Recent results in quantum chaos and its applications to nuclei and particles

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Contents

A. Relaño, J. M. G. Gómez, R. A. Molina, J. Retamosa, and E. Faleiro, *Phys. Rev. Lett.* **89**, 244102 (2002)

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C. Fernández-Ramírez and A. Relaño, *Phys. Rev. Lett.* **98**, 062001 (2007)

A. Relaño, *Phys. Rev. Lett.* **100**, 224101 (2008)

1 – Introduction

The concept of chaos in Classical Mechanics can not be easily carried to Quantum Mechanics

- In quantum systems with a classical analogue:
  Quantum mechanics $\xrightarrow{\hbar \to 0}$ Classical mechanics
- A quantum system is said to be regular when its classical analogue is integrable and it is said to be chaotic when its classical analogue is chaotic
- But many quantum systems have no classical analogue!
- The spectral fluctuations are the key property characterizing order and chaos in quantum systems

2 – Quantum Chaos and Random Matrix Theory

- Berry and Tabor, *Proc. R. Soc. London* **A356**, 375 (1977)

  The spectral fluctuations of a quantum system whose classical analogue is fully integrable are well described by Poisson statistics, i.e. the successive energy levels are not correlated.

- Bohigas, Giannoni, and Schmit, *Phys. Rev. Lett.* **52**, 1 (1984)

  CONJECTURE: Spectra of time-reversal invariant systems whose classical analogs are K systems show the same fluctuation properties as predicted by GOE.

The most commonly used statistics to characterize spectral fluctuations are

- ➛ The nearest level spacing distribution **P(s)**
- ➛ The spectral rigidity $\boldsymbol{\Delta_3(L)}$ of Dyson and Mehta

For the sequence of unfolded levels

$$\varepsilon_i, \quad i = 1, \ldots, N$$

the nearest level spacing s_i is defined by

$$s_i = \varepsilon_{i+1} - \varepsilon_i$$

and the average values are

$$\langle s \rangle = 1$$
$$\langle \varepsilon_n \rangle = n$$

- For the uncorrelated levels or Poisson case,

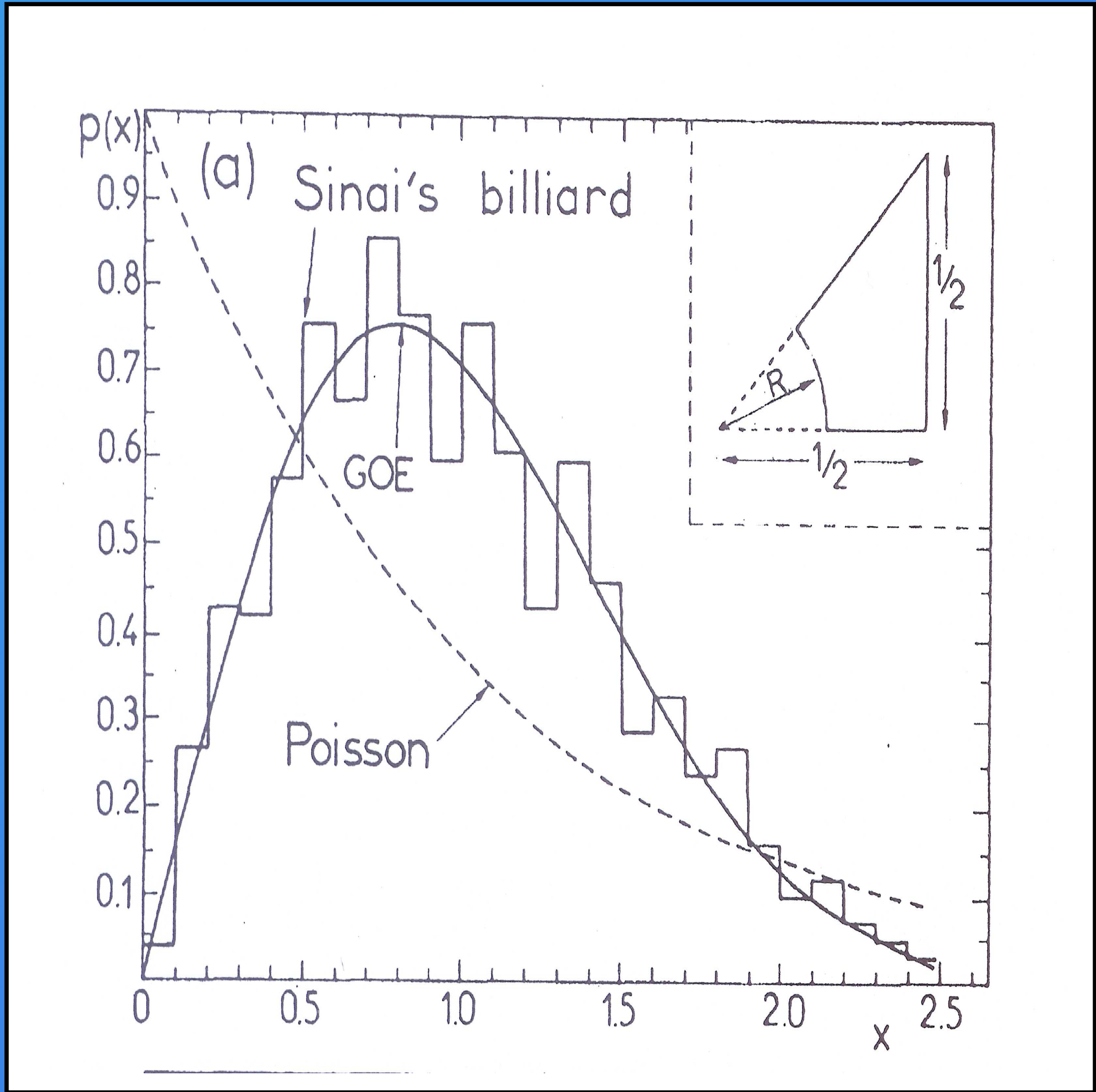
$$P(s) = e^{-s}, \quad P(0) = 1$$
$$\langle \Delta_3(L) \rangle = \frac{L}{15}$$

