Chaos, Thermalization and Statistical Features of Complex Nuclei

Vladimir Zelevinsky

NSCL/ Michigan State University

Supported by NSF and Binational Science Foundation USA-Israel

Workshop on Level Density and Gamma Strength

Oslo, May 13, 2009

OUTLINE

1. Introduction: Many-body quantum chaos
2. Nuclear case
3. Thermodynamics and chaos
4. Pairing phase transition
5. Ground state spin with random interactions
6. Predominance of prolate deformations
7. Summary

THANKS

- B. Alex Brown (NSCL, MSU)
- Mihai Horoi (Central Michigan University)
- Declan Mulhall (Scranton University)
- Valentin Sokolov (Budker Institute)
- Alexander Volya (Florida State University)

Chaotic motion in nuclei

* Mean field (one-body chaos)
* Strong interaction (mainly two-body)
* High level density
* Mixing of simple configurations
* Destruction of quantum numbers,
  (still conserved energy, J,M;T,T3;parity)
* Spectral statistics – Gaussian Orthogonal Ensemble
* Correlations between classes of states
* Coexistence with (damped) collective motion
* Analogy to thermal equilibrium
* Continuum effects

MANY-BODY QUANTUM CHAOS AS AN INSTRUMENT

SPECTRAL STATISTICS – ***signature of chaos***

- ***missing levels***
- ***purity of quantum numbers***
- ***statistical weight of subsequences***
- ***presence of time-reversal invariance***

EXPERIMENTAL TOOL – ***unresolved fine structure***

- ***width distribution***
- ***damping of collective modes***

NEW PHYSICS

- ***statistical enhancement of weak perturbations (parity violation in neutron scattering and fission)***
- ***mass fluctuations***
- ***chaos on the border with continuum***

THEORETICAL CHALLENGES

- **order our of chaos**
- **chaos and thermalization**
- **new approximations in many-body problem**
- **development of computational tools**

CHAOS versus THERMALIZATION

L. BOLTZMANN – *Stosszahlansatz* = **MOLECULAR CHAOS**

N. BOHR - *Compound nucleus* = **MANY-BODY CHAOS**

N. S. KRYLOV - *Foundations of statistical mechanics*

L. Van HOVE – *Quantum ergodicity*

L. D. LANDAU and E. M. LIFSHITZ – "*Statistical Physics*"

Average over the equilibrium ensemble should coincide with the expectation value in a generic individual eigenstate of the same energy – the results of measurements in a closed system do not depend on exact microscopic conditions or phase relationships if the eigenstates at the same energy have similar macroscopic properties

TOOL: MANY-BODY QUANTUM CHAOS

CLOSED MESOSCOPIC SYSTEM

at high level density

Two languages: ***individual wave functions***
thermal excitation

* Mutually exclusive ?
* Complementary ?
* Equivalent ?

Answer depends on thermometer

FAMILY OF ENTROPIES FOR A MESOSCOPIC SYSTEM

- **THERMODYNAMIC** (*Boltzmann*)

$\rho(E) \propto \exp(S_{\rm th})$

- **QUASIPARTICLE** (*Landau Fermi-liquid*)

$S^{\alpha}_{\rm s.p.} = -\Sigma_i \{ n^{\alpha}_i \ln(n^{\alpha}_i)$
$+ (1 - n^{\alpha}_i) \ln(1 - n^{\alpha}_i) \}$

- **INFORMATION** (*Shannon*)

$|\alpha\rangle = \Sigma_k C^{\alpha}_k |k\rangle, \quad S^{\alpha}_{\rm inf} = -\Sigma_k \{ |C^{\alpha}_k|^2 \ln |C^{\alpha}_k|^2 \}$

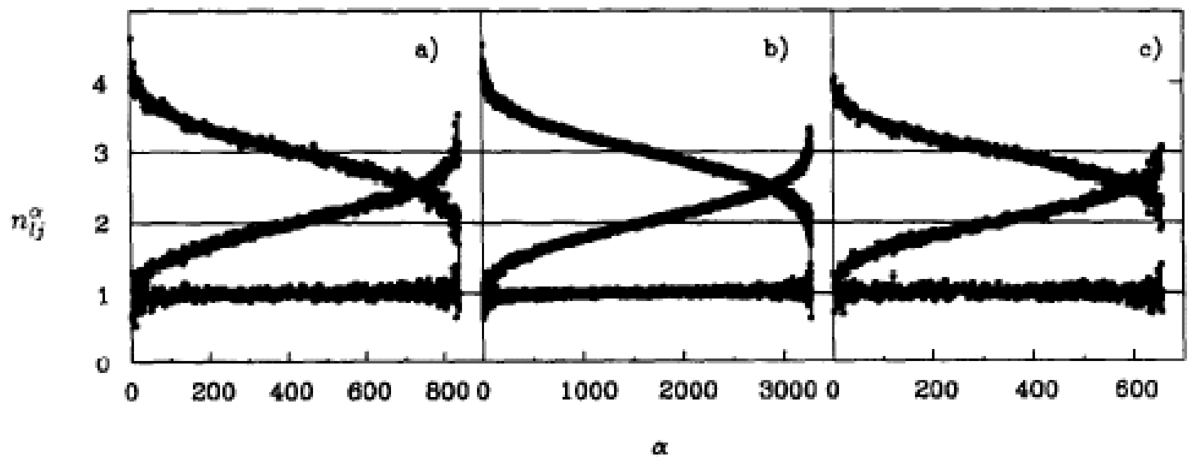
$$\langle n_i \rangle_E = [e^{(\epsilon_i - \mu)/T_{\rm s.p.}} + 1]^{-1}$$

Temperature T(E)

$$T_{\rm th} = \left(\frac{dS_{\rm th}}{dE} \right)^{-1}$$

$$T_{\rm inf} = \left(\frac{d\bar{S}_{\rm inf}}{dE} \right)^{-1}$$

T(s.p.) and T(inf) = for individual states !



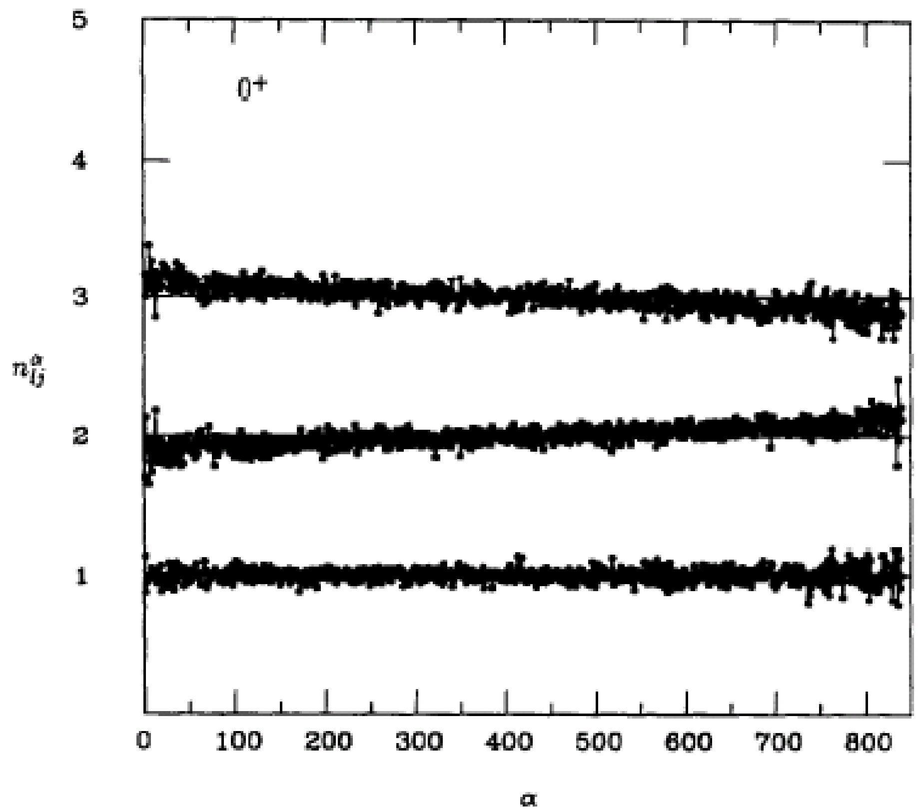
J=0 *J=2* *J=9*

Single – particle occupation numbers

Thermodynamic behavior
identical in all symmetry classes

FERMI-LIQUID PICTURE

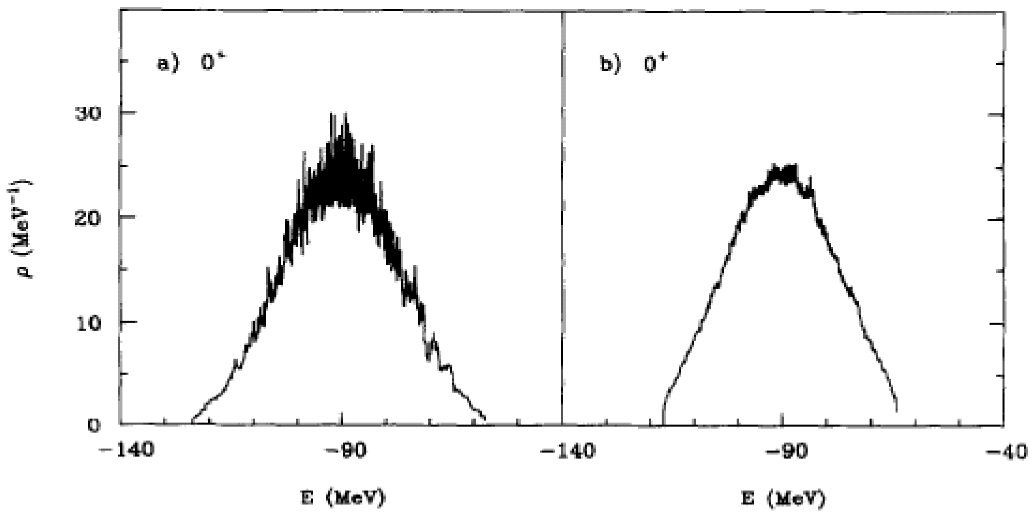
J=0



Artificially strong interaction (factor of 10)

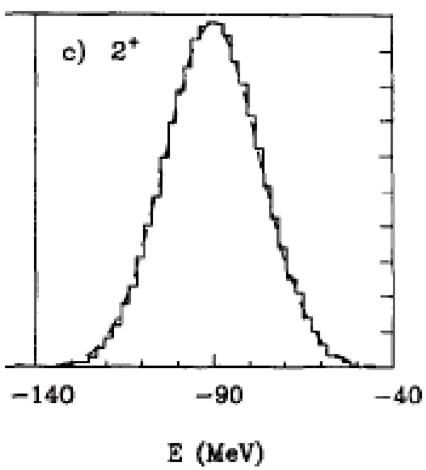
Single-particle thermometer cannot resolve spectral evolution

Shell model level density (28Si, J=0, T=0)

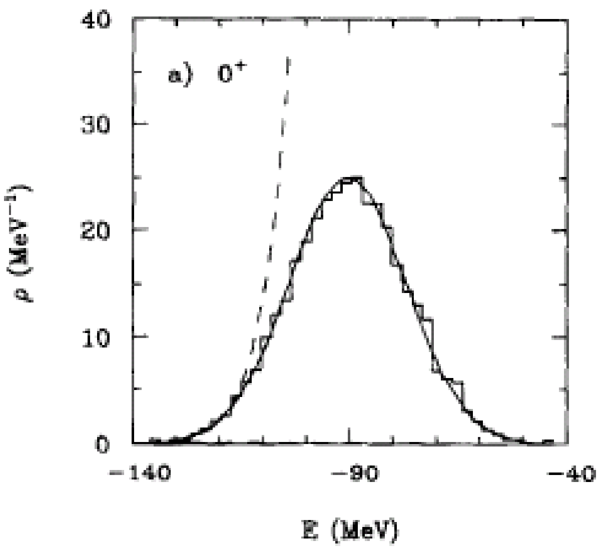


Averaging over
(a) 10 levels
(b) 40 levels

(*distorted edges*)

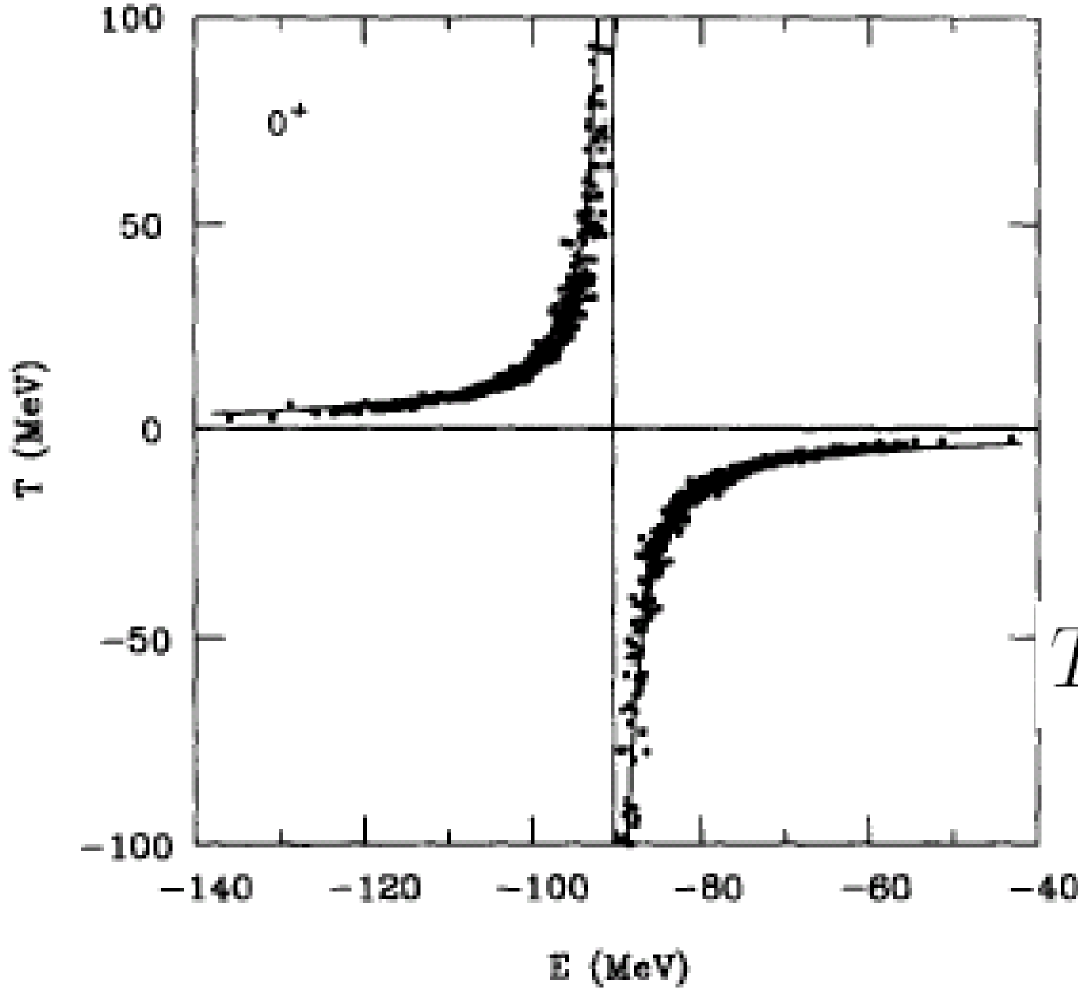


J = 2, T = 0



Shell model versus Fermi-gas

a = 1.4/MeV
a (F-G) = 2/MeV
(**two parities**?)



