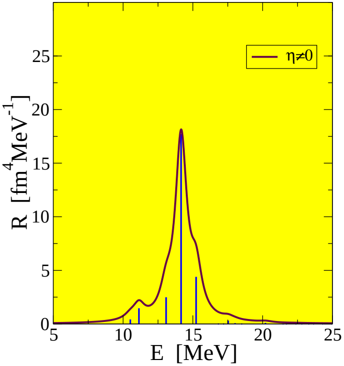

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- Smearing parameter η is needed to produce a width.
- Truncation of the space involves additional parameters and sets limitations.
- Large amount of terms contributing in R^0 (>2000).

Free response function

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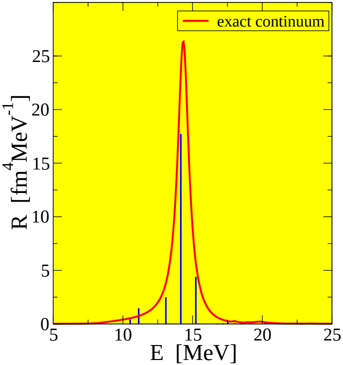

$$G(\mathbf{r}, \mathbf{r}'; \mathbf{E}) = \sum_\kappa \begin{cases} |w_\kappa(\mathbf{r})\rangle\langle \mathbf{u}^*_\kappa(\mathbf{r}')| & \mathrm{r} > \mathrm{r}' \\ |u_\kappa(\mathbf{r})\rangle\langle \mathbf{w}^*_\kappa(\mathbf{r}')| & \mathrm{r} < \mathrm{r}' \end{cases}$$

$$|u\rangle = r\begin{pmatrix} j_l(kr) \\ \frac{\kappa}{|\kappa|}\frac{E-V-S}{k} j_{\bar{l}}(kr) \end{pmatrix} \qquad |w\rangle = r\begin{pmatrix} h_l^{(1)}(kr) \\ \frac{\kappa}{|\kappa|}\frac{ikr}{E+2m} h_l^{(1)}(kr) \end{pmatrix}$$

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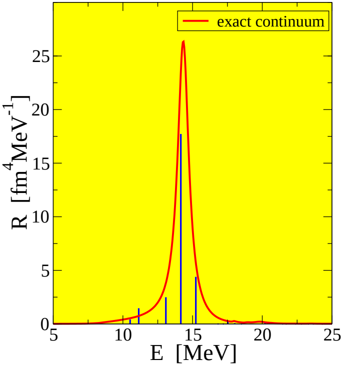
- Small amount of terms contributing in R^0 ($<$50)



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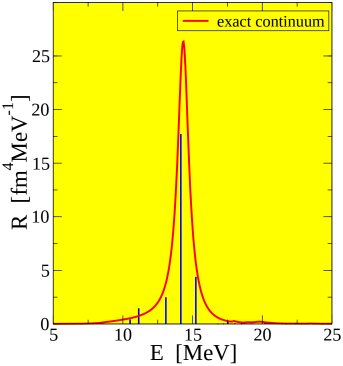

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- Antiparticle-hole pairs are included effectively.

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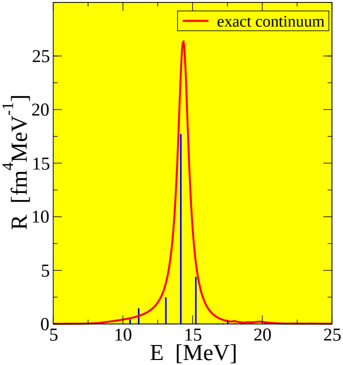

- Small amount of terms contributing in R^0 ($<$50)
- Antiparticle-hole pairs are included effectively.
- Resonance width is reproduced without smearing parameter (escape width $\Gamma^\uparrow$)