- For the Gaussian orthogonal ensemble (GOE),

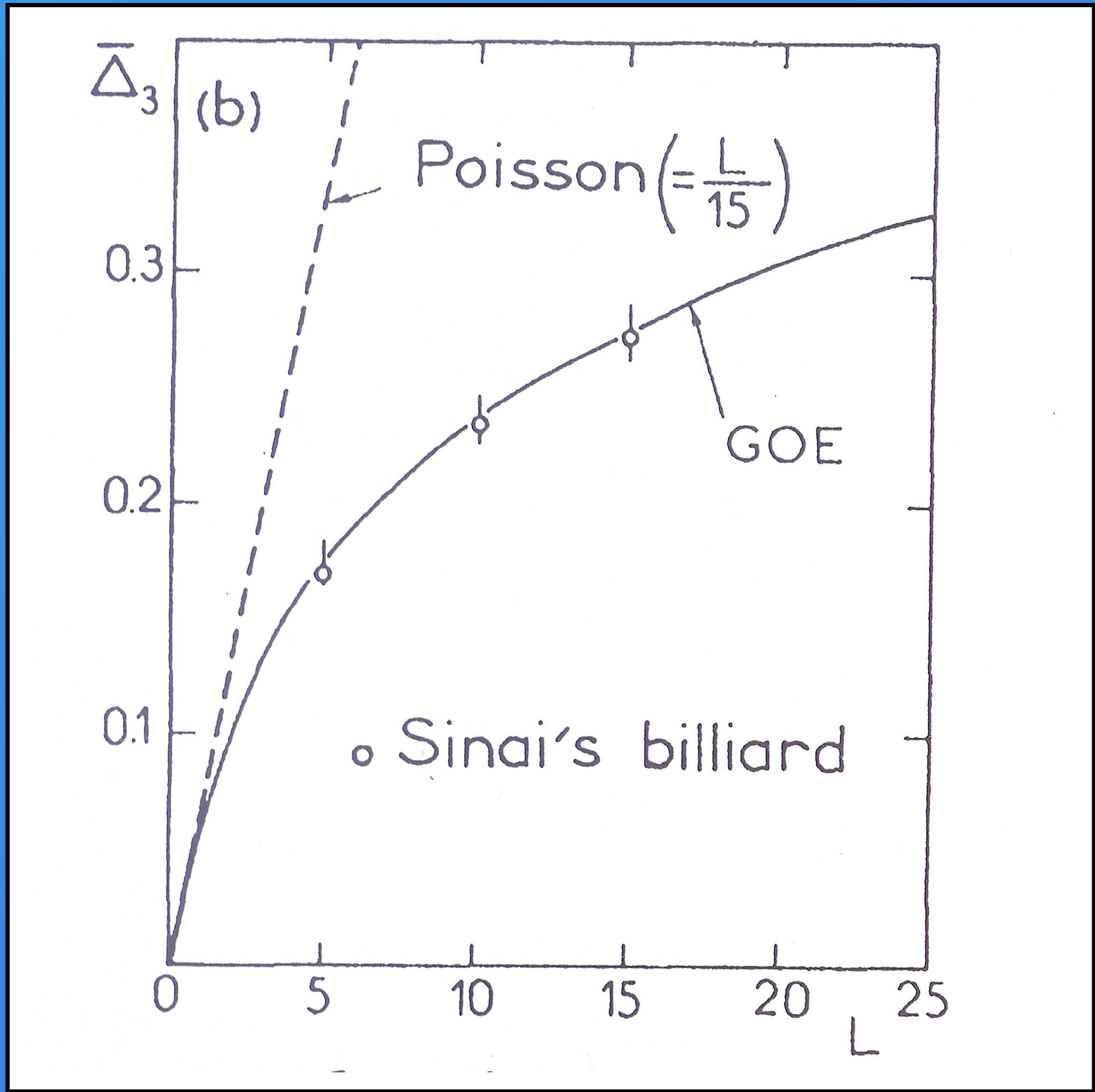
$$P(s) = \frac{\pi}{2} s \exp\left(-\frac{\pi}{4} s^2\right), \quad P(0) = 0$$

$$\langle \Delta_3(L) \rangle = \frac{1}{\pi^2} \log(L) + b + \mathcal{O}(L^{-1}), \;\; L \gg 1$$

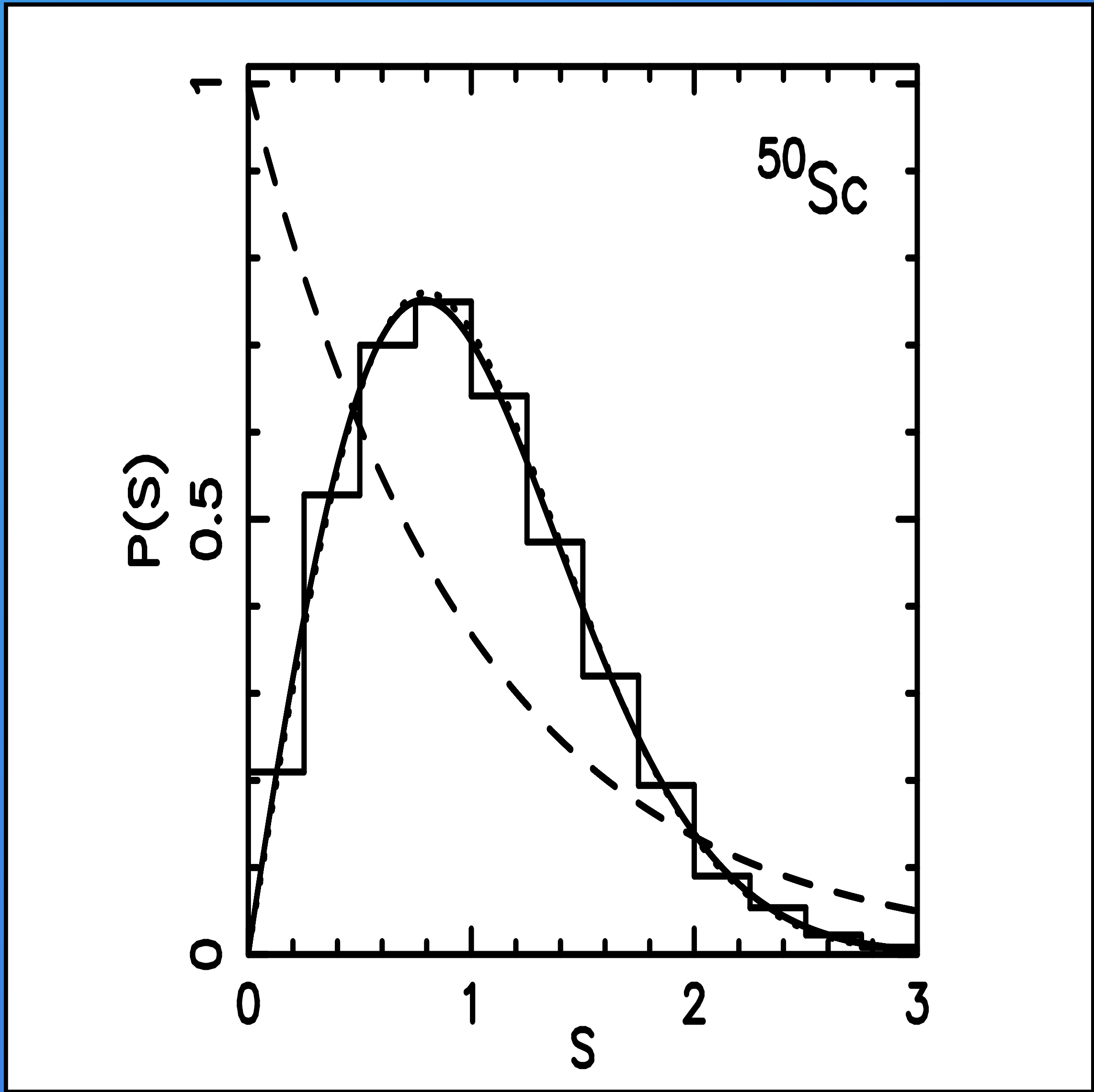
$P(s)$ distribution for the Sinai billiard



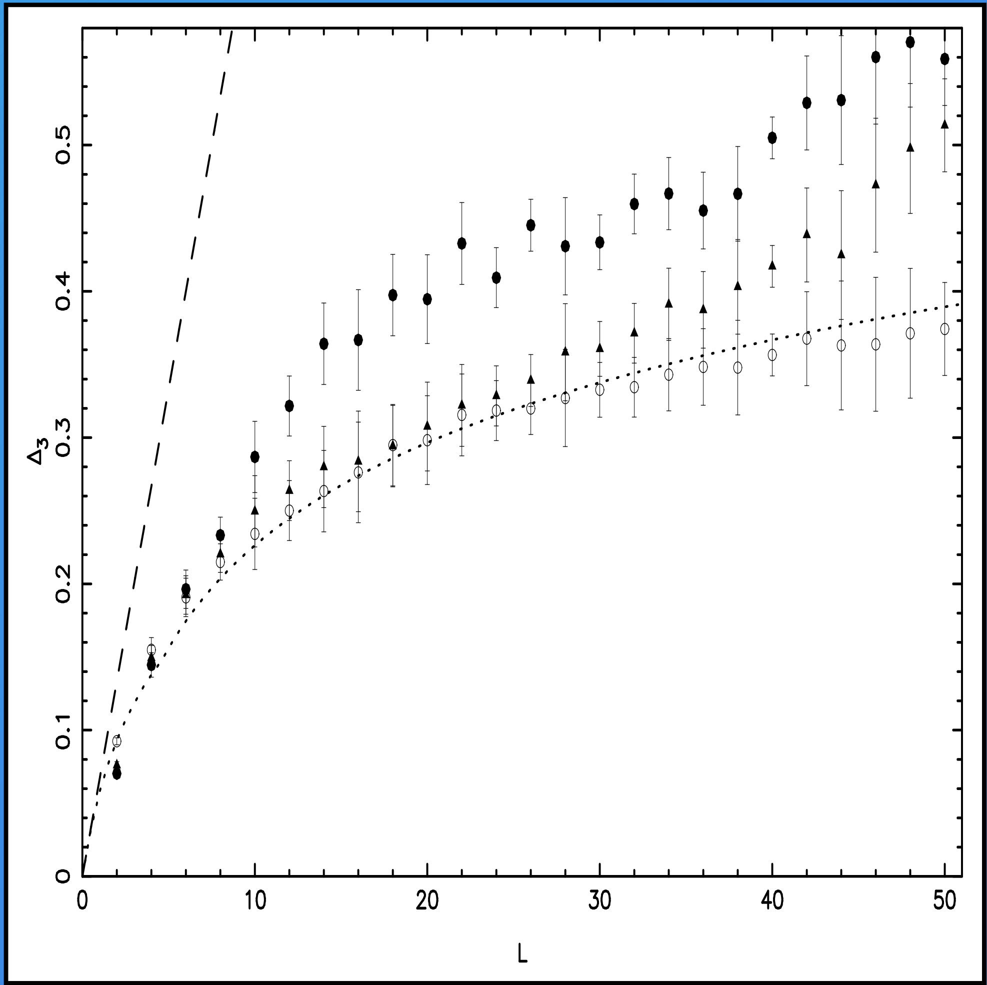
$\langle \Delta_3(L) \rangle$ for the Sinai billiard