Gaussian level density

CENTROID E_0

WIDTH σ_E

$$T_{\rm th} = \sigma_E^2/(E_0 - E)$$

839 states (28 Si)

EFFECTIVE TEMPERATURE of INDIVIDUAL STATES

From occupation numbers in the shell model solution (dots)
From thermodynamic entropy defined by level density (lines)

MEASURING COMPLEXITY

Eigenstate $|\alpha\rangle$ in a shell model basis $|k\rangle$

$|\alpha\rangle = \sum_k C_k^\alpha |k\rangle$

Information entropy

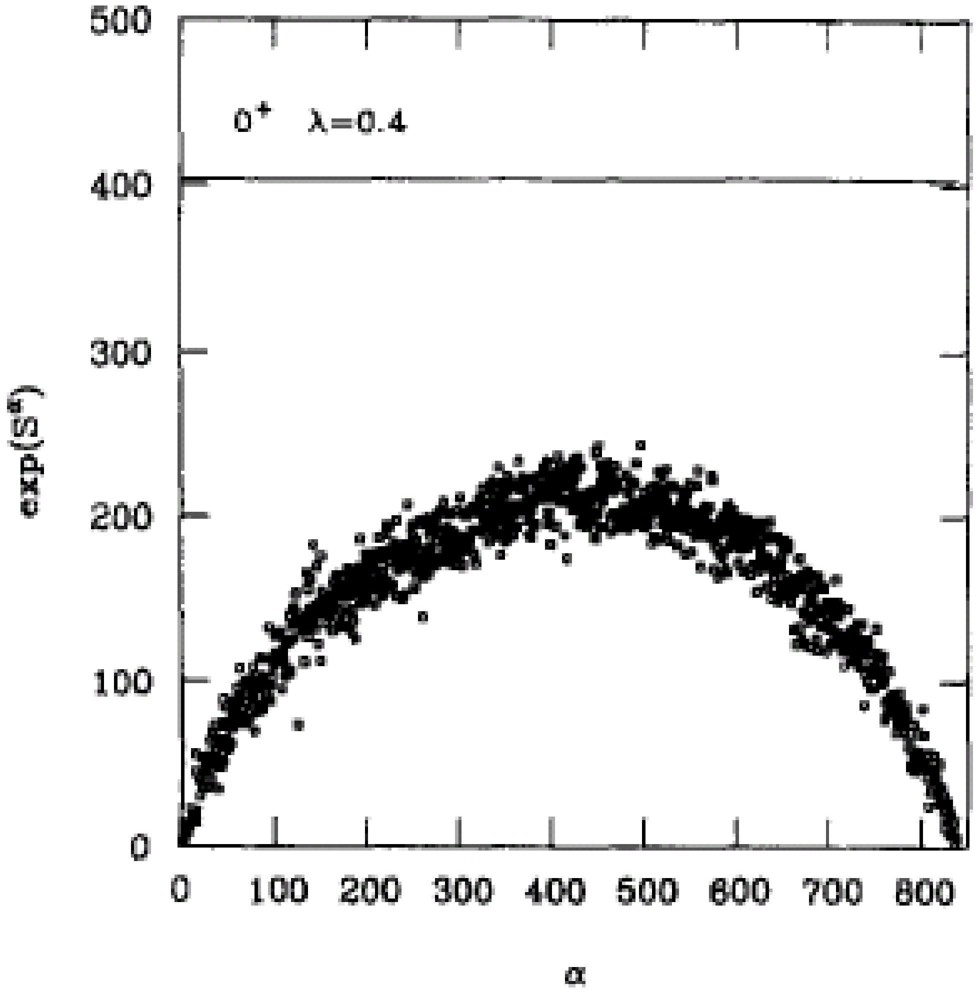
$S^\alpha = -\sum_k |C_k^\alpha|^2 \ln |C_k^\alpha|^2$

No mixing: $S^\alpha \to 0$

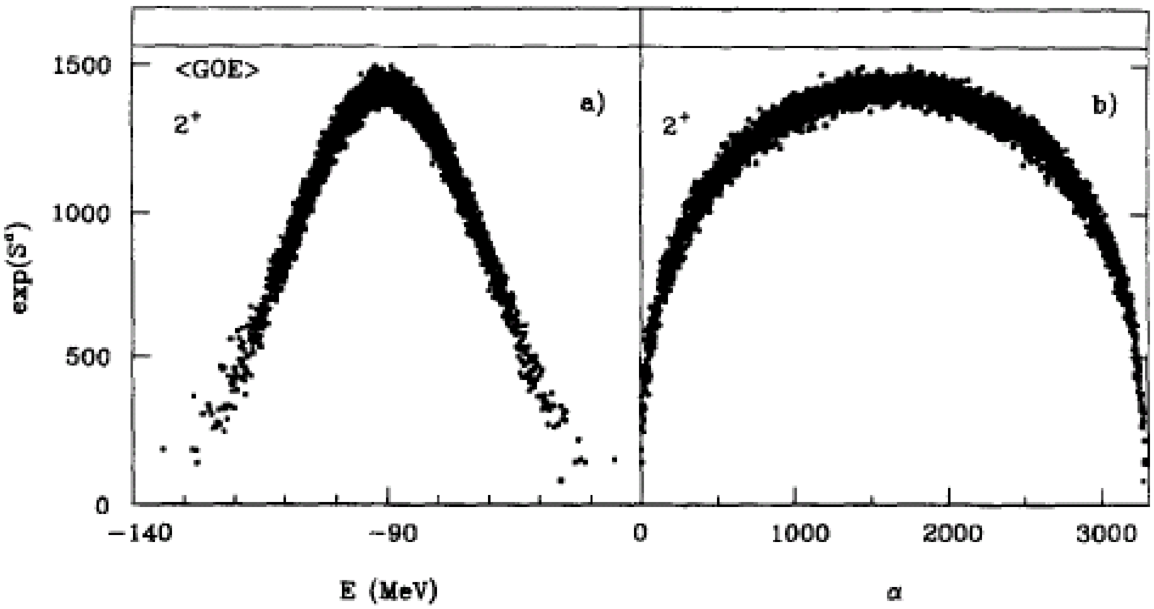
"Microcanonical" mixing: $S^\alpha \to \ln N$

GOE: $\overline{S^\alpha} = \ln(0.48N)$

Information entropy is basis-dependent
- special role of mean field



INFORMATION ENTROPY AT WEAK INTERACTION

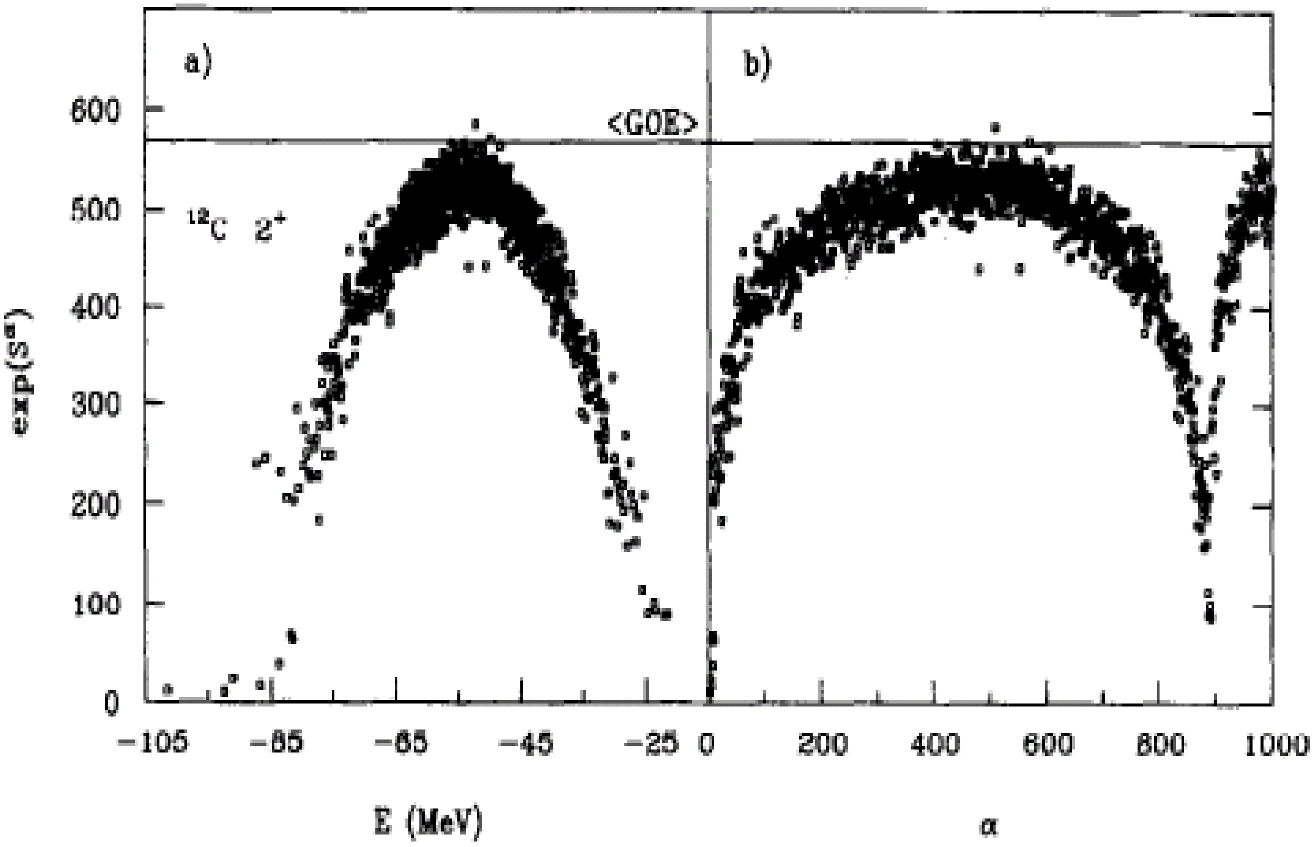


INFORMATION ENTROPY of EIGENSTATES

(a) function of energy; (b) function of ordinal number

ORDERING of EIGENSTATES of GIVEN SYMMETRY

SHANNON ENTROPY AS THERMODYNAMIC VARIABLE



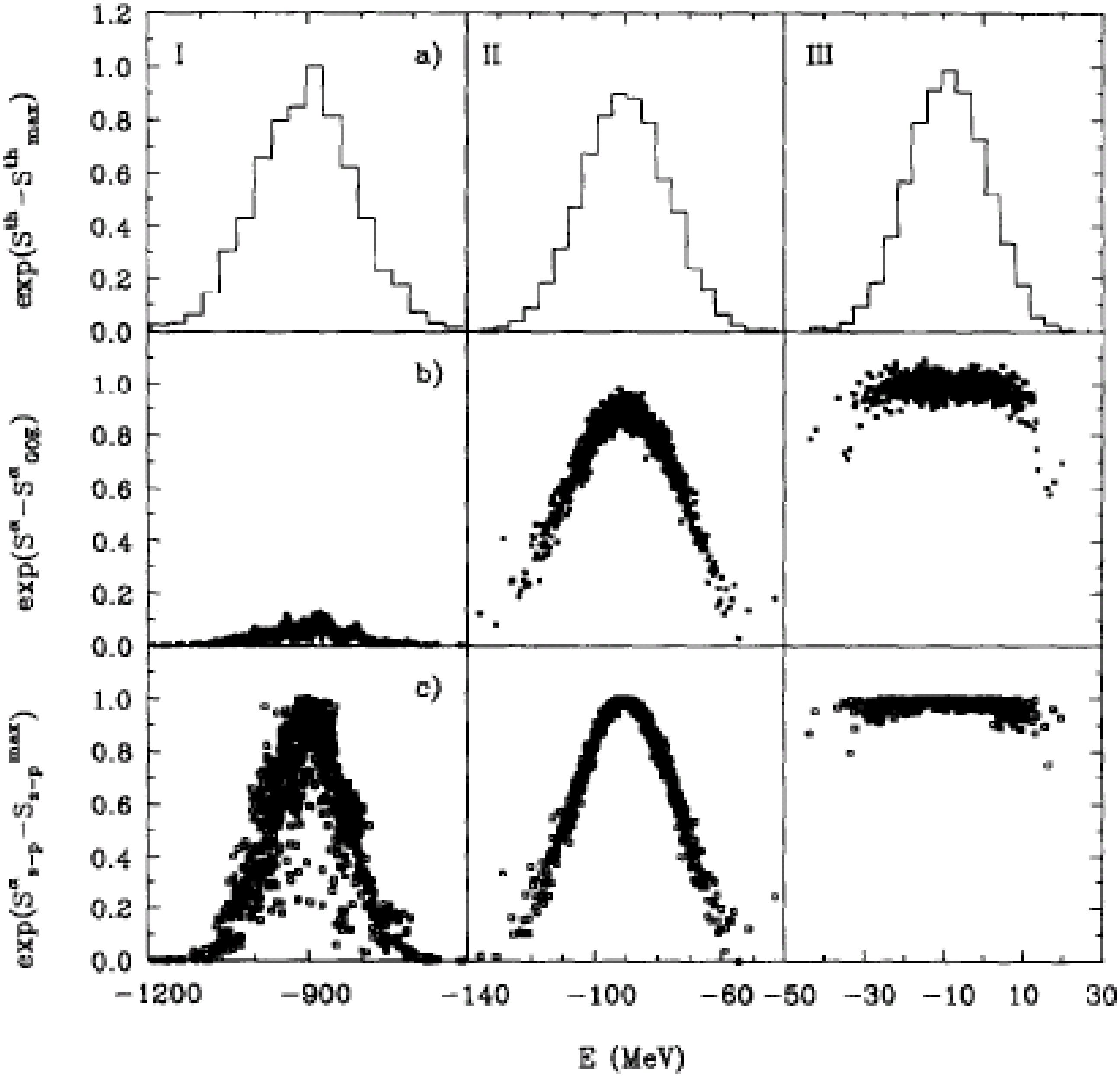
^{12}C

1183 states

Smart information entropy

(separation of center-of-mass excitations of lower complexity shifted up in energy)