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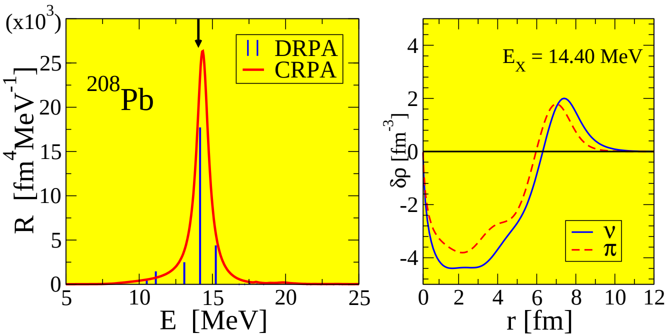
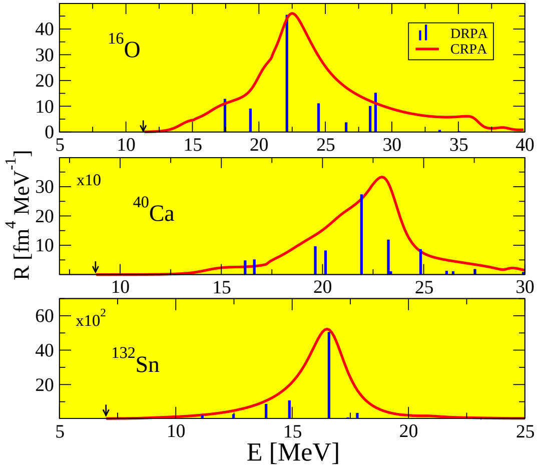
Isoscalar Monopole Resonance



	CRPA	DRPA	Exp.
$\bar{E}$ [MeV]	14.40	14.17	13.96$\pm$0.2
Γ [MeV]	0.95		2.88 $\pm$0.2
	CRPA	DRPA	TRK
m_1 $[10^5 MeV fm^4]$	5.448	5.446	5.453

Isoscalar Monopole Resonance



J. Daoutidis and P. Ring PRC 80, 024309 (2009)

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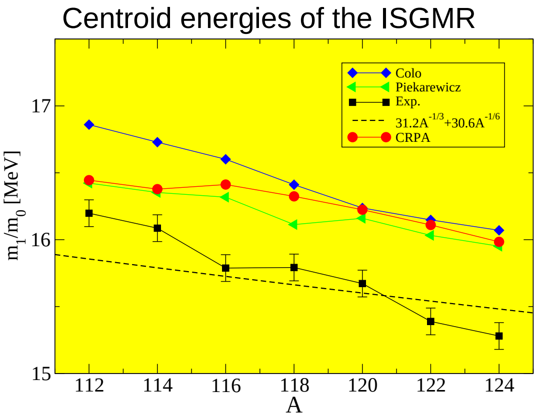
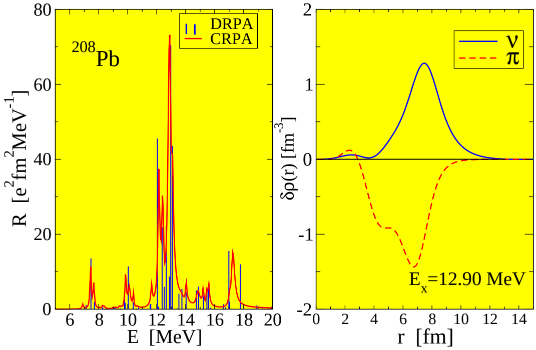


Isovector Dipole Resonance



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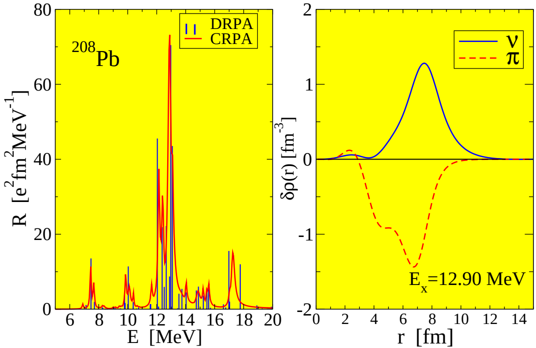
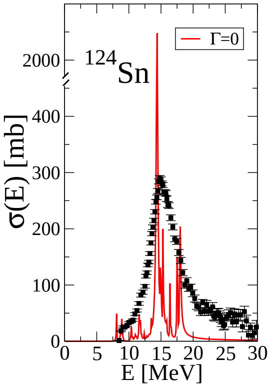


5-10 times faster numerical calculations!