$P(s)$ distribution for the nucleus ^{50}Sc (shell model)



$\langle \Delta_3(L) \rangle$ for various nuclei



- ● Ca
- △ Sc
- ○ Ti

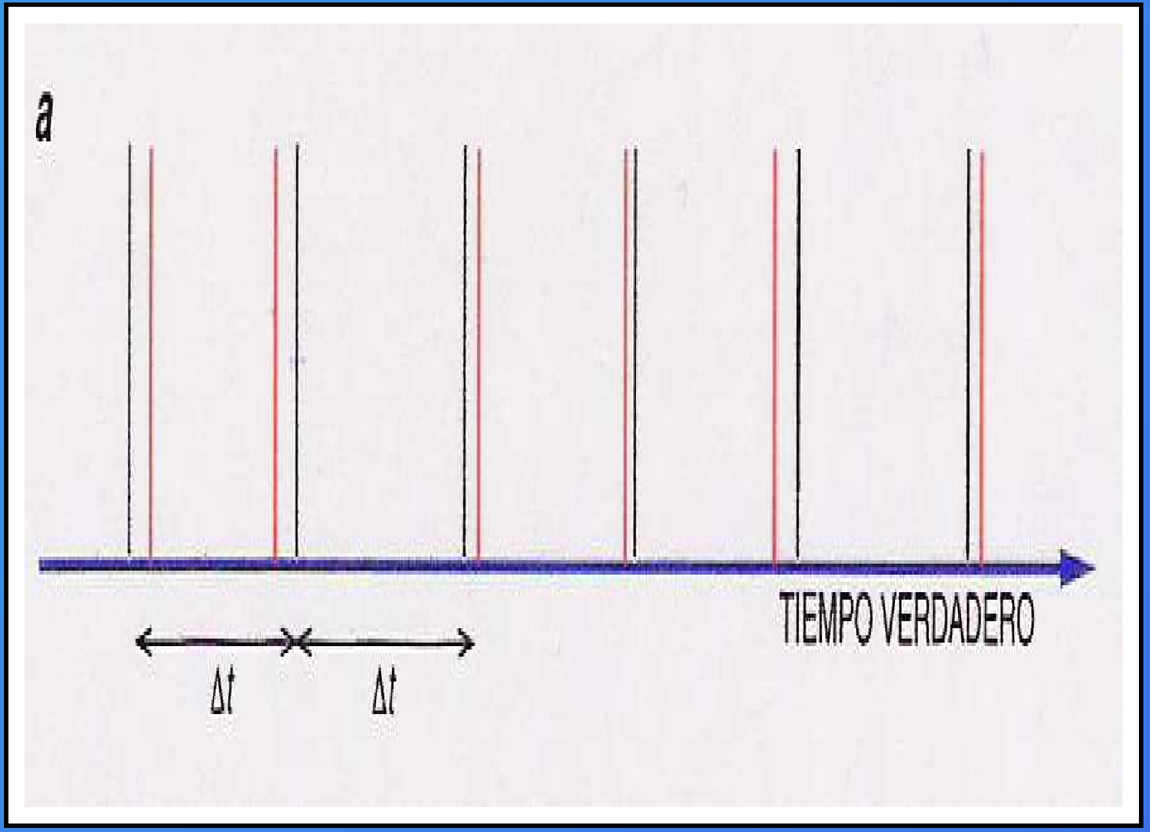
Dashed line: Poisson

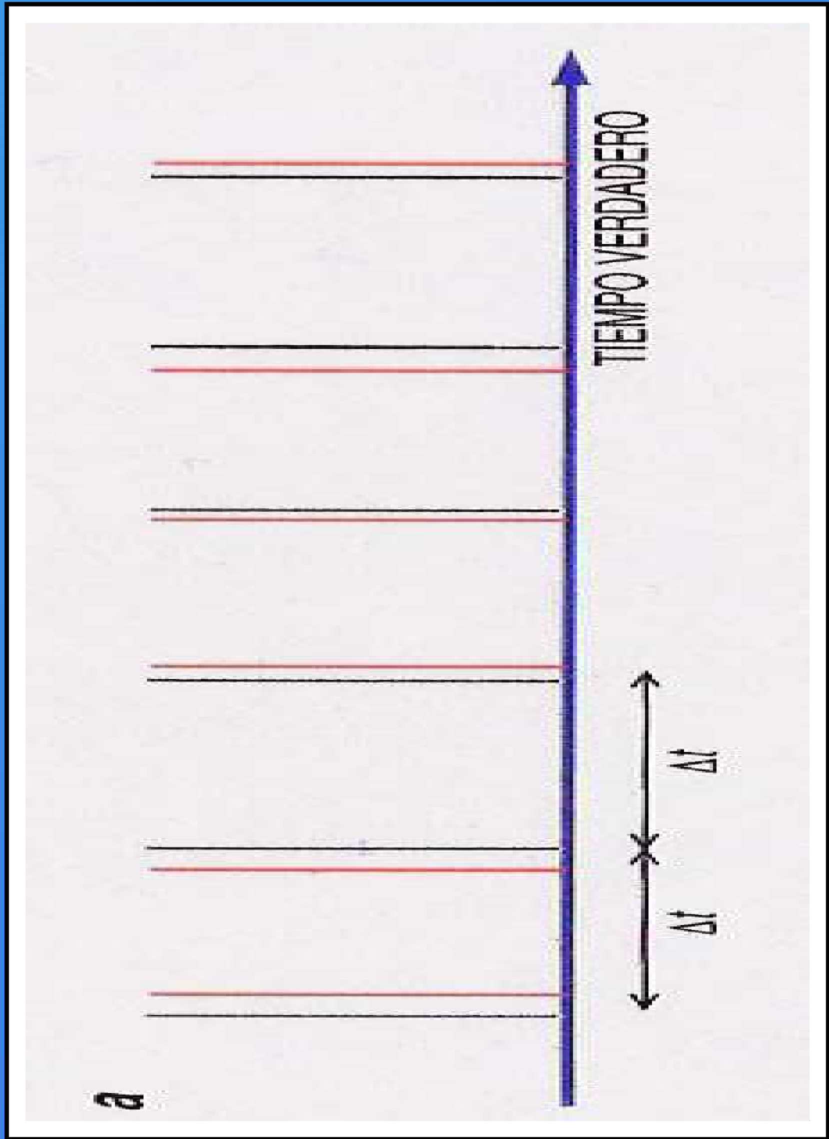
Dotted line: GOE

3 – Chaos and $1/f$ Noise

In 2002 a new approach to quantum chaos was proposed. The original idea came from the observation of a similarity existing between a discrete time series and the energy spectrum of quantum systems. This similarity is clearly exemplified by the comparison of the ticking signal of a clock and the unfolded energy levels of a quantum system.