CROSS-SHELL MIXING WITH SPURIOUS STATES



Exp (S)
Various measures

Level density

Information Entropy in units of S(GOE)

Single-particle entropy of Fermi-gas

Interaction: 0.1 1 10

Invariant correlational entropy as signature of phase transitions

$$|\alpha(\lambda)\rangle = \sum_k C_k^\alpha(\lambda)|k\rangle.$$

Eigenstates in an arbitrary basis
(Hamiltonian with random parameters)

$$\rho_{kk'}^\alpha(\lambda) = \overline{C_k^\alpha C_{k'}^{\alpha *}}$$

Density matrix of a given state
(averaged over the ensemble)

$$S^\alpha(\lambda) = -\mathrm{Tr}\{\rho^\alpha \ln(\rho^\alpha)\}$$

$$\lambda \in [\lambda, \lambda+\delta]$$

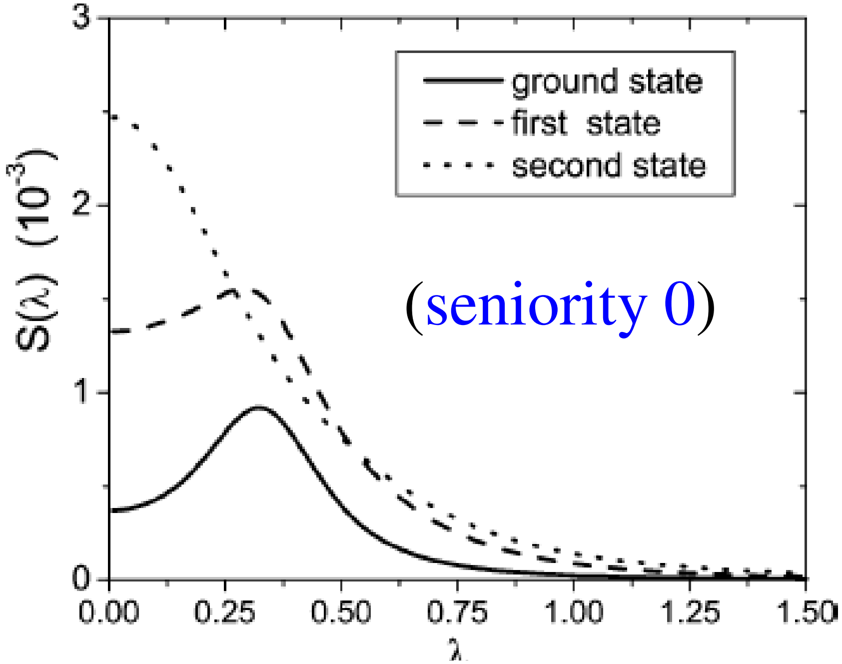
Correlational entropy
has clear maximum
at phase transition
(extreme sensitivity)

Pure state: eigenvalues of the density matrix are **1 (one) and 0 (N-1),**
S=0

Mixed state: between 0 and 1, **S up to ln N**

For two discrete points

$$r_\pm^\alpha = \frac{(1 \pm |\langle\alpha(\lambda)|\alpha(\lambda')\rangle|)}{2}$$



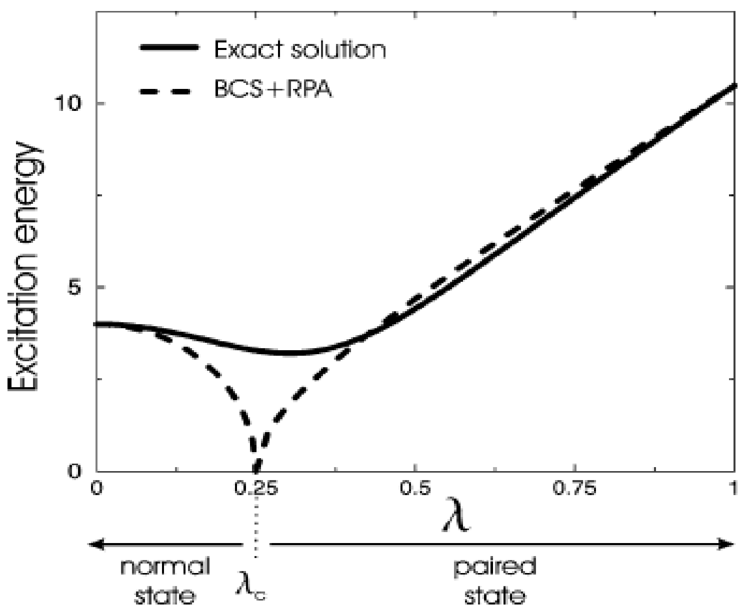
Model of two levels with pair transfer

Capacity 16 + 16, N=16

Critical value 0.3

(in BCS ¼)

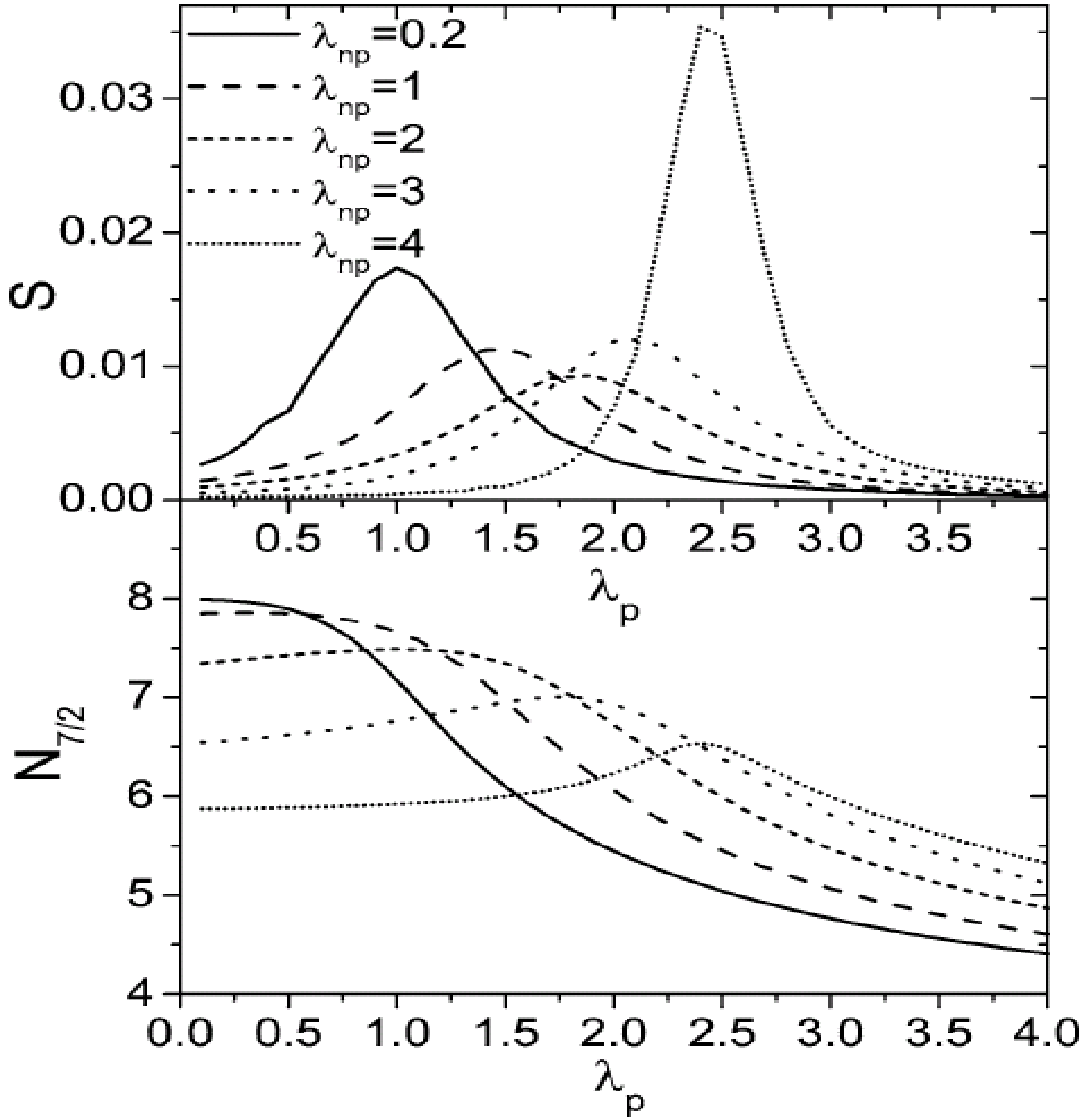
Averaging interval 0.01



First excited state "pair vibration"

No instability in the exact solution

Softening at the same point 0.3

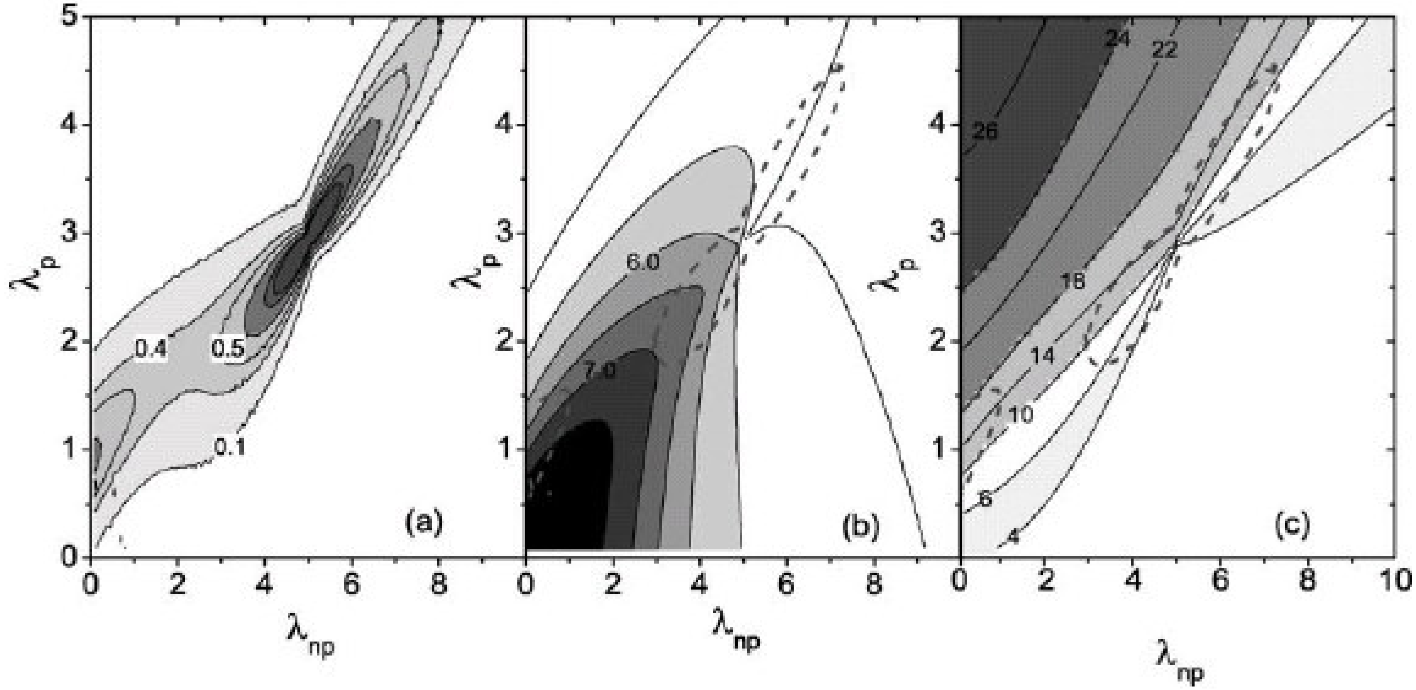


Shell model 48Ca

Ground state invariant entropy; phase transition depends on non-pairing interactions

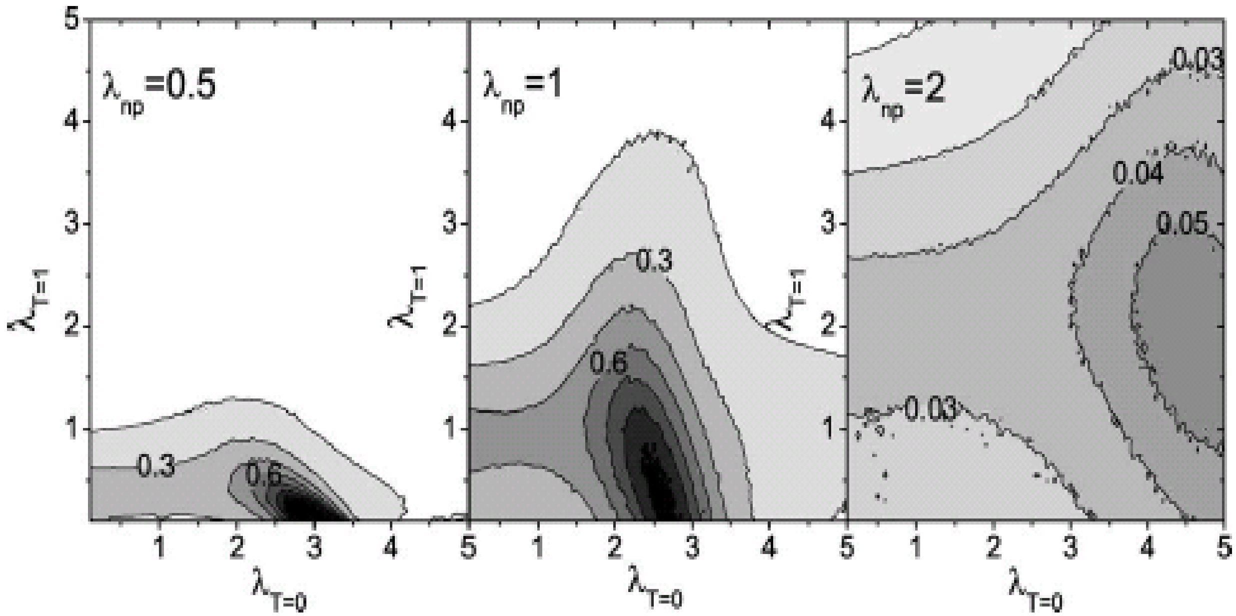
Occupancy of f7/2 shell

Correlation energy ~ 2 MeV



48 Ca

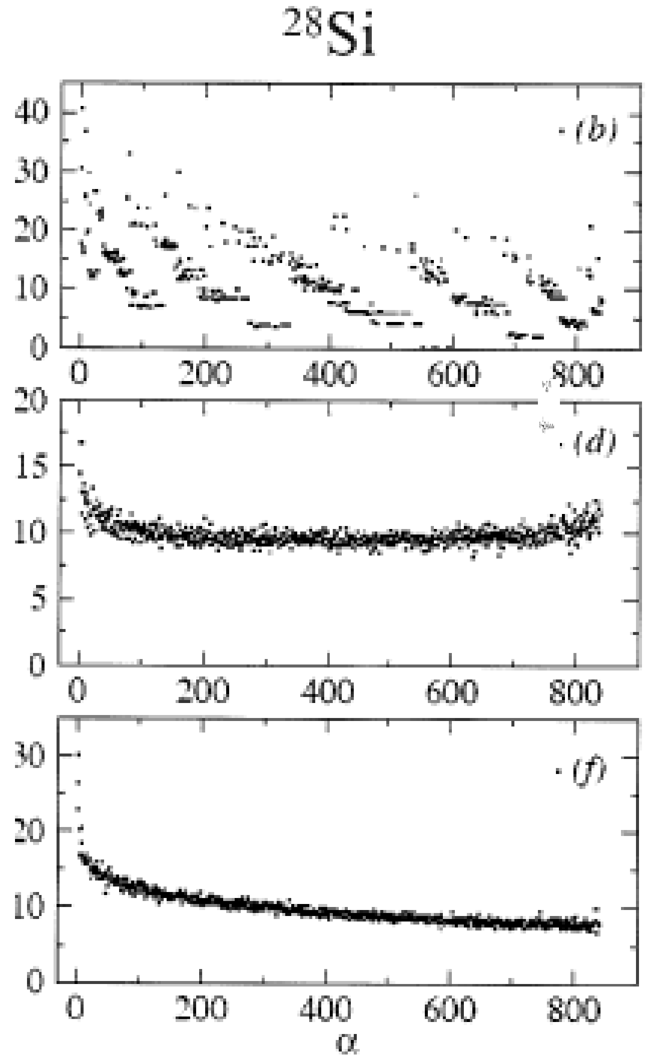
(a) Invariant entropy and the line of phase transitions
(b) Occupancy of the f7/2 orbital
(c) Effective number of T=1 pairs