Isovector Dipole Resonance

Coupling to more complex configurations (spreading width):

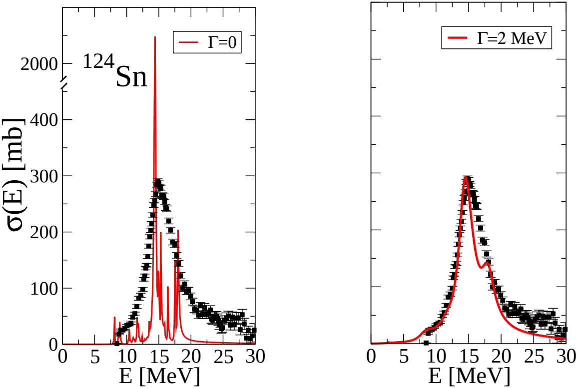
$$G(E) \rightarrow G(\omega + i\Gamma), \qquad \Gamma = 0$$



Isovector Dipole Resonance

Coupling to more complex configurations (spreading width):

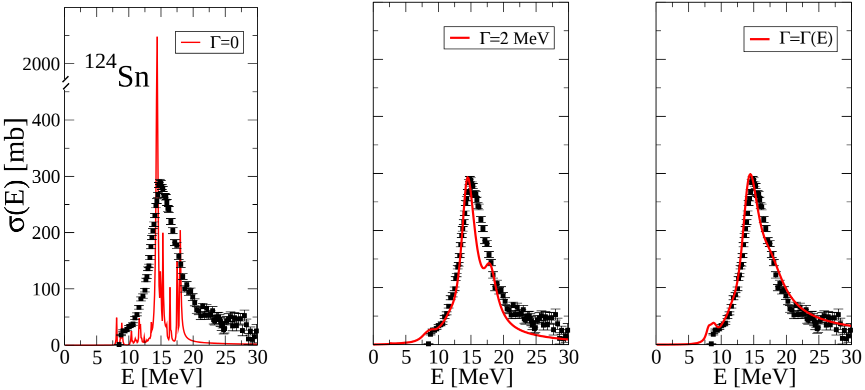
$$G(E) \rightarrow G(\omega + i\Gamma), \qquad \Gamma = constant$$



Isovector Dipole Resonance

Coupling to more complex configurations (spreading width):

$$G(E) \rightarrow G(\omega + i\Gamma), \qquad \Gamma(E) = \frac{E^2 + 4\pi^2 T^2}{E^2_{GDR}}$$



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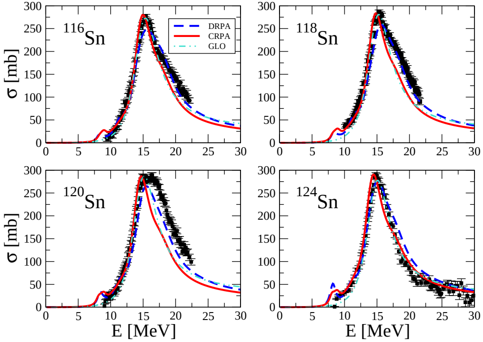
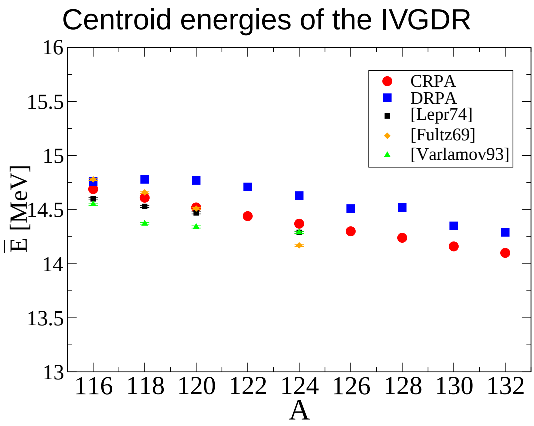


Exp.: Lepretre et. al. NPA 219,39 (1974)