Schematic comparison of an atomic clock (red) and an ideal clock (black)




TIEMPO VERDADERO
Δt
Δt
a

There is an analogy between a time series and a quantum energy spectrum, if time t is replaced by the energy E of the quantum states.

We define the statistic δ_n as a signal,

$$\delta_n = \sum_{i=1}^{n} (s_i - \langle s \rangle) = \varepsilon_{n+1} - \varepsilon_1 - n$$

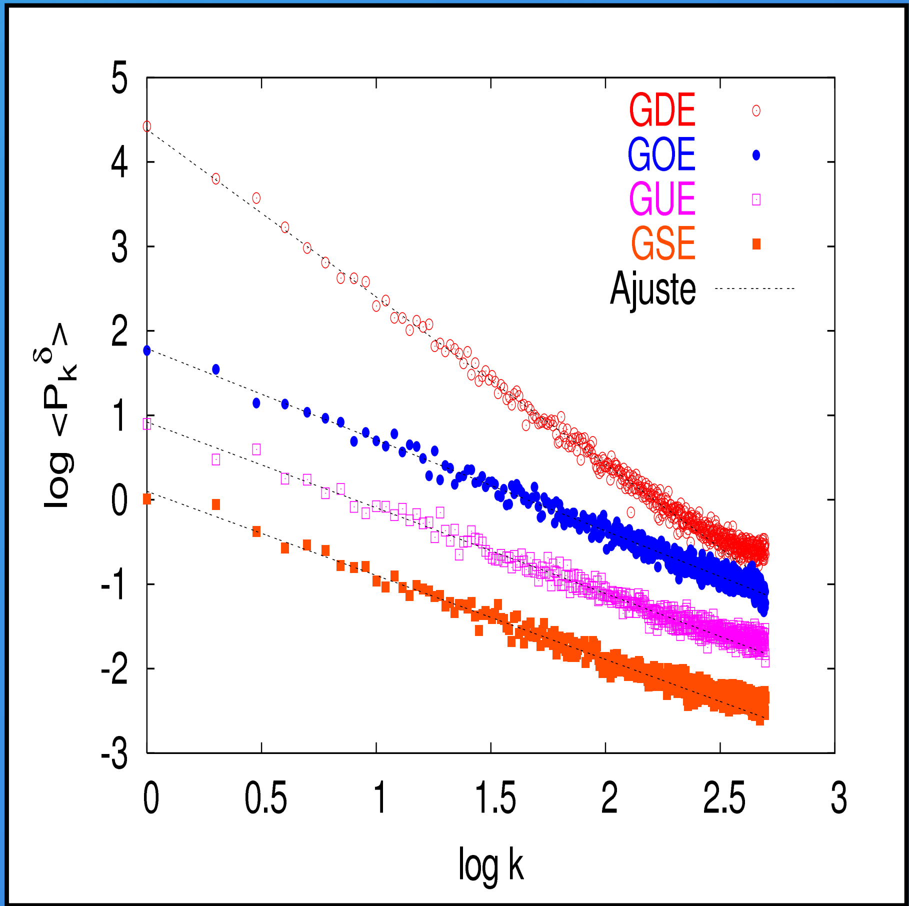
and the discrete power spectrum is

$$S(k) = \left| \widehat{\delta_k} \right|^2 ,$$

where $\widehat{\delta_k}$ is the Fourier transform of δ_n,

$$\widehat{\delta_k} = \frac{1}{\sqrt{N}} \sum_n \delta_n \exp\left(\frac{-2\pi i k n}{N} \right)$$

and N is the size of the series.



GDE: $\alpha = 2.00$

GOE: $\alpha = 1.08$

GUE: $\alpha = 1.02$

GSE: $\alpha = 1.00$

CONJECTURE: The energy spectra of chaotic quantum systems are characterized by $1/f$ noise.

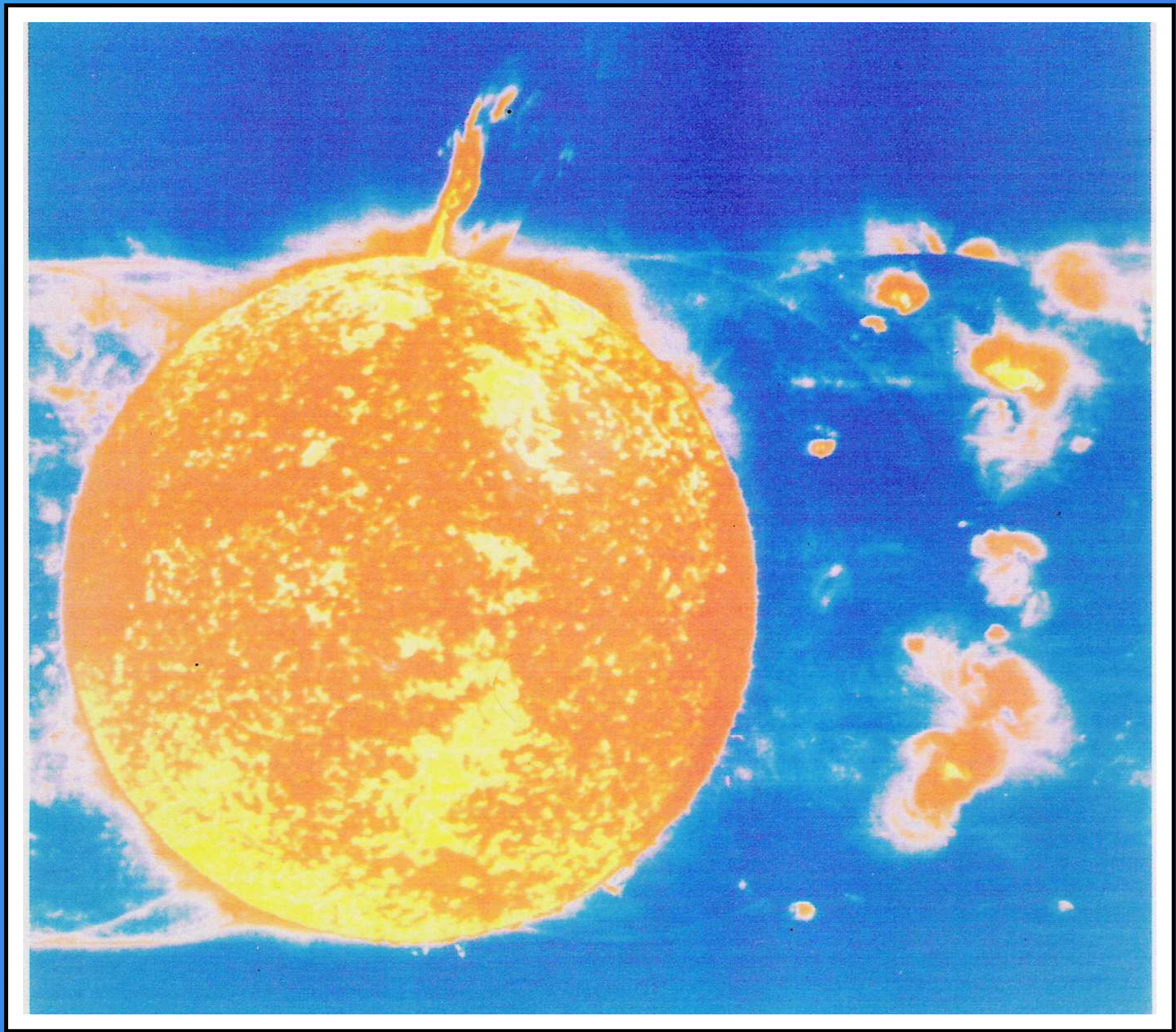
A. Relaño, J. M. G. Gómez, R. A. Molina, J. Retamosa, and E. Faleiro, *Phys. Rev. Lett.* **89**, 244102 (2002)

