Single Particle Level Densities in Tetrahedral and Octahedral Nuclear Shapes.

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May 12, 2009

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About the Method of Calculations in this Work:

Phenomenological Mean-Field Approach

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Technical Advantages:

- Strong predictive power in various applications;
- Reliability with respect to extrapolations in Z and N

Its attractive aspects:

- The method describes numerous nuclear phenomena with precision allowing for comparisons with experimental data
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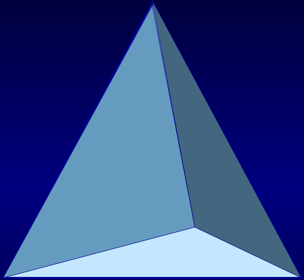
Non-Trivial Competing Symmetries in Nuclei: $\mathrm{T_d}$

The *tetrahedral group* denoted $\mathrm{T_d}$ and related hypothetical symmetries

A tetrahedron has four equal walls. Its shape is invariant with respect to 24 symmetry elements. Tetrahedron is *not* invariant with respect to the inversion. Of course nuclei cannot be represented by a sharp-edge pyramid



... the lowest order expansion:

$$\mathcal{R}(\vartheta,\varphi) = R_0\, c(\{\alpha\})[1 + \alpha_{3,+2}\, Y_{3,+2}(\vartheta,\varphi) + \alpha_{3,-2}\, Y_{3,-2}(\vartheta,\varphi)]$$

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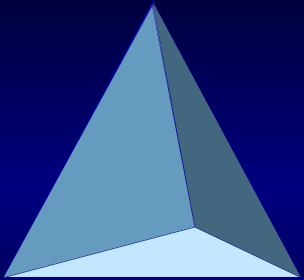

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Nuclear Tetrahedral Shapes - 3D Examples

Illustrations below show the tetrahedral-symmetric surfaces at three increasing values of rank $\lambda = 3$ *deformations* t_1*: 0.1, 0.2 and 0.3:*

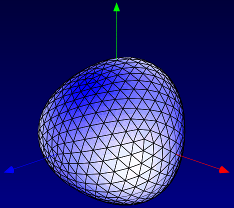


Figure: $t_3 = 0.1$

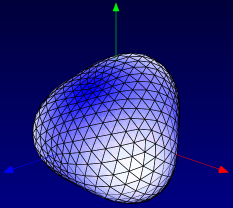


Figure: $t_3 = 0.2$

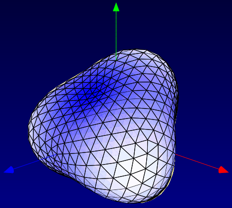


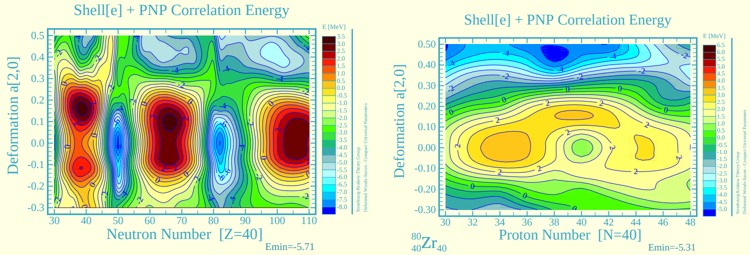
Figure: $t_3 = 0.3$

Observations:

- There are infinitely many tetrahedral-symmetric surfaces
- Nuclear 'pyramids' do not resemble pyramids!

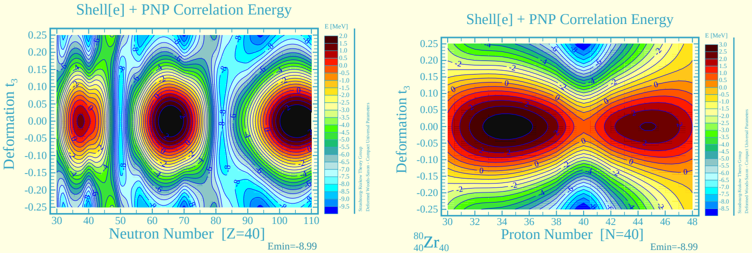
Quadrupole-Deformation Driving Shell-Effects

Deformation-driving shell-effects: here for the "usual" quadrupole deformation [introducing the way of illustration] Final deformation will be the result of a sum of the proton and neutron shell and pairing energies and the macroscopic energy.