24 Mg

Isovector against isoscalar pairing
Dependence on non-pairing interactions
(phase transitions smeared,
absolute values of entropy suppressed)

Critical value for T=0 phase transition: ~ 3 ***/Bertsch, 2009/***



PAIR CORRELATOR

(b) Only pairing

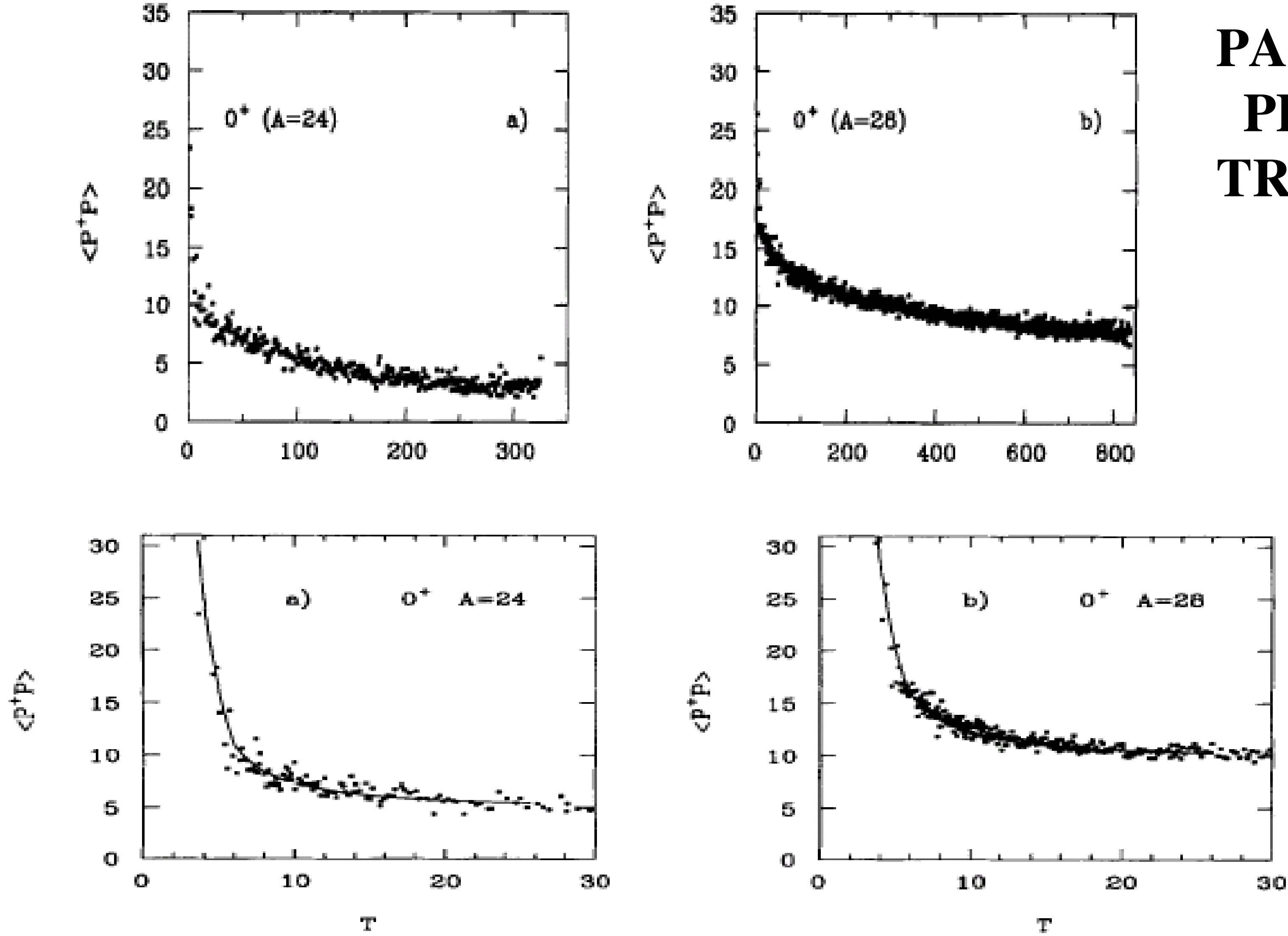
(d) Non-pairing interactions

(f) All interactions

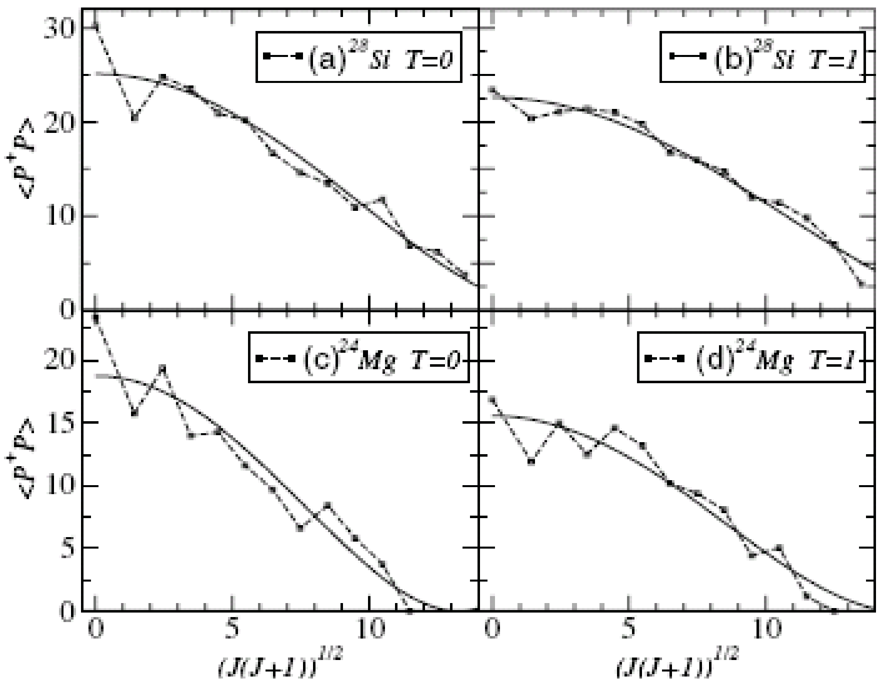
$$\mathcal{H}_P = \sum_{t=0,\pm 1} P_t^\dagger P_t$$

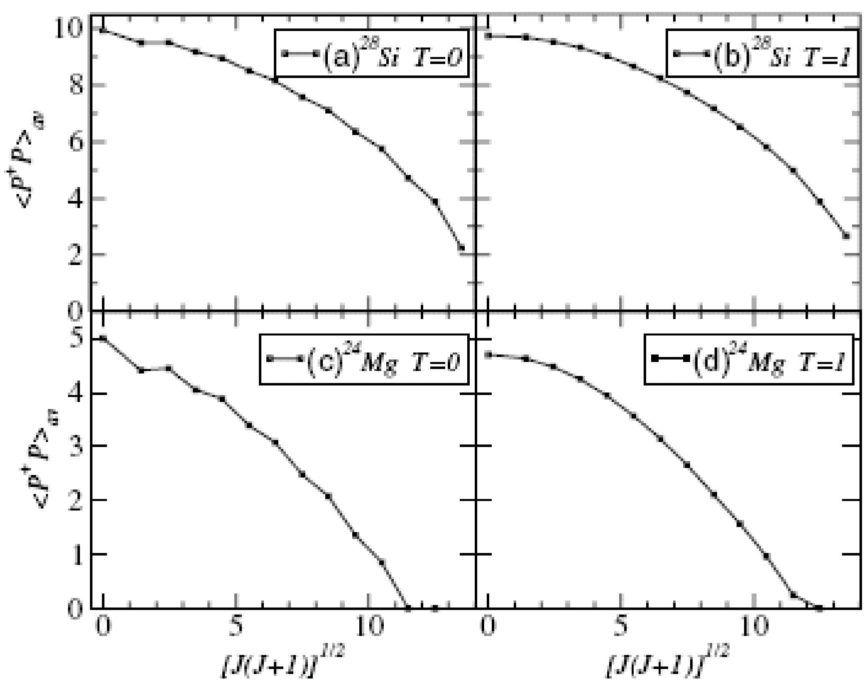
$$P_t = \frac{1}{\sqrt{2}} \sum_j [a_j a_j]_{J=0,T=1,T_3=t}$$

PAIRING PHASE TRANSITION



PAIR CORRELATOR as a THERMODYNAMIC FUNCTION





Pair correlator as a function of J

Yrast states

$$\langle \mathcal{H}_P(J) \rangle = \langle \mathcal{H}_P(0) \rangle \left[1 - \frac{J(J+1)}{B} \right]^2 .$$

Average over all states

Old semiclassical theory
(Grin'& Larkin, 1965)

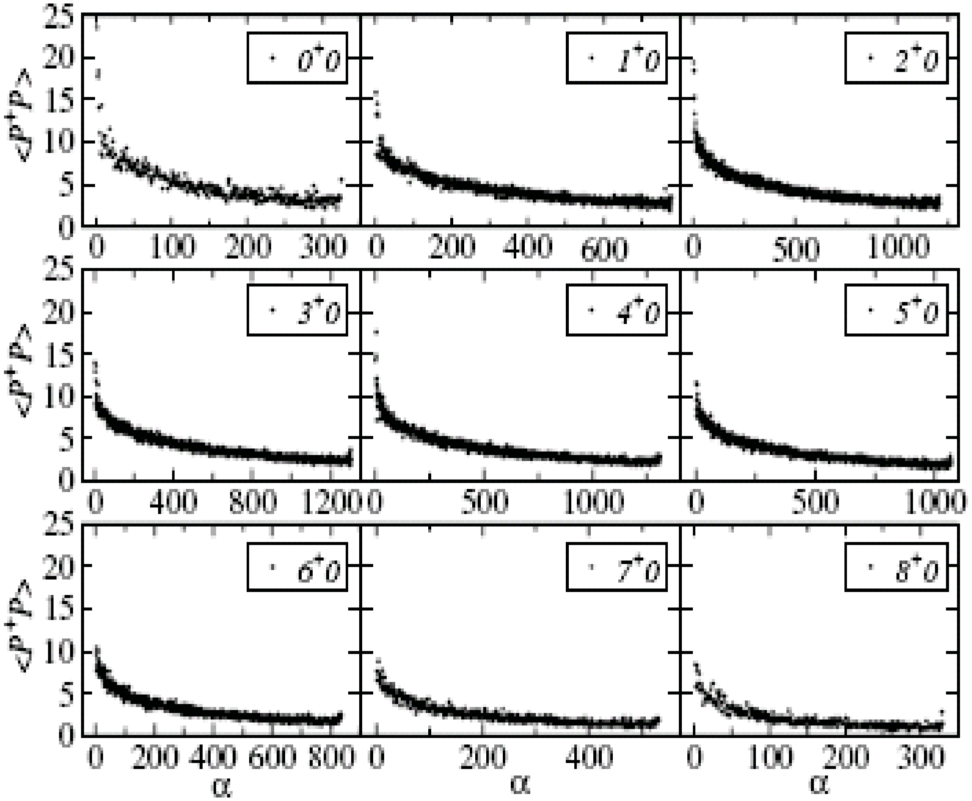
$$\Delta(J) \approx \Delta(0) \left[1 - \frac{J(J+1)}{J_c^2} \right]$$

$$J_c = a \frac{\Delta(0) I_r}{l_0}$$ **(too small)**

Geometry of orbital space rather than Coriolis force

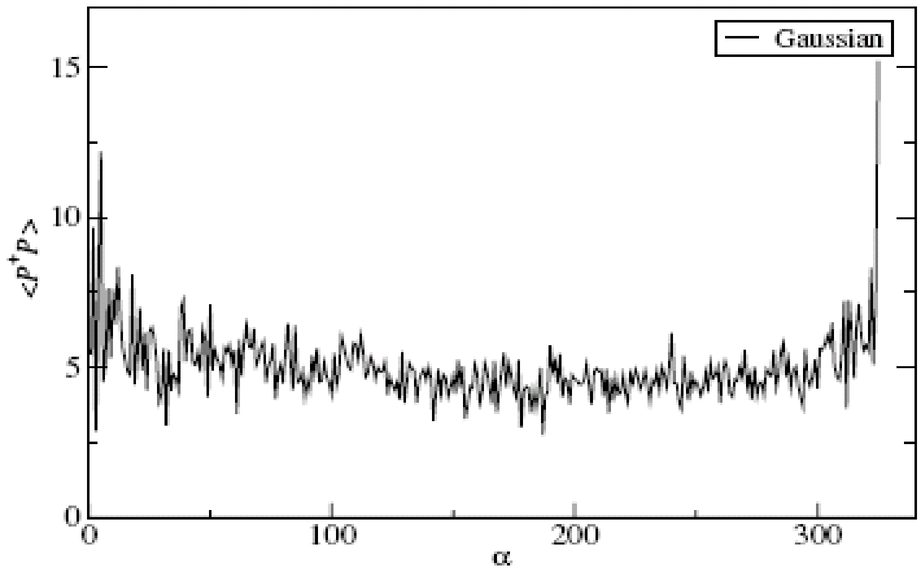
GLOBAL BEHAVIOR

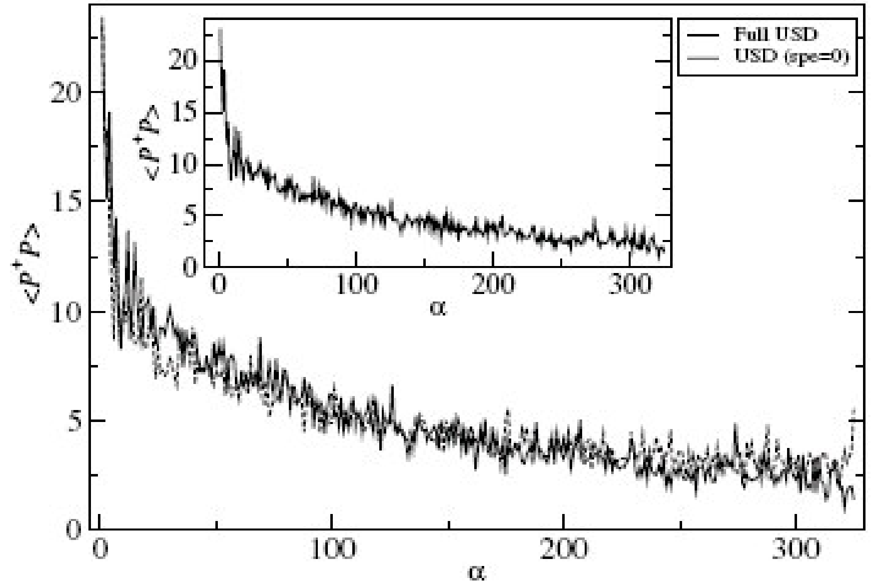
Pair correlator in 24 Mg for all states of various spins