Features of the conjecture

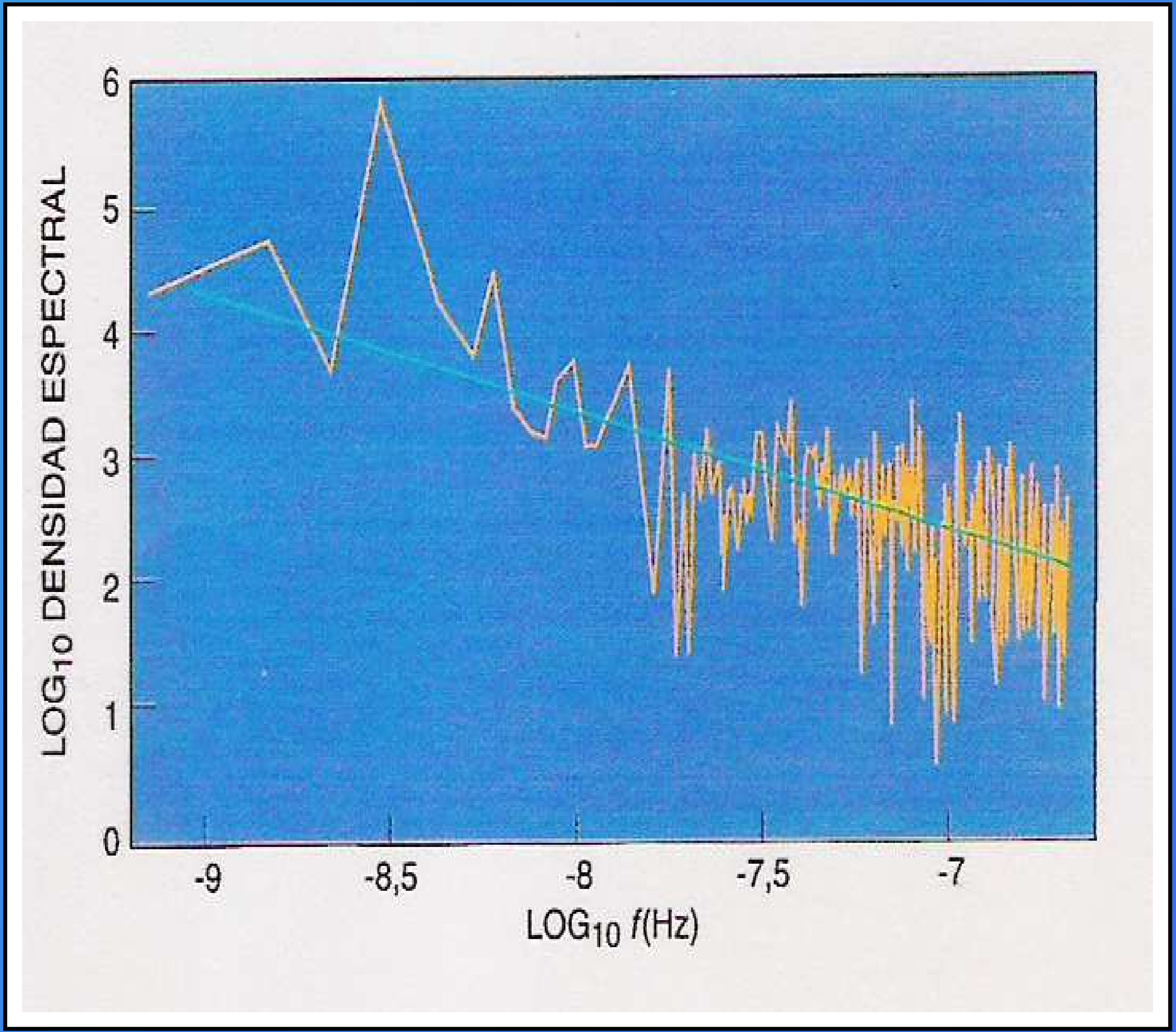
- The $1/f$ noise is an intrinsic property characterizing the chaotic spectrum by itself, without any reference to the properties of other systems such as GOE.
- The $1/f$ feature is universal, i.e. this behavior is the same for all kinds of chaotic systems, independently of their symmetries: either time-reversal invariant or not, either of integer or half-integer spin.

- The $1/f$ noise characterization of quantum chaos includes these physical systems into a widely spread kind of systems appearing in many fields of science, which display $1/f$ fluctuations:
  - Sunspot activity
  - Flow of the Nile river (last 2000 years)
  - Bach music
  - Semiconductor devices
  - Healthy human heartbeat
  - Family extinction through time of all organisms
  - Light coming from white dwarfs

Sunspots activity

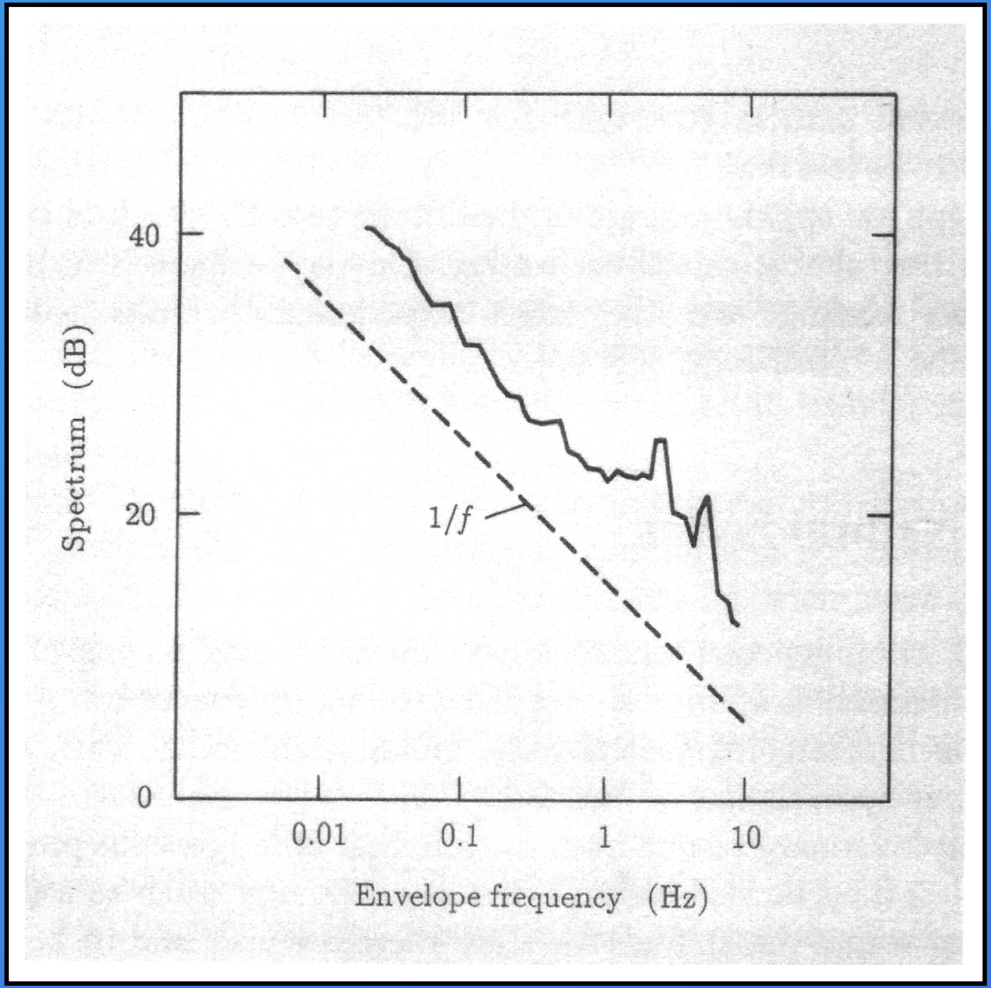


Power spectrum of the Sunspots time series



J. S. Bach's 1st Brandenburg Concerto

Power spectrum of the loudness fluctuations



RMT formula for $S(k)$

$$\langle S(k)\rangle_\beta = \frac{N^2}{4\pi^2}\left[\frac{K_\beta(k/N)-1}{k^2}+\frac{K_\beta(1-k/N)-1}{(N-k)^2}\right] + \frac{1}{4\sin^2\left(\frac{\pi k}{N}\right)}+\Delta, \qquad k=1,2,\ldots,N-1, \quad N\gg 1$$

$$\beta = \begin{cases} 0 & \text{Poisson} \\ 1 & \text{GOE} \\ 2 & \text{GUE} \\ 4 & \text{GSE} \end{cases} \qquad \Delta = \begin{cases} -\dfrac{1}{12}, & \text{for chaotic systems} \\ 0, & \text{for integrable systems} \end{cases}$$