Isovector Dipole Resonance



Isovector Dipole Resonance

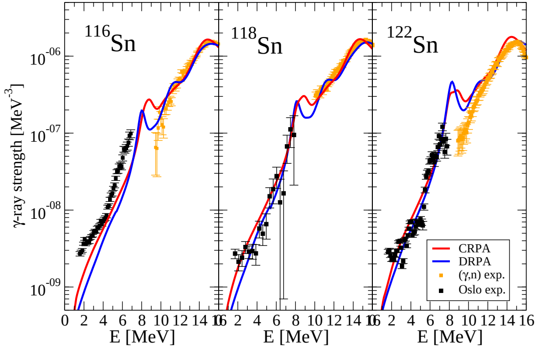
Performance of different point-coupling parameterizations

		DD-PC1	PC-F1	Expt. [MeV]
^{70}Zn	E_0	17.50	16.70	17.25 $\pm$ 0.08
	$\bar{E}$	16.00	15.86	15.68 $\pm$ 0.02
^{94}Zr	E_0	16.60	15.60	16.67 $\pm$ 0.07
	$\bar{E}$	15.90	15.58	16.00 $\pm$ 0.01
^{124}Sn	E_0	15.40	14.40	14.67 $\pm$ 0.08
	$\bar{E}$	14.99	14.70	14.34 $\pm$ 0.02
^{130}Te	E_0	15.30	14.60	14.53 $\pm$ 0.13
	$\bar{E}$	14.96	14.66	14.27 $\pm$ 0.01
^{138}Ba	E_0	15.20	14.40	15.29 $\pm$ 0.15
	$\bar{E}$	14.89	14.55	14.64 $\pm$ 0.01
^{144}Sm	E_0	15.10	14.50	15.37 $\pm$ 0.13
	$\bar{E}$	15.39	14.58	14.77 $\pm$ 0.02
^{208}Pb	E_0	13.60	12.80	13.50 $\pm$ 0.19
	$\bar{E}$	14.40	14.03	13.96 $\pm$ 0.20

→ play video. . .

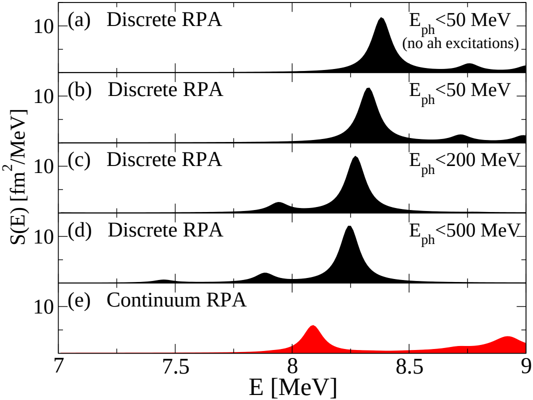
Pygmy Dipole Resonance

Conparison with new experimental results



J. Daoutidis and S. Goriely, in Prep.
Experiment: Toft et. al. PRC81, 64311 (2010) and PRC83, 44320 (2011)

Pygmy Dipole Resonance

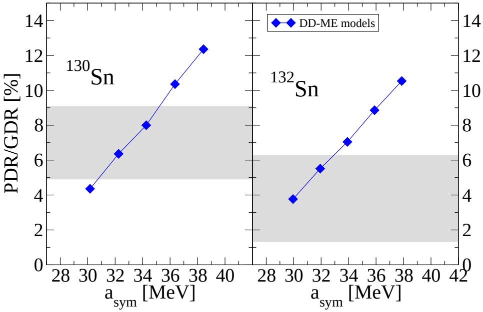


J. Daoutidis and P. Ring PRC 83, 044303 (2011)

asymmetry energy

Can the pygmy mode give information of the nuclear matter asymmetry energy a_{sym}?

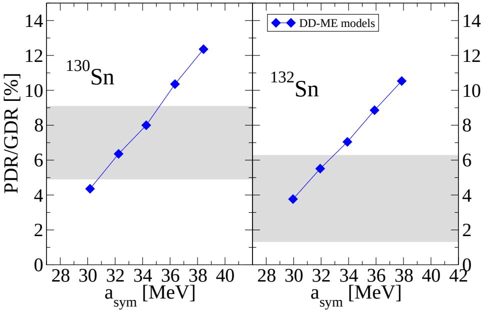
asymmetry energy



$$a_{sym} = 32.0 \pm 1.8\text{MeV}$$

A. Klimkiewicz et al, Phys. Rev. C76, 051603 (2007).

asymmetry energy



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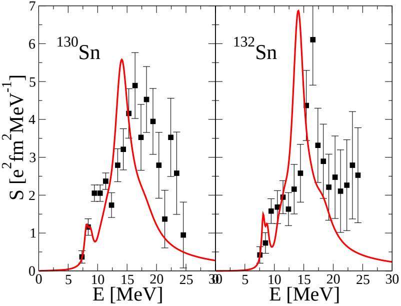
$$a_{sym} = 32.0 \pm 1.8\text{MeV}$$