Comparison of the quadrupole-deformation driving forces in function of neutron number for Z=40 [left] and in function of the proton number for N=40 [right].

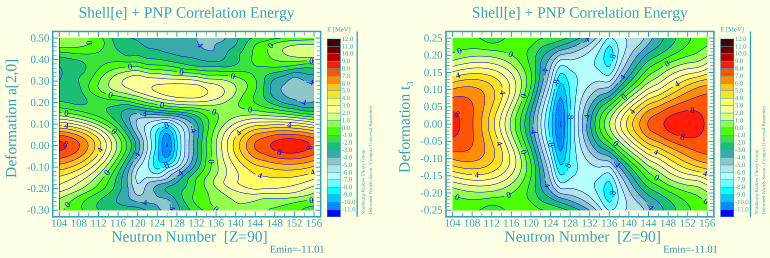
Nuclei around Zirconium: a series of magic numbers (the largest of possible astrophysical interest) N=30, 40, 56, 64, 70 and 96 where there appear big values of tetrahedral deformations.



Tetrahedral symmetry driving shell-effects [notice that the neutron axis is much more extended as compared to the proton axis].

Tetrahedral-Deformation Driving Shell-Effects

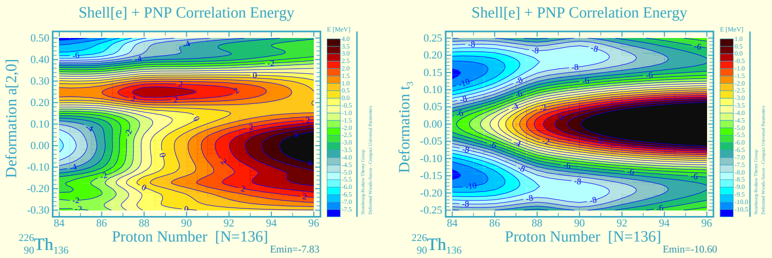
Around Z=90, starting from N=132 there is an important gain in energy down to -8 MeV (!) when allowing for tetrahedral shapes



Comparison of the quadrupole driving shell-effects [left] competing with tetrahedral one [right] around Thorium isotopes.

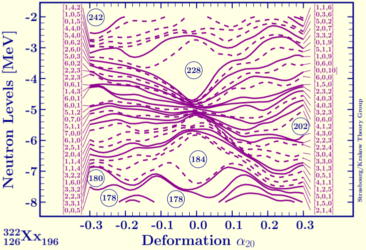
Tetrahedral-Deformation Driving Shell-Effects

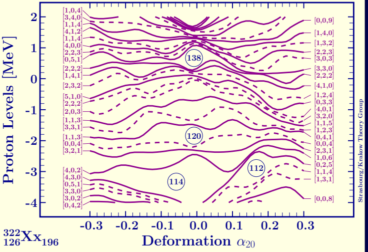
Huge tetrahedral symmetry driving *proton* shell-effects for Z between 84 and 96 (-8-to-10) MeV gain



Comparison of the quadrupole-shape driving shell-effects [left] with the competing tetrahedral one [right] at $N \sim 136$.

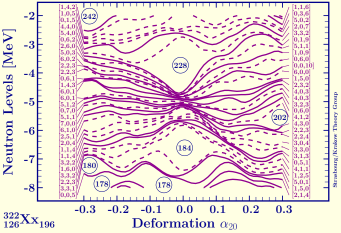
Super-Heavy Nuclei: Quadrupole Shell-Effects

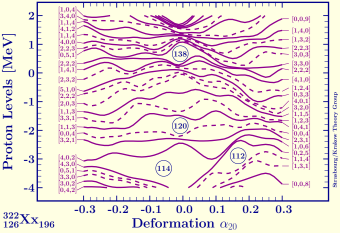


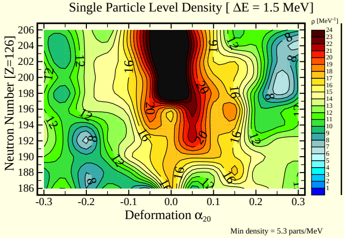


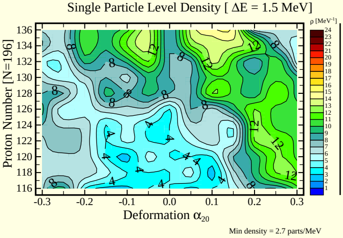
Single particle energy levels for neutrons [left] and protons [right] in function of the quadrupole deformation.