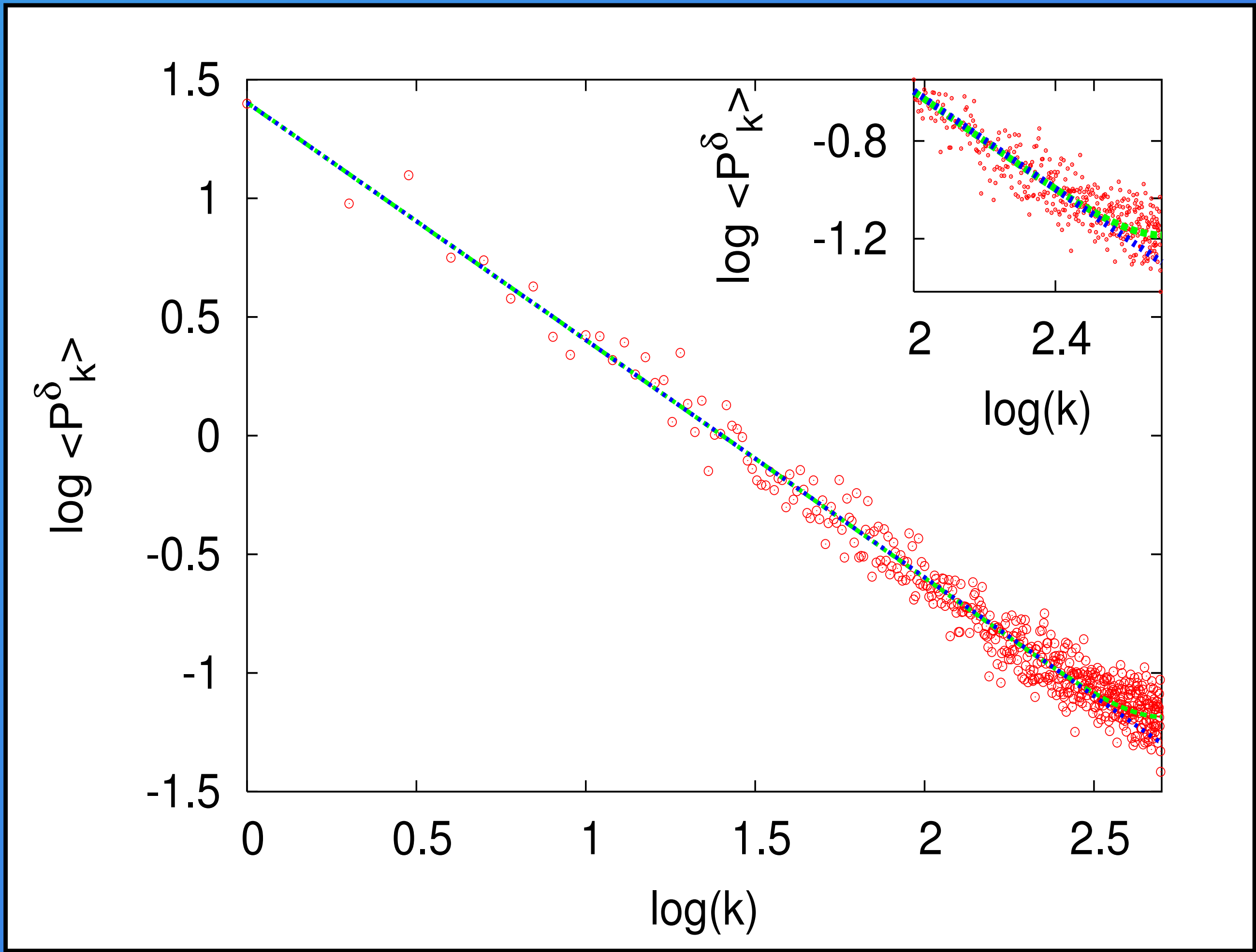
E. Faleiro, J. M. G. Gómez, R. A. Molina, L. Muñoz, A. Relaño, and J. Retamosa, *Phys. Rev. Lett.* **93**, 244101 (2004)

RMT formula for small frequencies

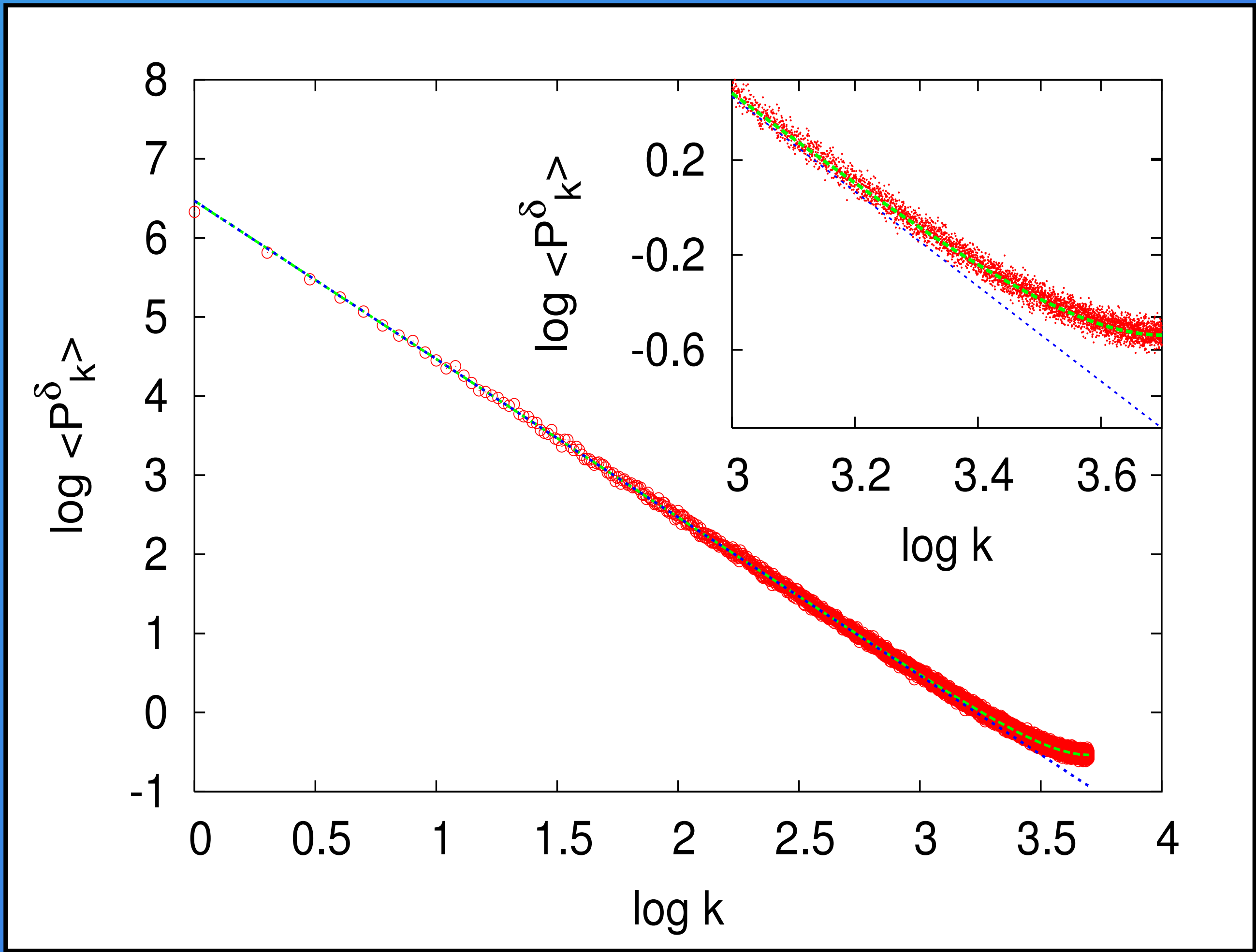
Spectral form factor: $K_\beta(\tau) \simeq \frac{2\tau}{\beta}, \qquad \tau \ll 1.$

$$\langle S(k)\rangle_\beta = \begin{cases} \frac{N}{2\beta\pi^2 k}, & \text{for chaotic systems} \Rightarrow 1/f \ \text{ noise} \\ \frac{N^2}{4\pi^2 k^2}, & \text{for integrable systems} \Rightarrow 1/f^2 \ \text{ Brown noise} \end{cases}$$