- Pygmy strength integrated up to 11 MeV,
- The calculated strength is shifted about 1 MeV compared to the measured one to account for the theoretical uncerteinties.

asymmetry energy

Things are not so simple...

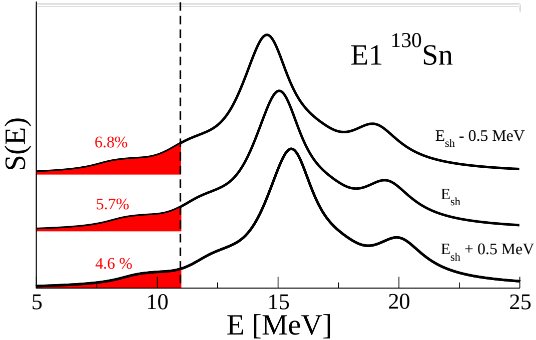


	^{130}Sn	^{132}Sn
$\bar{E}[MeV]$	16.35$\pm$0.50	16.1$\pm$0.70

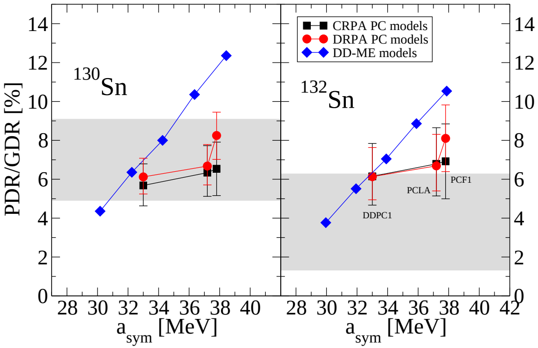
P. Adrich et al, Phys. Rev. Lett. 95, 132501 (2005).

asymmetry energy

The large error bar of the centroid energies should be considered in the shift.



asymmetry energy



asymmetry energy

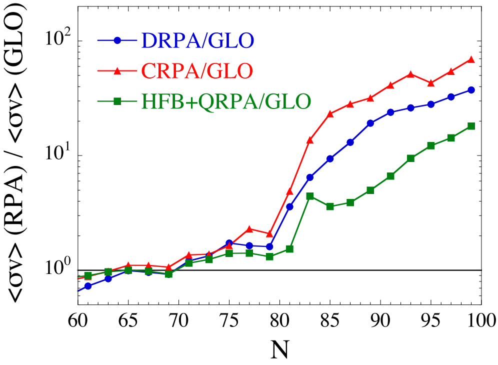
Can the pygmy mode give information of the nuclear matter asymmetry energy a_{sym}?

-We need improved experimental data to answer

Reaction Rates

Hauser-Feshbach statistical method

$$\langle \sigma v \rangle = \left(\frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E e^{-\frac{E}{kT}} \, dE$$



Summary

1. Relativistic DFT plus QRPA allows to calculate excitation strength in a fully-self consistent way
2. Continuum QRPA calculations are now possible.
   They appear to be important for
   - the study of soft modes, sensitive to basis trancation.
   - quantitive improvement of the collective properties
   - determination of the escape width of the resonances
   - considerably reducing the numerical effort

Summary

1. Relativistic DFT plus QRPA allows to calculate excitation strength in a fully-self consistent way
2. Continuum QRPA calculations are now possible. They appear to be important for
   - the study of soft modes, sensitive to basis trancation.
   - quantitive improvement of the collective properties
   - determination of the escape width of the resonances
   - considerably reducing the numerical effort
3. Gamow-Teller resonances for astrophysical purposes (beta-decay rates, r-process path)
4. Apply Relativistic Hartree Bogoliubov theory to treat pairing correlations at the cases where BCS fails (drip lines, halo nuclei)
5. Extend to include deformed nuclei

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Thank you