Central part of the spectrum is well described by statistical model with mean occupation numbers

J=0, T=0 states in 24 Mg





Realistic single-particle energies + random interactions
(Gaussian matrix elements with zero mean and the same variance as in realistic interaction)
Enhancement – for the states of lowest complexity

Degenerate s.-p. energies + realistic interactions
Growing level density quickly leads to chaos
In the absence of the mean-field skeleton, pairing works for lowest states only

RESULTS

- **Regular behavior of pair correlator in a mesoscopic system**
- **Long tail beyond “phase transition”**
- **Similar picture for all spin and isospin classes**
- **In the middle – semiclassical picture with average occupation numbers of single-particle orbitals**
- **Pairing is considerably influenced by non-pairing interaction**
- **Are the shell model results generic?**
  - **exact solution**
  - **rotational invariance**
  - **isospin invariance**
  - **well tested at low energy**
  - **with growing level density leads to many-body quantum chaos in agreement with random matrix theory**
  - **loosely bound systems and effects of continuum**

DO WE UNDERSTAND
ROLE of INCOHERENT INTERACTIONS ?

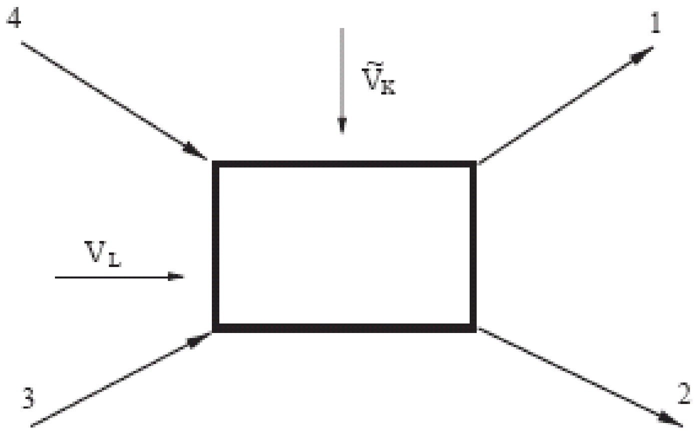
- Ground state predominantly J=0 (even A)

- Ordered structure of wave functions ?

- New aspects of quantum chaos:

- **correlations between different symmetry sectors governed by the same Hamiltonian**
- **geometry of a mesoscopic system**
- **“random” mean field**
- **effects of time-reversal invariance**
- **exploration of interaction space**
- **manifestations of collective phenomena**

ORDER FROM RANDOM INTERACTIONS ?



$$H_{int} = \sum_{L\Lambda, tt_3; \{j\}} V_{Lt}(j_1 j_2; j_3 j_4) P^{\dagger}_{L\Lambda, tt_3}(j_1 j_2) P_{L\Lambda, tt_3}(j_3 j_4)$$

$$P_{L\Lambda, tt_3}(j_1 j_2) = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}} [a_{j_1} a_{j_2}]_{L\Lambda, tt_3}$$

$$P^{\dagger}_{L\Lambda, tt_3}(j_1 j_2) = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}} [a^{\dagger}_{j_2} a^{\dagger}_{j_1}]_{L\Lambda, tt_3}$$

FULL ROTATIONAL INVARIANCE

FERMI-STATISTICS

$$P_{L\Lambda, tt_3}(j_1 j_2) = (-)^{j_1+j_2+L+t} P_{L\Lambda, tt_3}(j_2 j_1)$$

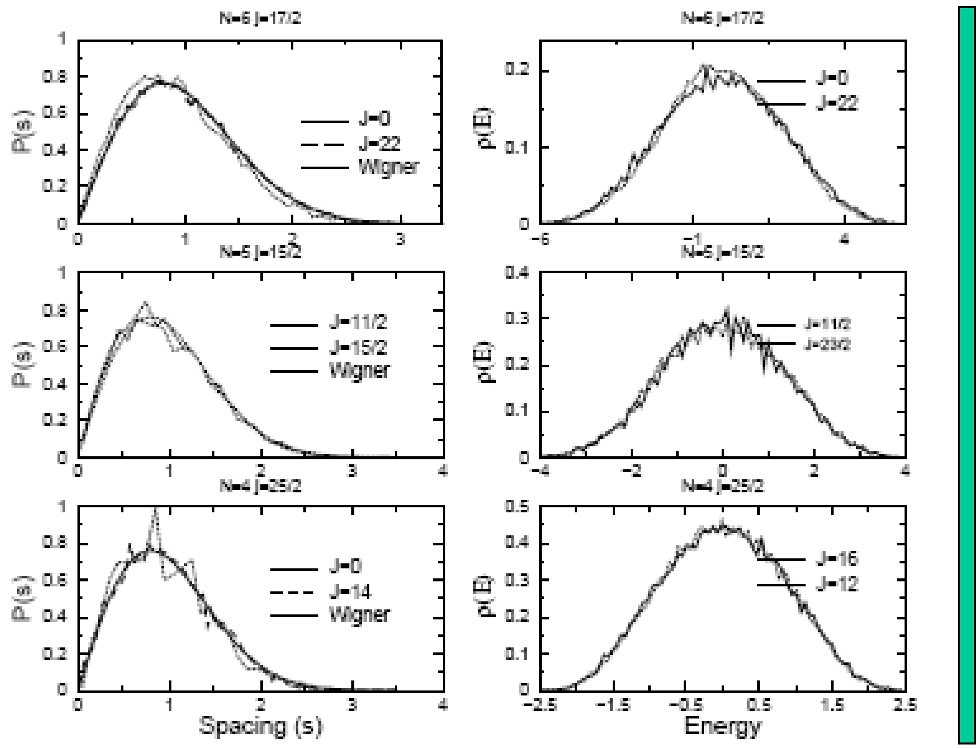
RANDOM AMPLITUDES V(L)

SYMMETRIC ENSEMBLE

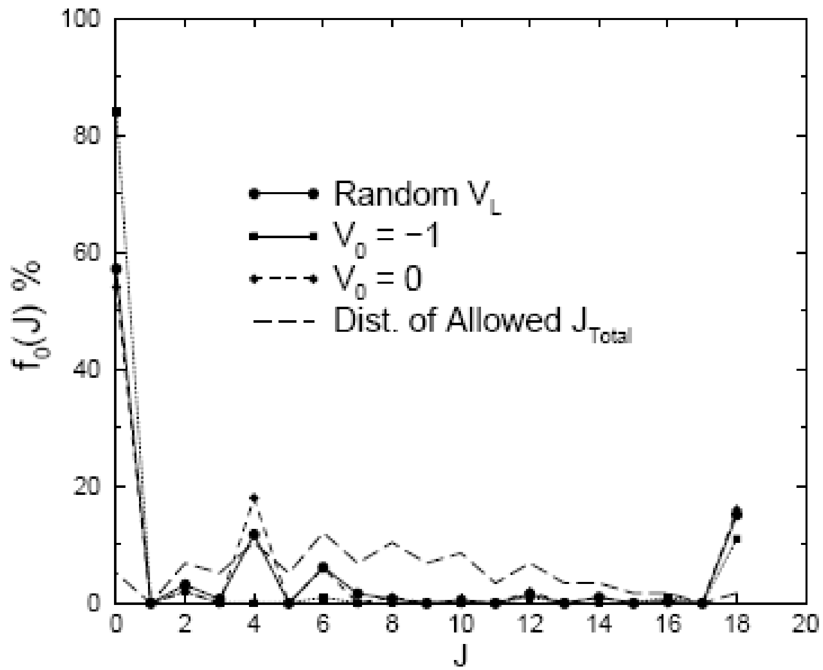
STATISTICS of GROUND STATE SPINS ?

C.W. Johnson, G.F. Bertsch, D.J. Dean, Phys. Rev. Lett. 80 (1998) 2749.

Non-equivalence of particle-particle and particle-hole channels

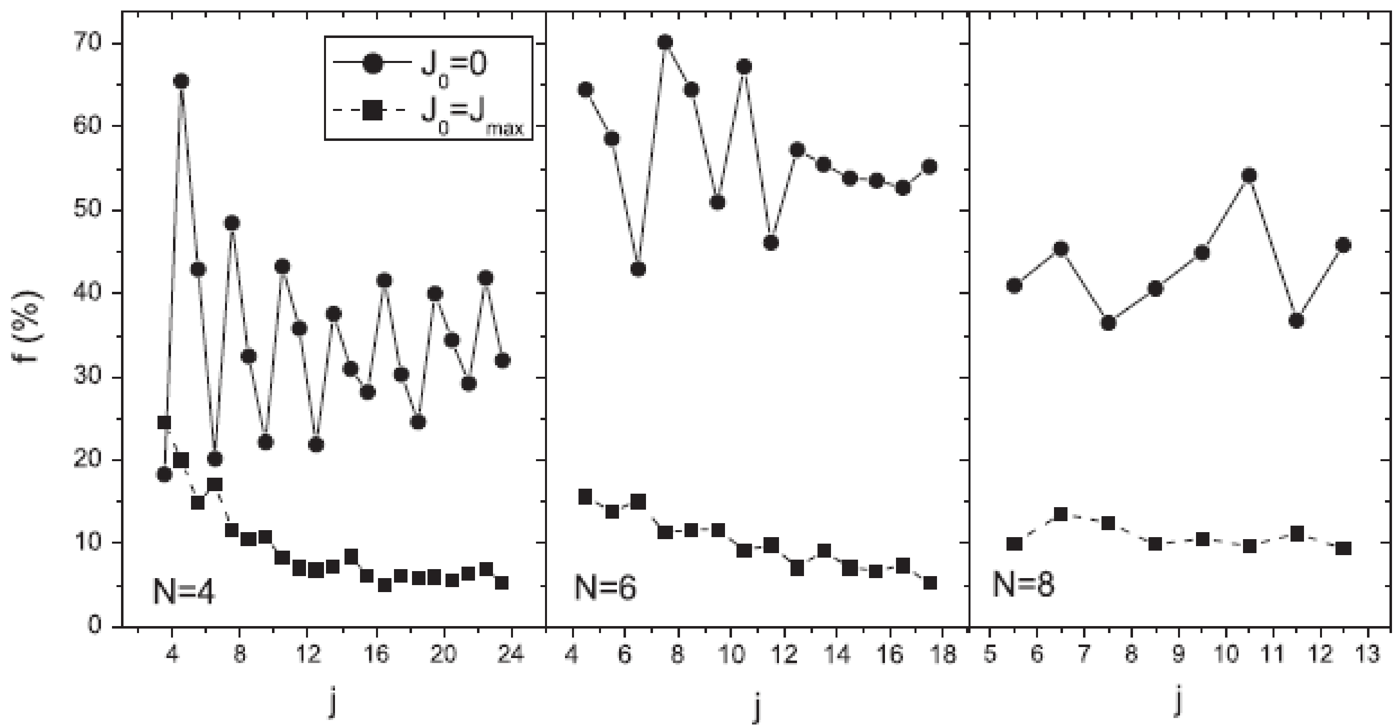


Spectra are chaotic:
Gaussian level density,
Wigner-Dyson level spacing distribution,
Exponential distribution of
off-diagonal many-body matrix elements
(average over many realizations)



Distribution of ground state spins

6 particles, j=11/2



Fraction of ground states of

spin J=0 and J=J(max)

(single j model)

sd-SHELL MODEL ^{24}Mg

(a) degenerate $\epsilon_{\mathrm{s.p.}}$, 63 random m. e.
$J_0 = 0,\ T_0 = 0$ **59.1%; overlap 2%**

(b) realistic $\epsilon_{\mathrm{s.p.}}$, 63 random m.e.
$J_0 = 0,\ T_0 = 0$ **49.3%; overlap 5.3%**

(c) realistic $\epsilon_{\mathrm{s.p.}}$ and 6 pairing m.e., 57 random m.e.
$J_0 = 0,\ T_0 = 0$ **67.8%; overlap 10.6%**

(d) degenerate $\epsilon_{\mathrm{s.p.}}$, 6 random pairing m. e.
$J_0 = 0,\ T_0 = 0$ **92.2%; overlap 5.2%**