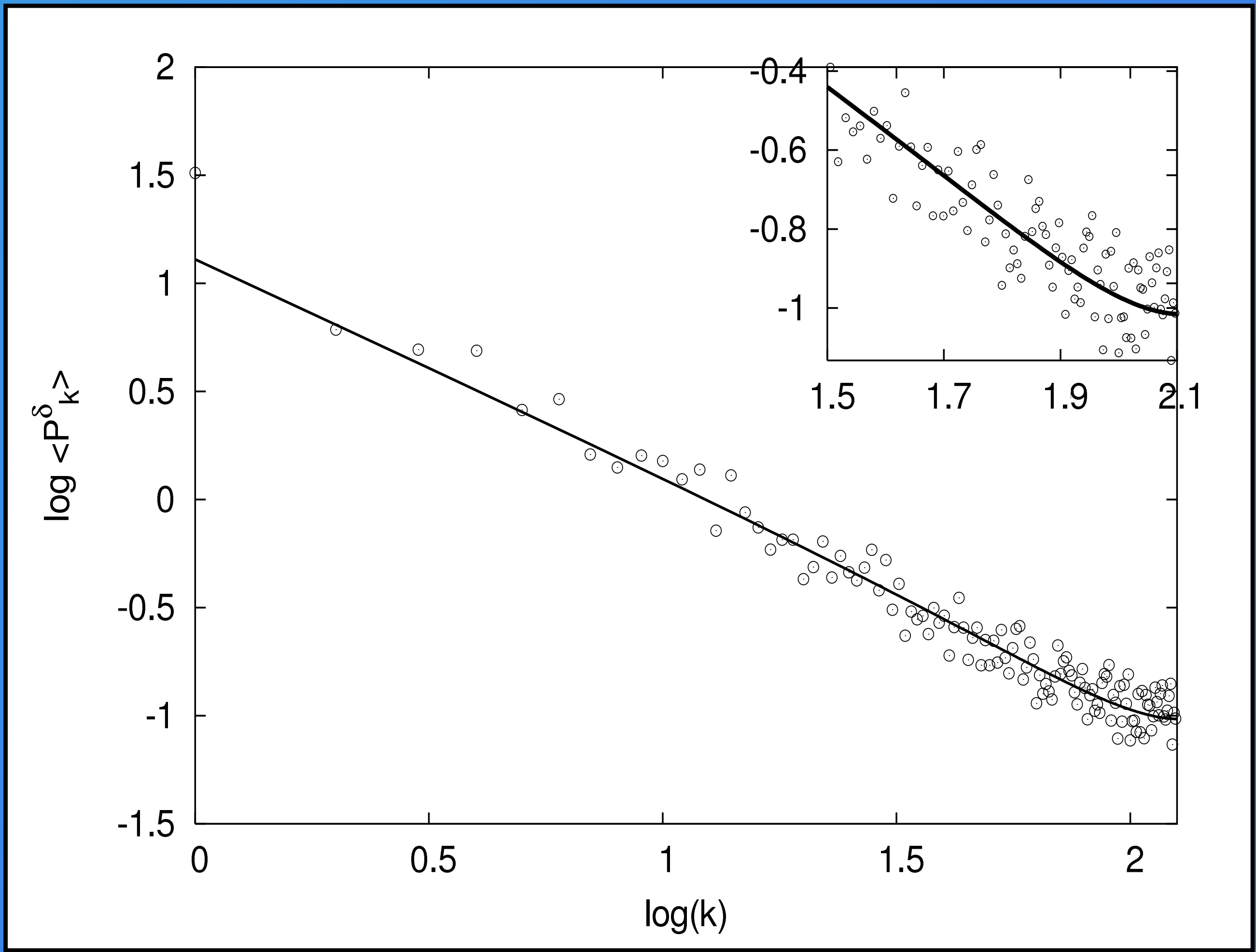
Theoretical vs numerical $\langle S(k) \rangle$ for GUE



Theoretical vs numerical $\langle S(k)\rangle$ for a rectangular billiard



Theoretical vs numerical $\langle S(k) \rangle$ for ^{34}Na



Order to chaos transition

- The transition from order to chaos has been studied in several simple quantum systems with a classical analogue, like the Robnik billiard (Gómez *et al.*, *Phys. Rev. Lett.* **94**, 084101 (2005)), the coupled quartic oscillator, and the quantum kicked top (Santhanam and Bandyopadhyay, *Phys. Rev. Lett.* **95**, 114101 (2005)).

  In all these cases, a $1/f^{\alpha}$ noise is observed throughout the order to chaos transition, with α varying smoothly from $\alpha = 2$ to $\alpha = 1$.

- Recently a random matrix model based on tunneling between chaotic and regular parts of phase space, has been proposed to explain this behavior (A. Relaño, *Phys. Rev. Lett.* **100**, 224101 (2008)).

4 – Applications

Nuclear spectra with missing levels and mixed symmetries

- When the spectrum is not perfect, the spectral statistics do not coincide with RMT predictions: missing levels and mixed symmetries in nuclear spectra deviate statistics from GOE behavior towards Poisson
- General case:
  - φ_i: fraction of observed levels in the i-th sequence
  - $\eta_i = \overline{\rho_i}(\epsilon) / \left(\sum_{i=1}^{l} \overline{\rho_i}(\epsilon) \right)$: fractional densities ($l$ = number of sequences)

R. A. Molina *et al.*, *Phys. Lett. B* **644**, 25 (2007)

➢ Analytical derivation

$$\left\langle \mathcal{P}_k^{\delta} \right\rangle = \frac{N^2}{4\pi} \sum_{i=1}^{l} \eta_i \varphi_i \left[\frac{K_i\left(\frac{\varphi_i k}{N\eta_i}\right) - 1}{k^2} + \frac{K_i\left(\frac{\varphi_i (N-k)}{N\eta_i}\right) - 1}{(N-k)^2} \right. \\ \left. + \frac{1}{4\sin^2(\pi k/N)} + \langle \varphi \rangle^2 \Delta \right]$$

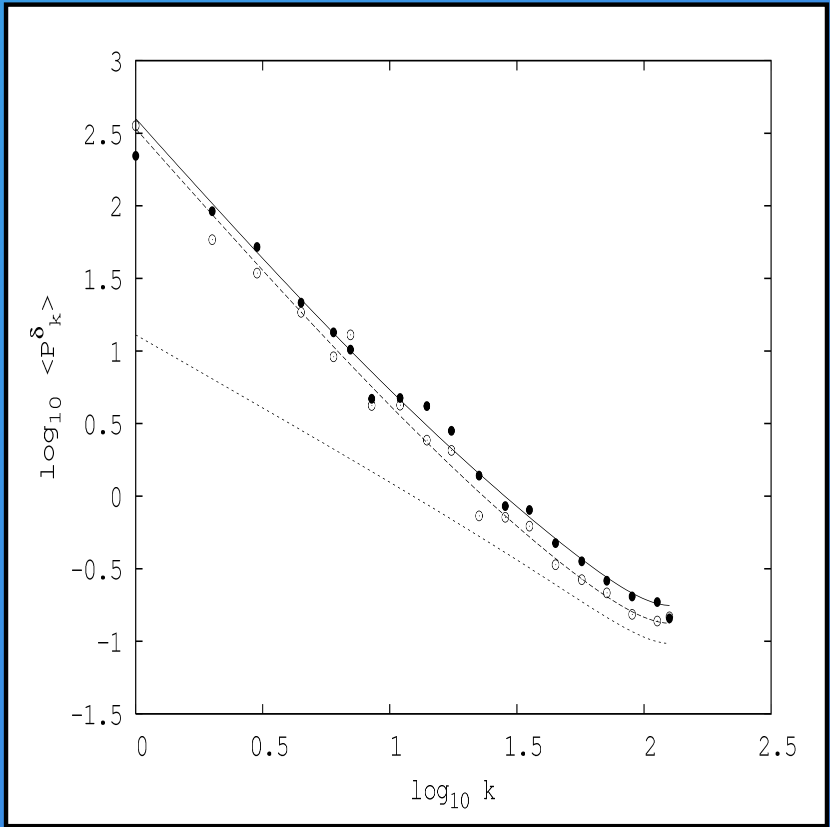
- Levels of sequence i are observed with probability φ_i
- A level belongs to sequence i with probability η_i

