Instantaneous Shape Sampling for Calculating the Electromagnetic Dipole Strength in Transitional Nuclei

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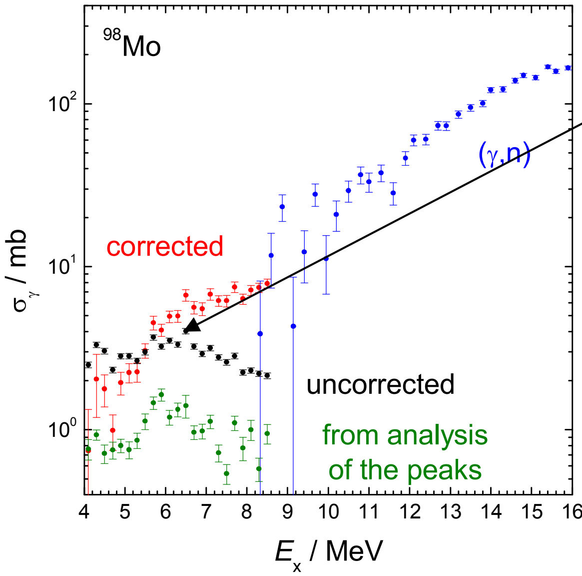
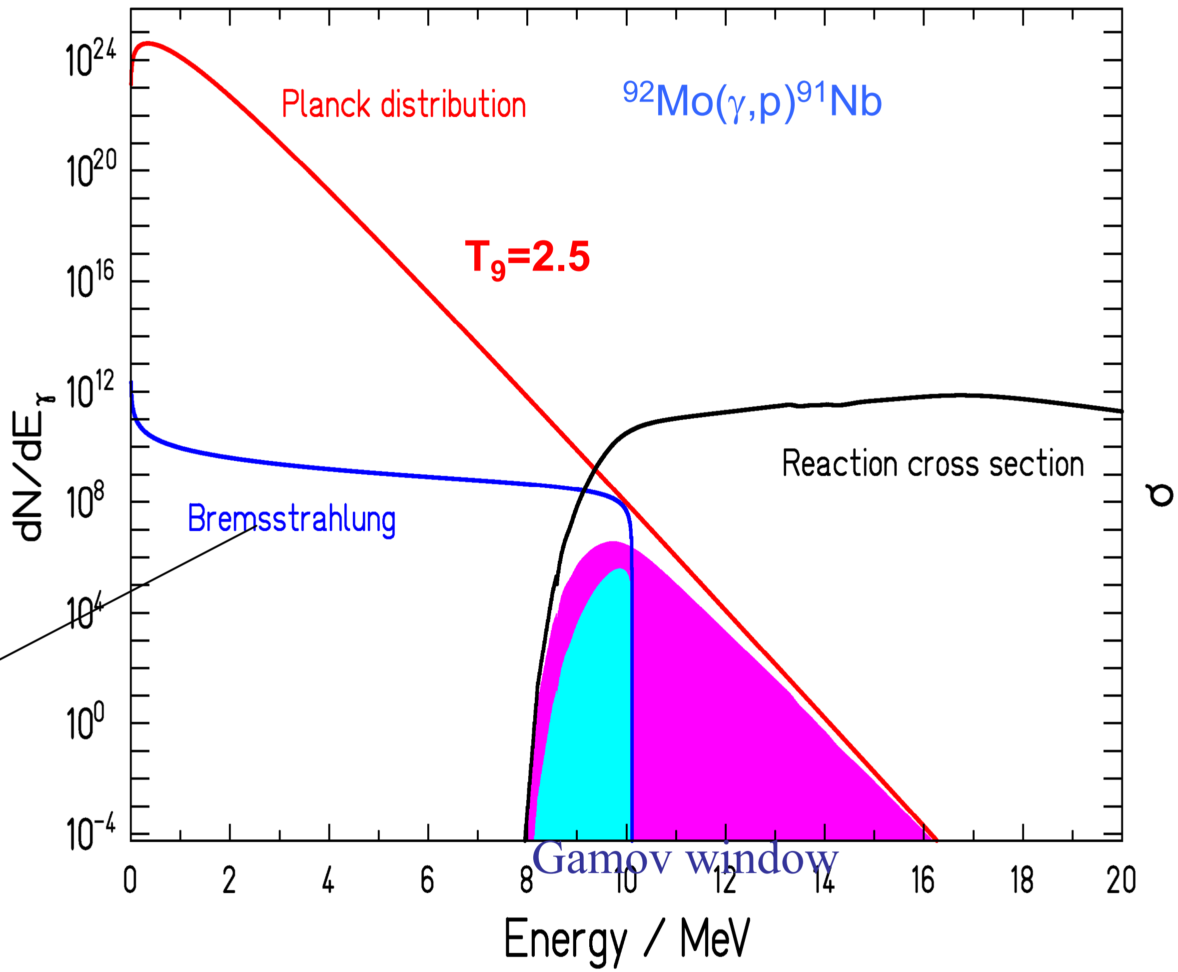
- I. Bentley (ND)