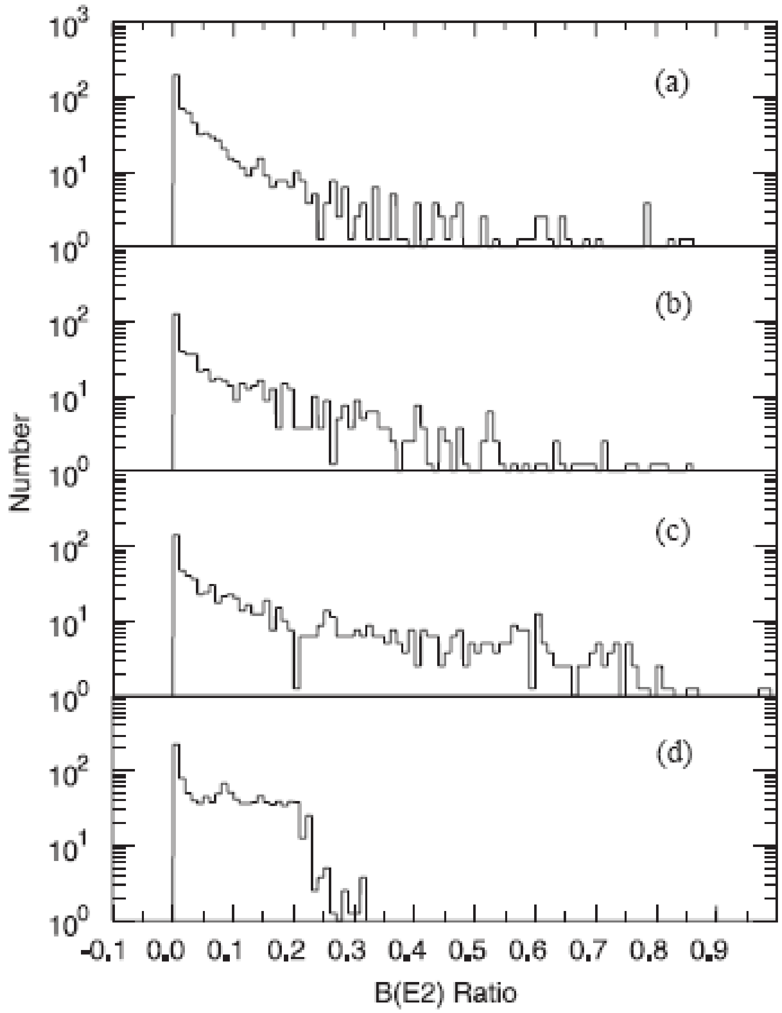
Many spins 1/2: $J_0 = 0,\ T_0 = 0$ **99%**

Quantum glass $J_0 \sim \sqrt{N}, \quad H = \sum_{12} J_{12}(\mathrm{s}_1 \cdot \mathrm{s}_2)$

GROUND STATE DISTRIBUTION (6 particles, j=21/2)

J	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
0	0.61	12.7	65.4	61.9	65.3	54.5	80.5	55.2	64.1
2	1.45	3.6		0.8	0.8			0.6	
4	2.38	5.4	1.9	2.5	2.7		1.0	3.7	2.2
5	2.15	1.8				9.1			
6	3.18	6.4	4.8	6.5	14.7	9.1	1.7	6.5	4.9
8	3.74	3.6	3.4	2.6	2.2		1.8	4.7	3.3
10	4.41	4.1	2.6	3.0	5.8	9.1	1.2	3.5	2.4
12	4.53	4.4			1.3			1.0	
13	4.07	2.6						0.6	0.6
16	4.49	3.0			0.7			0.7	
18	4.31	3.1	1.0	1.7				1.3	1.0
28	2.05	1.4	0.9	1.2		9.1		0.9	1.0
33	0.94	0.9						0.6	
36	0.66	0.9		0.7					
42	0.19	0.5	0.8	0.6				0.9	0.7
46	0.05	0.3	0.8	1.0				0.8	0.7
48	0.05	0.3	11.8	11.5	1.4	9.1	9.2	12.4	11.7

(a) Natural multiplicity
(b) Boson approximation
(c) Uniform V(L) from –1 to +1
(d) Gaussian V(L), dispersion 1
(e) Uniform V(L) scaled 1/(2L+1)
(f) [Zhao *et al*., 2002]
(g) Uniform V(L) except V(0)=-1
(h) As (g) but V(0)=+1
(i) As (g) and (h) but V(0)=0



$$\frac{B(E2;0\Rightarrow 2_1^+)\ \mathrm{stat}}{B(E2;0\Rightarrow 2_1^+)\ \mathrm{S.M.}\ ^{24}\mathrm{Mg}}$$

Degenerate orbitals

63 random m.e.

Realistic orbitals

63 random m.e.

Realistic orbitals and 6 pairing m.e., 57 random

Degenerate orbitals,

6 random pairing m.e.

Do we understand the role of incoherent interactions in many-body physics ?

-- Random interactions prefer ground state spin 0

-- Probability of maximum spin enhanced

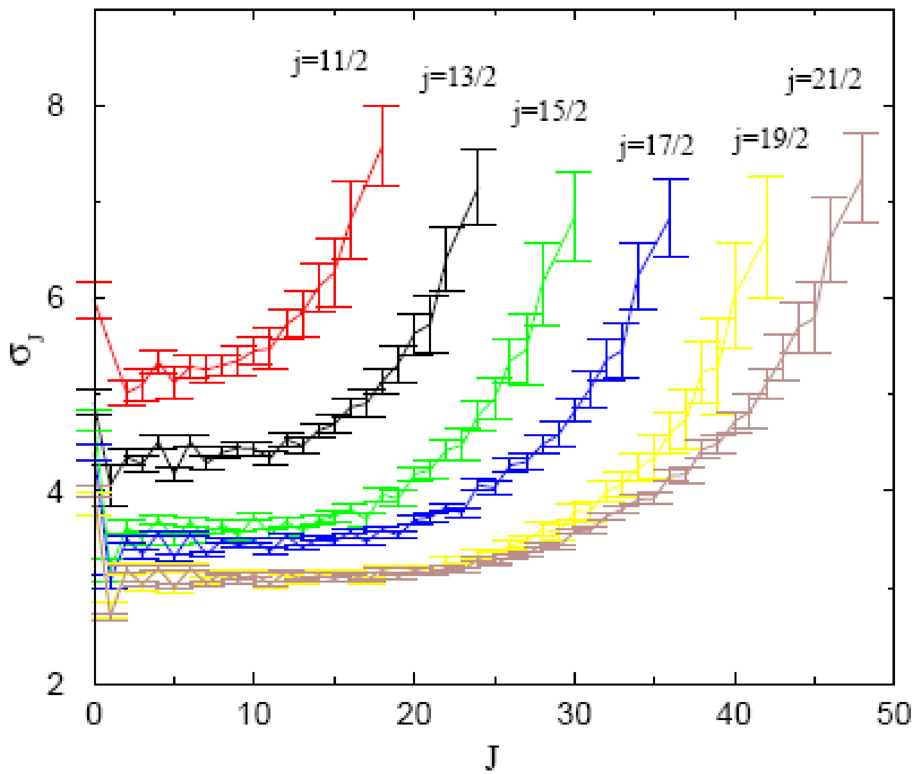
-- Ordered wave functions? Collectivity?

-- New aspect of quantum chaos: correlations between the symmetry classes

-- Geometric chaoticity of angular momentum coupling

-- Bosonization of fermion pairs?

-- Role of time-reversal invariance



Widths of level distributions in the J-class for a single-j model

(6 *particles*)

IDEA of GEOMETRIC CHAOTICITY

Angular momentum coupling as a random process

Bethe (1936) j(a) + j(b) = J(ab)
+ j(c) = J(abc)
+ j(d) = J(abcd)
… = J

Many quasi-random paths

Statistical theory of parentage coefficients ?

Effective Hamiltonian of <u>classes</u>

$$\tilde{H} = H_0 + H_2\mathbf{J}^2 + H_4\mathbf{J}^4 + \ \ldots$$

$$+H_0'\mathbf{T}^2 + \ H_2'\mathbf{T}^2\mathbf{J}^2 \ldots$$

Interacting boson models, quantum dots, …

STATISTICAL THEORY

Every set of random parameters

corresponds to the mean field

with occupation numbers n_m

$$H = \sum_{L\Lambda} V_L P^{\dagger}_{L\Lambda} P_{L\Lambda} \Rightarrow E(\{n_m\}) = \frac{1}{2} \sum_{mm'} V_{mm'} \langle n_m n_{m'} \rangle$$

$$V_{mm'} = 2 \sum_{L\Lambda} (C^{L\Lambda}_{jm\, jm'})^2; \quad \langle n_m n_{m'} \rangle \approx n_m n_{m'}$$

$$\tilde{E} = \frac{1}{2} \sum_{mm'} V_{mm'} n_m n_{m'} - \mu \sum_m n_m - \gamma \sum_m m n_m$$

$$E(N, M) = \frac{1}{2} [\mu(N, M) + \gamma(N, M) M]$$

MINIMIZATION:

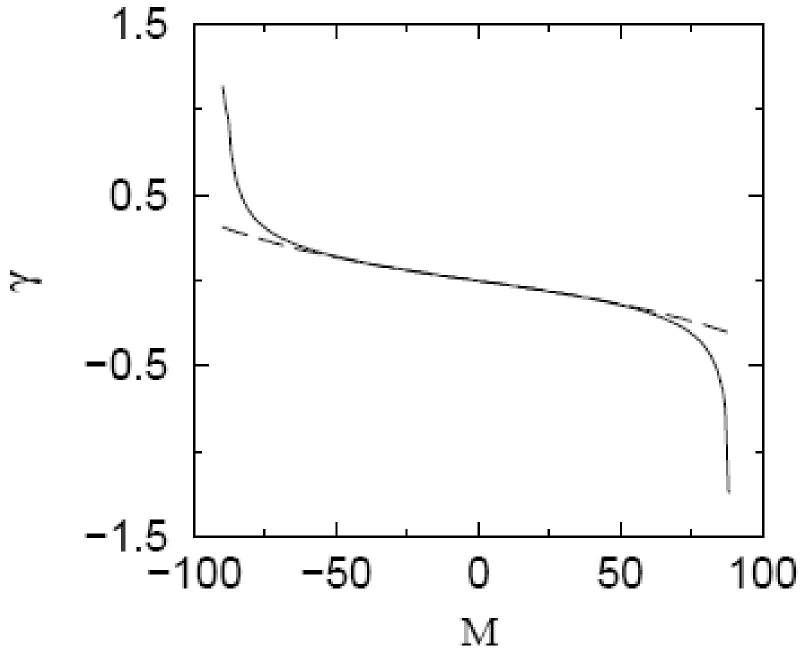
$$n_m = \bar{n} + (mM)/(\Omega \langle m^2 \rangle)$$

$$\mp (\bar{n} - 1/2)/[\bar{n}(1 - \bar{n})] \, [M^2/(\Omega^2 \langle m^2 \rangle^2)] \, (m^2 - \langle m^2 \rangle^2)$$

$$\langle H\rangle_{NM} = \frac{N^2}{\Omega^2}\sum_L (2L+1)V_L$$

$$+M^2\frac{3}{2\mathbf{j}^4\Omega^2}\sum_L (2L+1)V_L\,(\mathbf{L}^2-2\mathbf{j}^2)$$

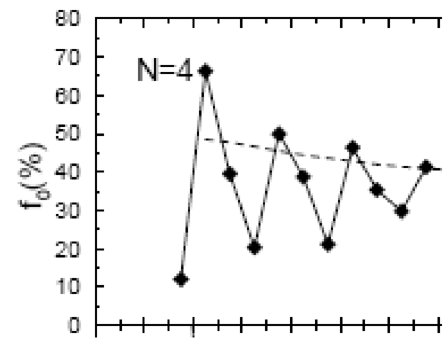
$$+M^4\frac{9\,(\Omega-2N)^2}{40\,\mathbf{j}^8(\Omega-N)^2N^2\Omega^2}\sum_L (2L+1)V_L$$
$$\times\left(3\mathbf{L}^4+3\mathbf{L}^2-12\mathbf{j}^2\mathbf{L}^2-6\mathbf{j}^2+8\mathbf{j}^4\right).$$



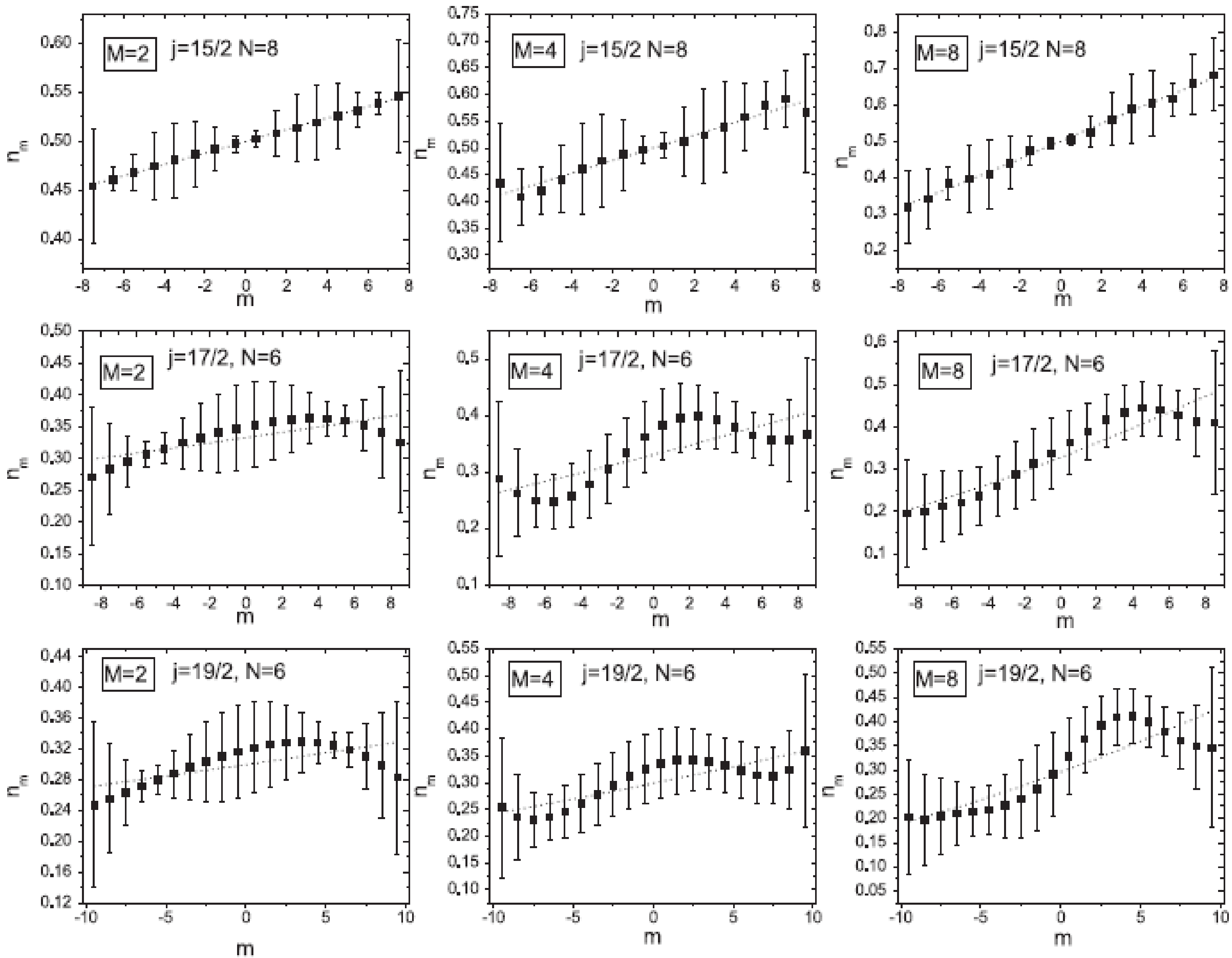
Effective Hamiltonian for N particles and given M=J explained by geometry:

j + j = L

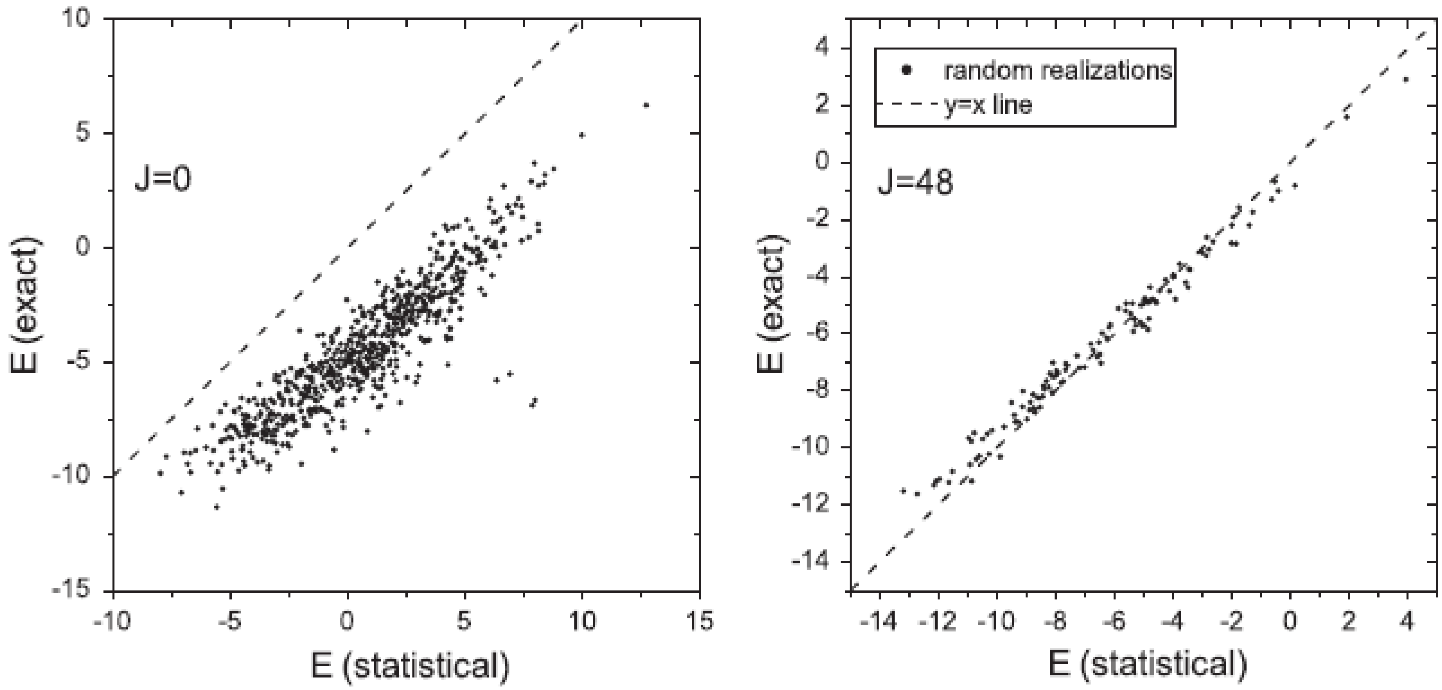
Cranking frequency is linear in M



Typical predictions for f(0)



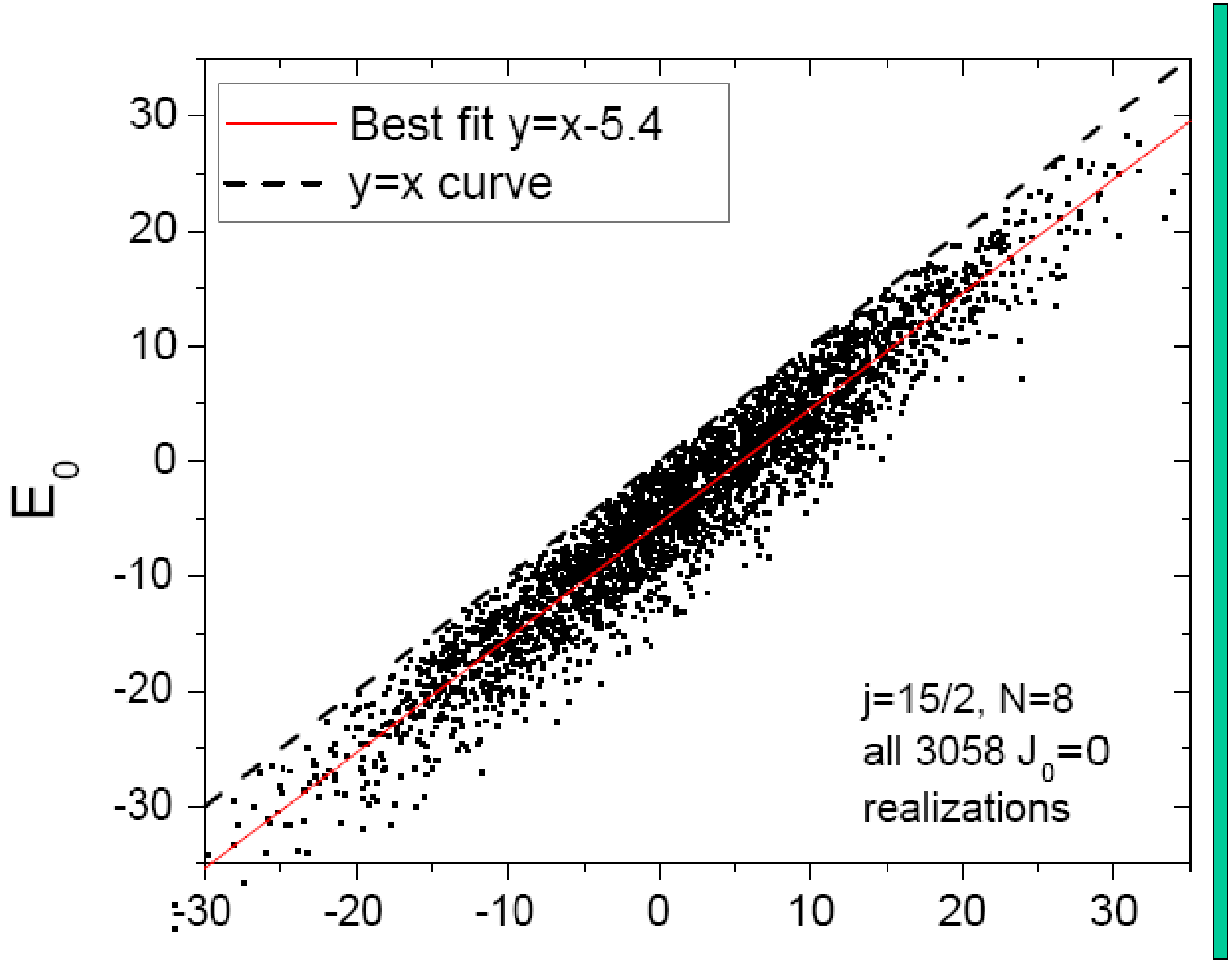
Dotted lines – statistical predictions for the state M=J



Predictions for energy of individual states with J=0 and J=J(max)

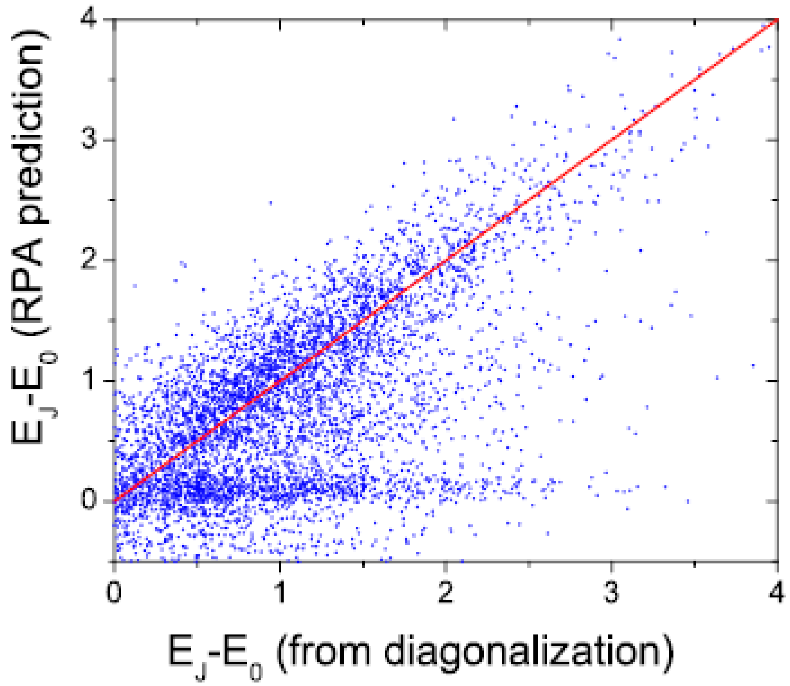
compared to exact diagonalization

(6 particles, j = 21/2)

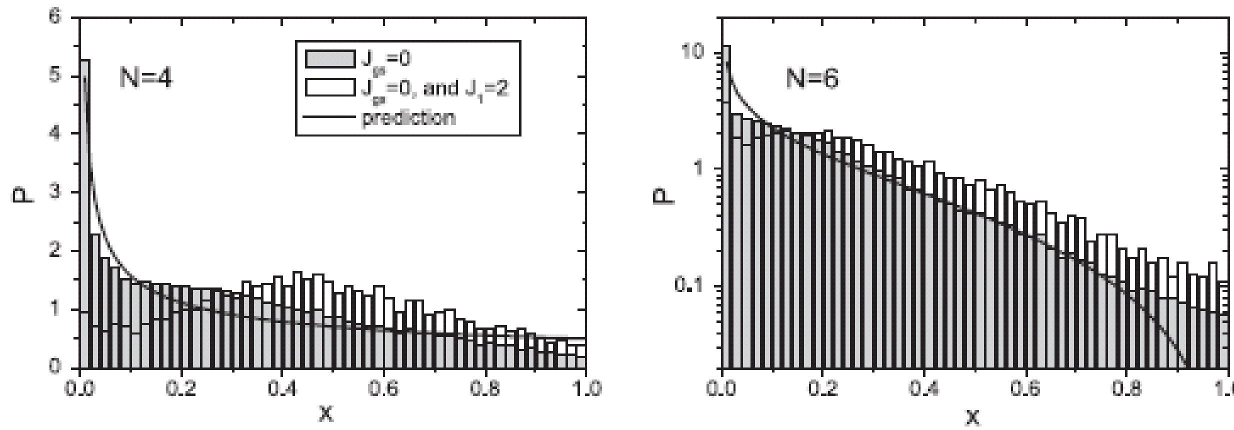


Best fit y=x-5.4
y=x curve
j=15/2, N=8
all 3058 J0=0
realizations
E0
30
20
10
0
-10
-20
-30

System: j=15/2, N=6



Collectivity of low-lying states



Distribution of overlaps $x = |\langle 0|s=0\rangle|^2, \quad 0 \leqslant x \leqslant 1$

|0> ground state of spin J = 0 in the random ensemble
|s = 0> fully paired state of seniority s = 0
4 particles on j = 15/2 – dimension d(0) = 3 (left)
6 particles on j = 15/2 – dimension d(0) = 4 (right)
Completely random overlaps:

$$P(d=3) \sim 1/\sqrt{x}, \qquad P(d=4) \sim (1-x)^{3/2}/\sqrt{x}$$

Collectivity out of chaos:

Johnson, Dean, Bertsch 1998
V.Z., Volya 2004
Johson, Nam 2007
Horoi, V.Z. 2009

Predominance of prolate deformations :

Teller, Wheeler 1938 – alpha-carcass
Bohr, Wheeler 1939 - liquid drop
Lemmer 1960 - extra kinetic energy of large orbital momenta
Castel, Goeke 1976 - the same in terms of collective energy
Castel, Rowe, Zamick 1990 - adding self-consistency
Frisk 1990 - single-particle level density
Arita et al. 1998 -
periodic orbits and their bifurcations
Deleplanque et al. 2004 –
Hamamoto, Mottelson 1991 - metallic clusters
2009 – surface properties of deformed field

"The nature of the parameters responsible for the prolate dominance has not yet been adequately understood"

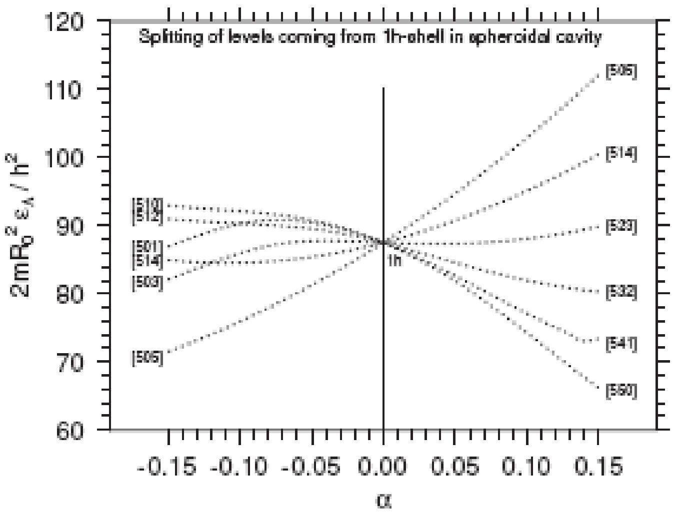


FIG. 5. Splitting of levels originating from the $1h$ shell in spheroidal cavity. The asymptotic quantum numbers $[N\ n_z\ \Lambda]$ are assigned to the levels on both prolate and oblate sides. See the text for details.