- Simplifications: $\begin{cases} \varphi_i = \varphi \quad \forall i \\ \eta_i = \eta = 1/l \quad \forall i \end{cases}$

$$\langle \mathcal{P}_k^\delta \rangle = \frac{N^2}{4\pi}\varphi \left[\frac{K\left(\frac{l\varphi k}{N}\right) - 1}{k^2} + \frac{K\left(\frac{l\varphi (N-k)}{N}\right) - 1}{(N-k)^2} \right.$$
$$\left. + \frac{1}{4\sin^2(\pi k/N)} + \varphi^2 \Delta \right]$$

- Can we obtain a good estimation of the parameters φ and l?

➛ **Numerics**: Shell model spectra



- Two mixed symmetries:

  $J = 3, 4 \quad (l = 2)$

- Incomplete sequences ($\varphi = 0.8$)
- Fit of φ and l for GOE:

  ⋆ Mixed sequences (filled circles) $\begin{cases} \varphi = 0.77 \pm 0.03 \\ l = 2.1 \pm 0.4 \end{cases}$

  ⋆ Pure sequences (open circles) $\begin{cases} \varphi = 0.80 \pm 0.03 \\ l = 1.1 \pm 0.3 \end{cases}$

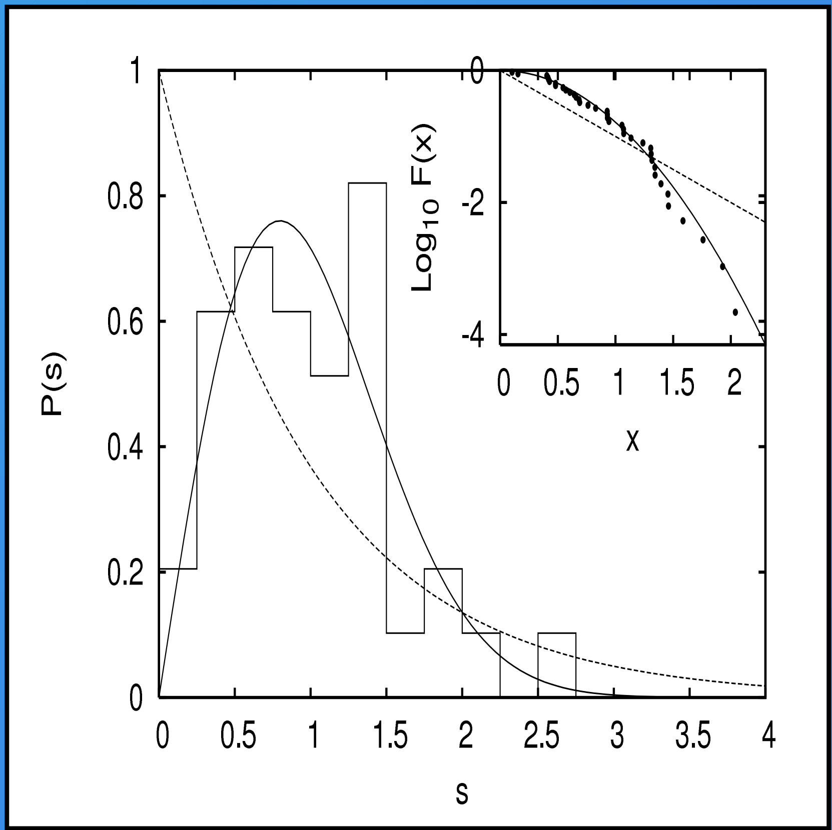
➛ **We obtain very good estimates of φ and l**

Baryon spectra

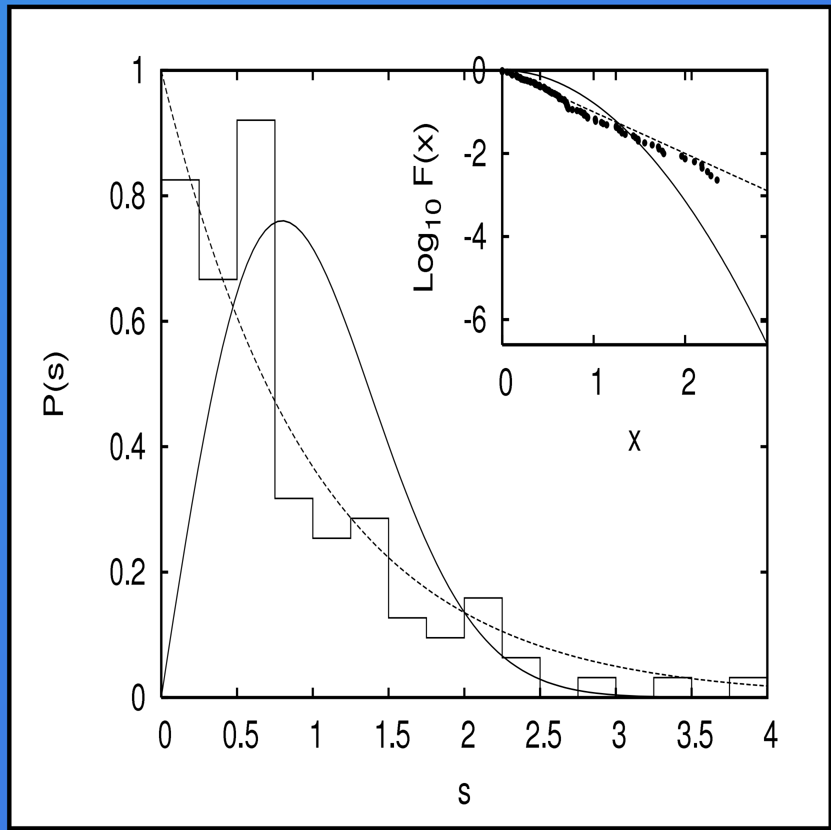
Problem: The number of baryons predicted by quark models is larger than what is observed experimentally.

- A spectral fluctuation analysis has been performed.
- The experimental $P(s)$ distribution is close to RMT. Quark model results are close to the Poisson distribution.

Experimental



Quark Model



$$F(x) = 1 - \int_0^x P(s)ds$$

- If the observed experimental spectra is incomplete, the experimental $P(s)$ distribution should be much closer to Poisson than the theoretical one. The situation is just the opposite.
- Present quark models are not able to reproduce the statistical properties of the experimental baryon spectrum.

5 – Concluding remarks

- The observation of a formal analogy between a time series and the energy level spectrum of a quantum system, and the characterization of the spectral fluctuations by means of the statistic δ_n, have opened a new field in the study of quantum chaos.
- The power spectrum $S(k)$ of δ_n is a simple statistic, easy to calculate and easily interpreted. It characterizes the fully chaotic or regular behavior of a quantum system by a single quantity, the exponent α of the $1/f^\alpha$ noise. This result is valid for all quantum systems, independently of their symmetries (time-reversal invariance or not, integer or half-integer spin, etc.).

- The power spectrum approach to spectral fluctuations can be used to detect the fraction of missing levels and the number of mixed symmetries in experimental spectra.