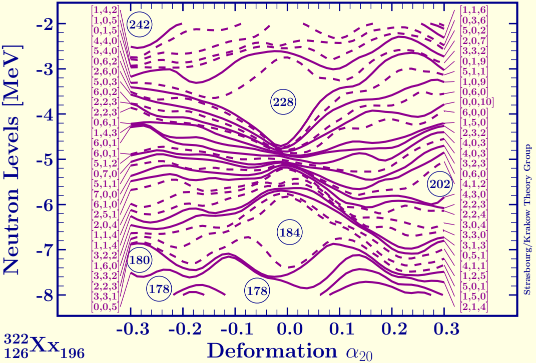
Super-Heavy Nuclei: Quadrupole Shell-Effects





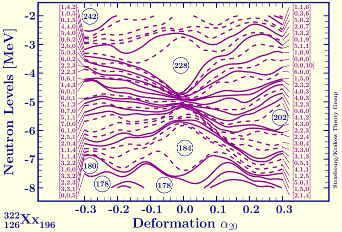


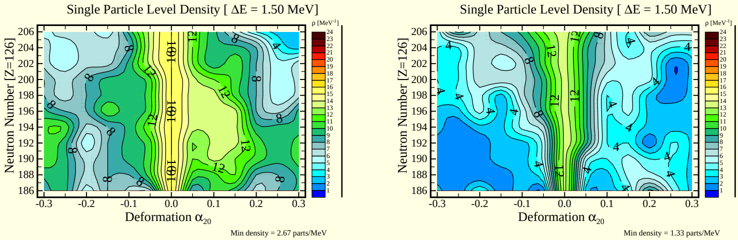




Single particle energy levels for neutrons in function of the quadrupole deformation. Continuous lines are positive parity and dashed lines are negative parity.

Super-Heavy Nuclei: Quadrupole Shell-Effects

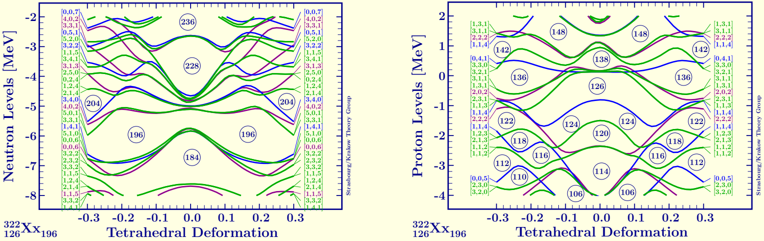




Density of single particle energy levels for negative parity [left] and positive parity [right] in function of the quadrupole deformation.

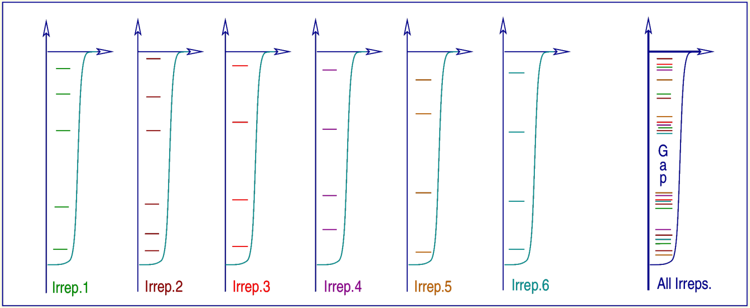
Super-Heavy Nuclei: Tetrahedral Shell-Effects

Parity is non-conserved here. There are three families of levels in total. Green lines correspond to 4-dimensional irreps - they are marked with double Nilsson labels.

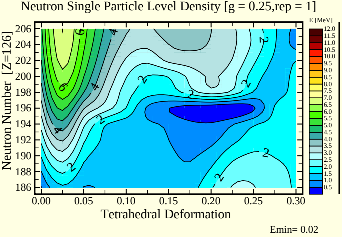


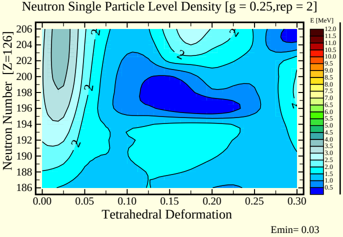
Single particle energy levels for neutrons [left] and protons [right] in function of the tetrahedral deformation.