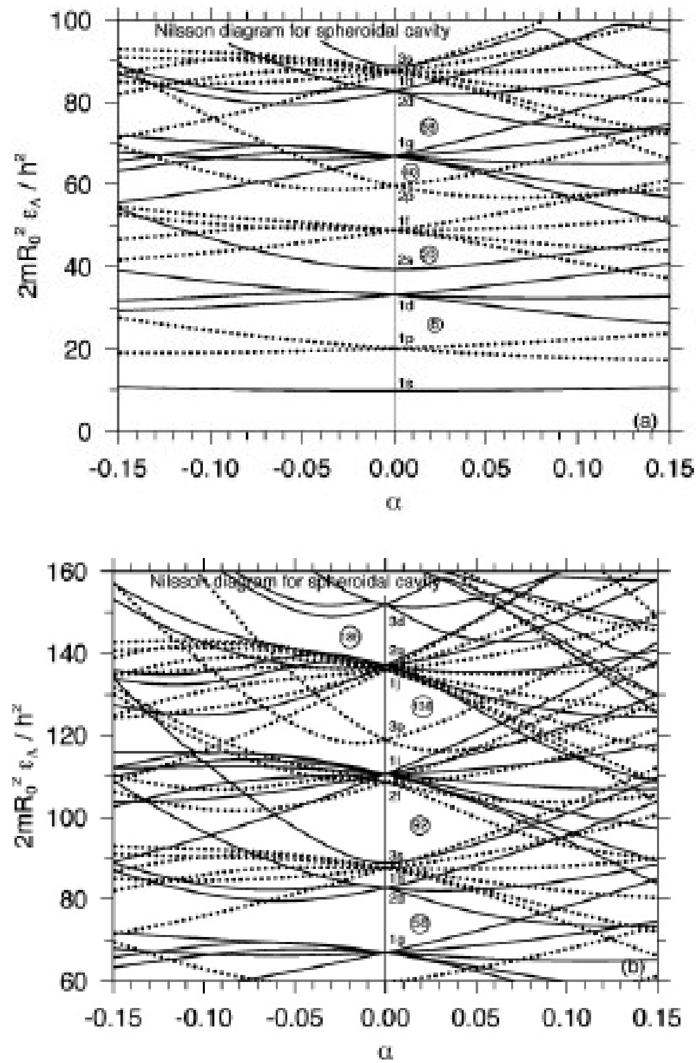
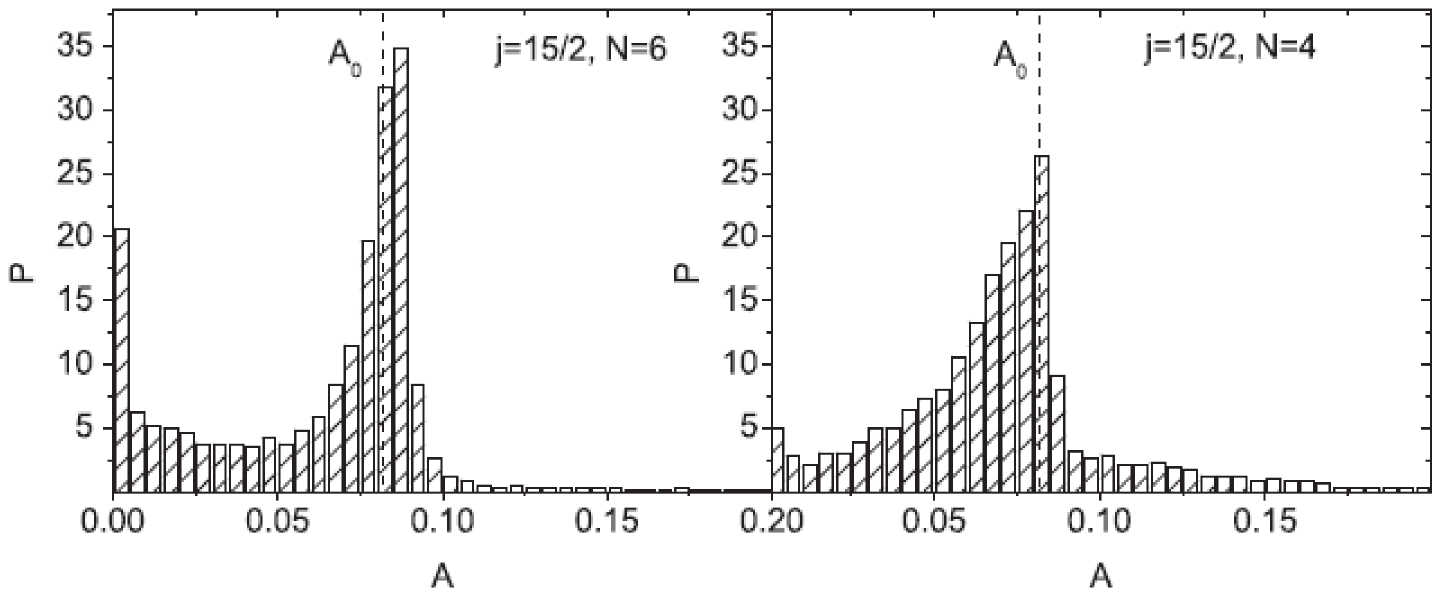


FIG. 3. (a) One-particle energies of spheroidal cavity as a function of deformation parameter. At spherical point $\alpha = 0$ the quantum numbers, $n\ell$, are written. The particle number of the system obtained by filling all lower-lying levels is written with a circle in several places. Positive-parity levels are plotted by solid curves, while negative-parity levels by dotted curves. (b) One-particle energies of spheroidal cavity as a function of deformation parameter, for the system larger than that plotted in (a).



ALAGA RATIO

$$A = \frac{Q^2(2_1^+)}{B(\mathrm{E2};0^+ \Rightarrow 2_1^+)}$$

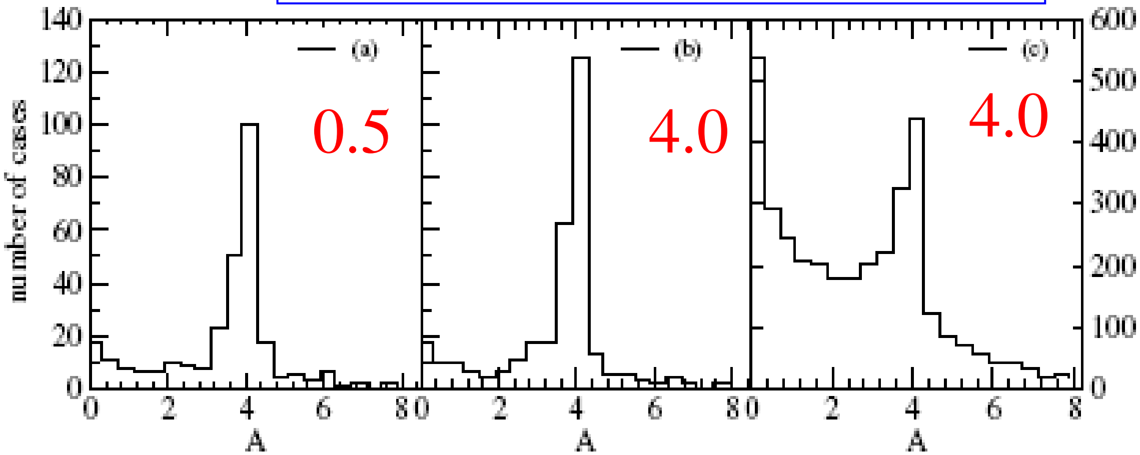
Spherical $A = 0$, rigid rotor $A = 4/49$

(**take sequences** J=0, J=2)

Distribution of Alaga ratio

N= 10 000

4 neutrons +
4 protons
0f7/2 + 1p3/2



Selection by E(4)/E(2)
[3.0, 3.6]

All cases
J=0, J=2

Interaction:
(a) weak
(b) strong

λ	$N(0,2)$	$\frac{N(Q<0)}{N(0,2)}$	$N(E4/E2)$	N_{rot}	$\frac{N_{prolate}}{N_{rot}}$
0.05	1398	0.62	50	3	1.00
0.5	3320	0.54	322	100	0.74
1.0	3846	0.52	354	100	0.70
1.5	4056	0.52	378	119	0.72
2.0	4129	0.52	371	122	0.74
3.0	4196	0.52	366	126	0.70
4.0	4233	0.52	367	125	0.71
10.0	4295	0.53	368	112	0.74

Here A(rot) = 4.10
Selection N(rot):
A between 3.90 and 3.30

$$Q(J) = Q_0 \frac{3K^2 - J(J+1)}{(J+1)(J+3)} \Rightarrow -Q_0 \frac{J}{J+3}$$

Selection N(prolate):
Q(2)<0

λ	$N(0,2)$	$\frac{N(Q<0)}{N(0,2)}$	$N(E4/E2)$	N_{rot}	$\frac{N_{prolate}}{N_{rot}}$
1.0	3156	0.55	289	39	0.77
2.0	3153	0.53	264	34	0.79
3.0	3140	0.52	266	34	0.82
4.0	3156	0.53	240	35	0.89

4 protons + 6 neutrons

N(rot) lower, N(prolate) higher

λ	$N(0,2)$	$\frac{N(Q<0)}{N(0,2)}$	$N(E4/E2)$	N_{rot}	$\frac{N_{prolate}}{N_{rot}}$
1.0	4569	0.54	322	120	0.73
2.0	4530	0.52	339	116	0.75
3.0	4490	0.52	349	119	0.76
4.0	4461	0.52	371	124	0.81

4 neutrons + 4 protons

1p3/2 + 0f7/2

(inverted sequence)

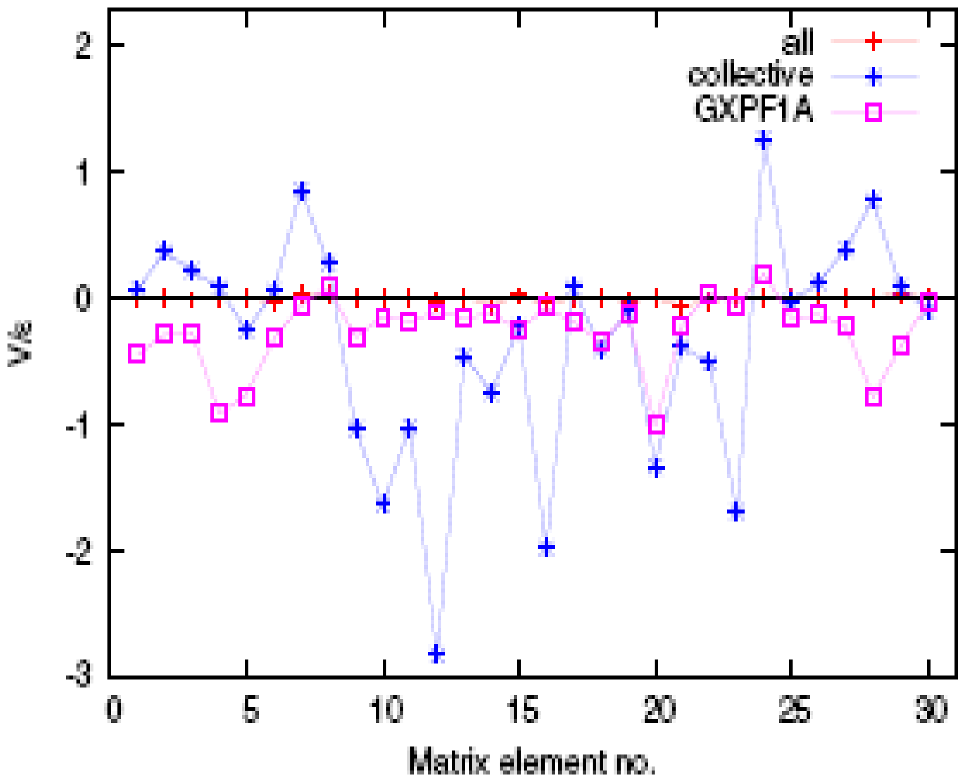
λ	$N(0,2)$	$\frac{N(Q<0)}{N(0,2)}$	$N(E4/E2)$	N_{rot}	$\frac{N_{prolate}}{N_{rot}}$
1.0	2170	0.43	176	55	0.07
2.0	2185	0.37	193	70	0.06
3.0	2212	0.35	188	73	0.05

4 neutrons + 4 protons

0f7/2 + 0g9/2

(opposite parity)

Strong interaction 4.0



	$\langle j_1 j_2 \vert V \vert j_3 j_4\rangle(JT)$	Full average	Prolate average
1	$\langle ff\vert V\vert ff\rangle(10)$	0.021	0.078
2	$\langle ff\vert V\vert ff\rangle(30)$	0.012	0.374
3	$\langle ff\vert V\vert ff\rangle(50)$	-0.007	0.227
4	$\langle ff\vert V\vert ff\rangle(70)$	0.007	0.089
5	$\langle ff\vert V\vert ff\rangle(01)$	0.008	-0.252
6	$\langle ff\vert V\vert ff\rangle(21)$	-0.020	0.062
7	$\langle ff\vert V\vert ff\rangle(41)$	0.026	0.869
8	$\langle ff\vert V\vert ff\rangle(61)$	0.034	0.282
9	$\langle ff\vert V\vert pf\rangle(30)$	0.004	-1.033
10	$\langle ff\vert V\vert pf\rangle(50)$	0.022	-1.630
11	$\langle ff\vert V\vert pf\rangle(21)$	0.006	-1.010
12	$\langle ff\vert V\vert pf\rangle(41)$	-0.010	-2.826
13	$\langle ff\vert V\vert pp\rangle(10)$	0.014	-0.451
14	$\langle ff\vert V\vert pp\rangle(30)$	-0.043	-0.739
15	$\langle ff\vert V\vert pp\rangle(01)$	0.025	-0.223
16	$\langle ff\vert V\vert pp\rangle(21)$	-0.036	-1.977
17	$\langle pf\vert V\vert pf\rangle(20)$	0.007	0.088
18	$\langle pf\vert V\vert pf\rangle(30)$	0.010	-0.393
19	$\langle pf\vert V\vert pf\rangle(40)$	-0.018	-0.092
20	$\langle pf\vert V\vert pf\rangle(50)$	0.004	-1.328
21	$\langle pf\vert V\vert pf\rangle(21)$	-0.052	-0.376
22	$\langle pf\vert V\vert pf\rangle(31)$	-0.019	-0.507
23	$\langle pf\vert V\vert pf\rangle(41)$	0.011	-1.685
24	$\langle pf\vert V\vert pf\rangle(51)$	-0.003	1.276
25	$\langle pf\vert V\vert pp\rangle(30)$	0.007	-0.023
26	$\langle pf\vert V\vert pp\rangle(21)$	0.014	0.133
27	$\langle pp\vert V\vert pp\rangle(10)$	0.003	0.400
28	$\langle pp\vert V\vert pp\rangle(30)$	0.003	0.779
29	$\langle pp\vert V\vert pp\rangle(01)$	0.054	0.102
30	$\langle pp\vert V\vert pp\rangle(21)$	0.005	-0.092

Matrix elements

9-12: pf mixing,

16 : quadrupole pair transfer,

20-24: quadrupole-quadrupole forces in particle-hole channel = formation of the mean field

QUESTIONS and PROBLEMS

- **Geometric chaoticity**
- **Extension to continuum:**
  - **level densities**
  - **correlations and fluctuations of cross sections**
  - **mesoscopic universal conductance fluctuations**
  - **dependence on intrinsic chaos**
  - **loosely bound nuclei**
- **Microscopic picture of shape phase transitions**
- **New approximations for large systems:**
  pairing + collective motion + incoherent chaos

Statistical Properties of Nuclear Structure

Vladimir Zelevinsky

NSCL/ Michigan State University

Supported by NSF

Lund, May 2009