Photo disintegration –Experimental situation

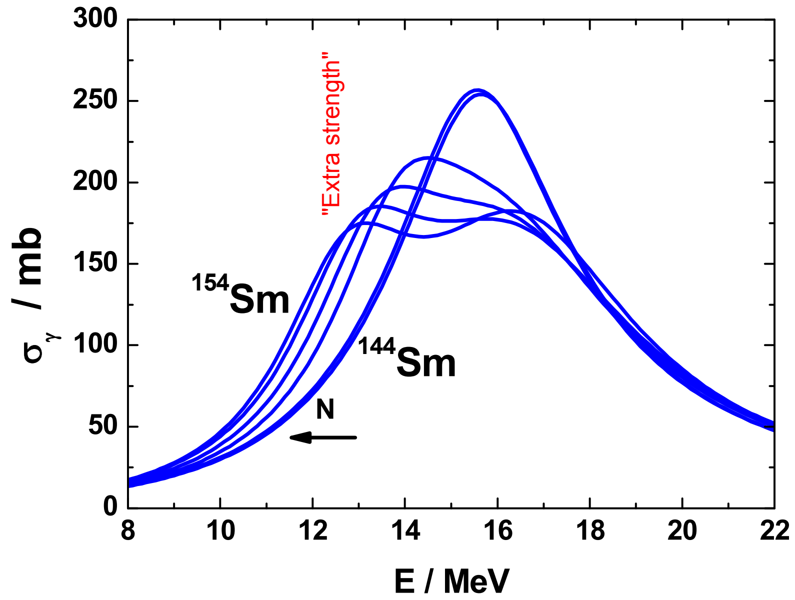
In hot stellar medium
for (γ,p) , (γ,n) , (γ,α)
is only a narrow
window available.

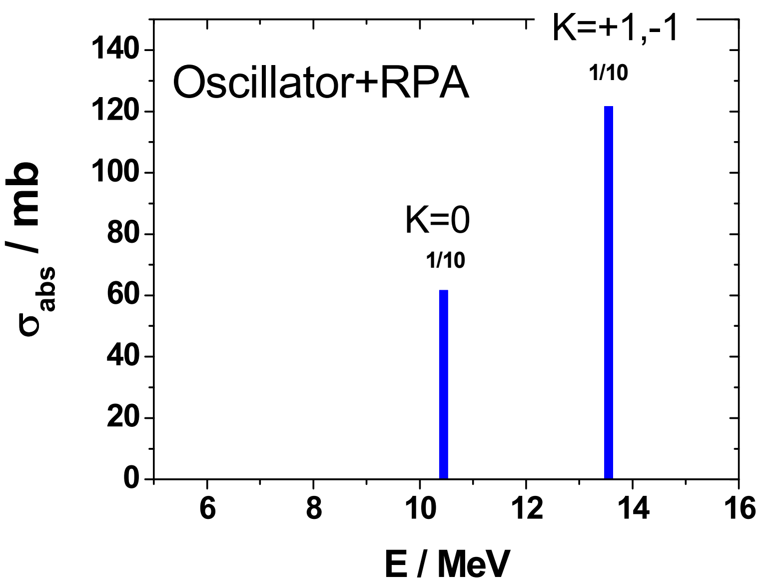
Dipole strength close to
the particle separation
energy is important