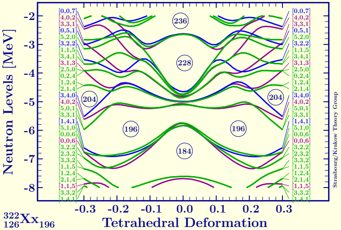
Symmetry Point-Group: Irreducible Representations

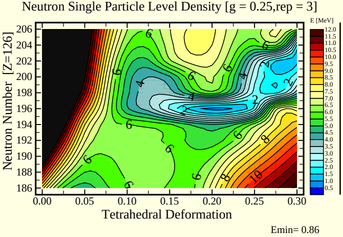


Consider a symmetry point-group with six irreducible representations (this is actually the case of the octahedral group). The single-particle levels are split into six families (octahedral case) or tree families (tetrahedral case), to each of which the Landau-Zener non-crossing rule applies.





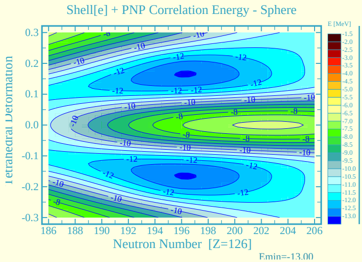




Density of single particle energy levels for first irreducible representation [left], second representation [right] and third irreducible representation [middle] in function of the tetrahedral deformation.

Tetrahedral-Deformation Driving Shell-Effects

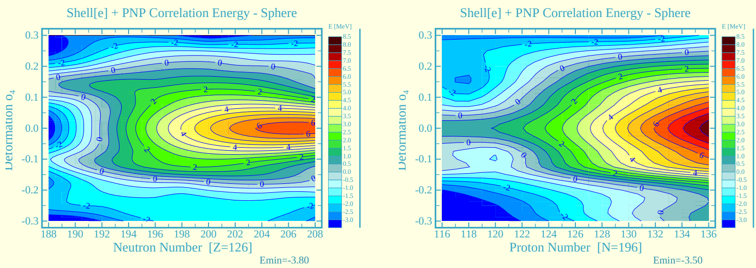
Nuclei around Z=126: 4 MeV energy gain with using tetrahedral deformations.



Tetrahedral symmetry driving shell-effects coming from the neutrons [left] for Z=126 and protons [right] for N=196.

Octahedral-Symmetry Driving Shell-Effects

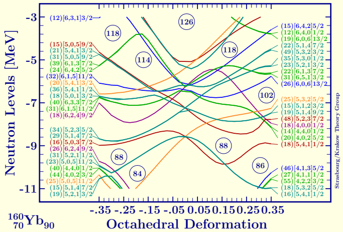
Huge octahedral deformation driving shell-effects at zero tetrahedral deformation.

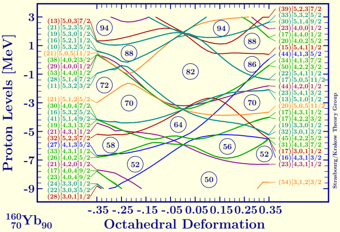


Octahedral-symmetry driving shell-effects coming from the neutrons [left] and protons [right] in Superheavy nuclei

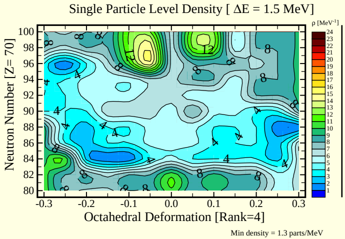
["academic" example since in these nuclei tetrahedral symmetry dominates]

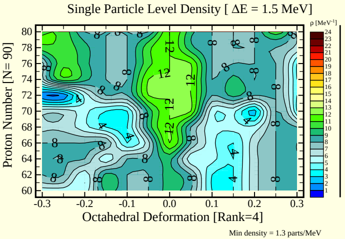
Super-Heavy Nuclei: Octahedral Shell-Effects





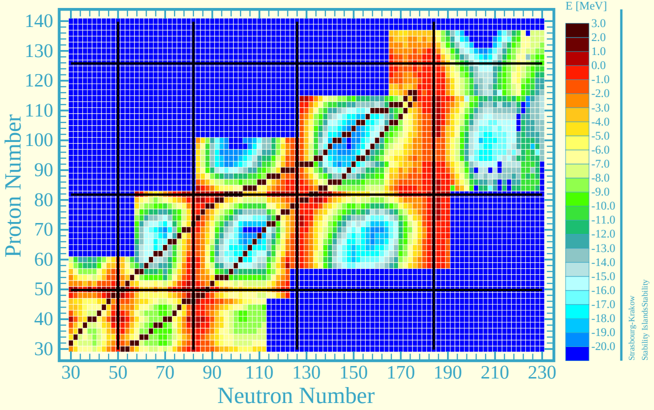
Single particle energy levels for neutrons [left] and protons [right] in function of the tetrahedral deformation.





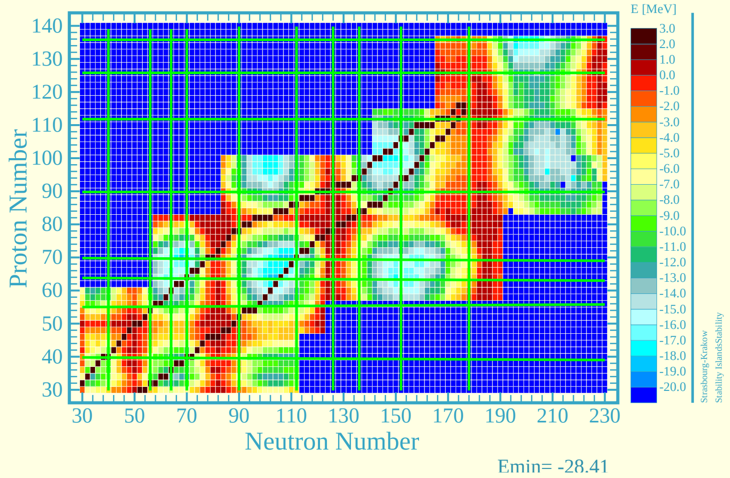
Density of single particle energy levels for neutrons [left] and protons [right] in function of the octahedral deformation.

Islands of Stability: Traditional α_{20} and α_{40}



Leading-order effects: Results of the shell-energy minimisation over α_{20}, α_{40}. Plotted: minimum shell-energy minus the shell energy at spherical shape. Black lines correspond to spherical magic numbers. Negative values tell us 'how much more inexpensive is being non-spherical'.

Islands of Stability: Tetrahedral and Octahedral Effects



Similar to the preceding but minimisation was done over tetrahedral and octahedral deformations. Straight lines give the positions of tetrahedral magic numbers. Observe apparent similarity of the 2 figs.

And so: What Are We Learning from the Two Rather Similar Diagrams?

- These diagrams apparently similar carry very different information

- While the first one gave the energy gain in terms of 'classical' quadrupole and hexadecapole deformation, the second one gives the energy gain in terms of high-rank symmetries

- The geometries are very different and so are the implied stability conditions, different fission paths and barriers etc.

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Changing a Representation

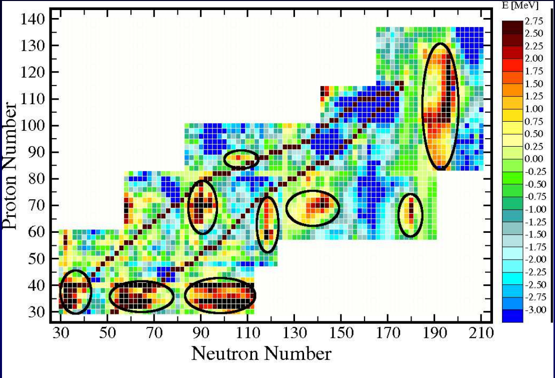

Figure: *Difference between shell energies for quadrupole+hexadecapole deformation minus tetrahedral+octahedral deformations from two previous diagrams. Positive result corresponds to the high-rank symmetries winning.*

Summary and Conclusions

- *We performed an exploratory set of calculations comparing the nuclear (in)stability with respect to 'traditional' axial symmetries as well as the 'high-rank' (tetrahedral and octahedral) symmetries*
- *We connect the problem of stability with the problem of single particle level densities*
- *Systematic comparisons lead to the prediction of extremely strong symmetry-driving shell effects favouring the new symmetries*
- *More complete calculations including total energies with more degrees of freedom are under way*

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Outlook

- *These more complete calculations will be used as the background for the cranking model calculations of properties of rotational bands with vanishing E2 transitions*
- *Similarly, predictions of the new super-heavy domains are being extended now using 4 dimensional calculations - cross-checked with the Hartree-Fock calculations*
- *Calculation of the electromagnetic transitons and branching ratios are under way*