Static deformation + Hydrodynamic/oscillator approximation



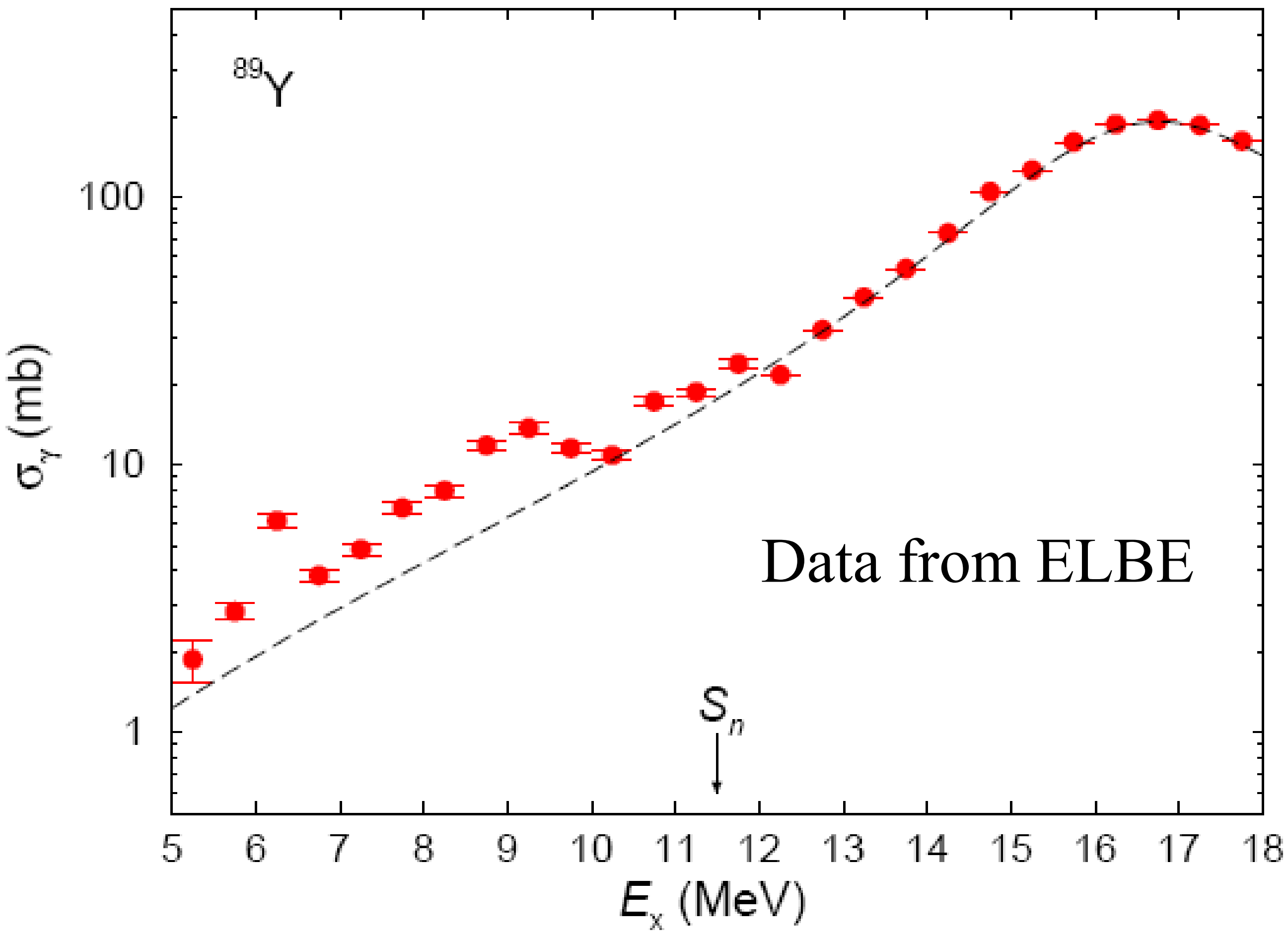


Lorentzian widths **$\Gamma_{K=\pm1} = 2\Gamma_{K=0}$ = 6 MeV**.
Shape parameter ε from Moeller&Nix
Deformation increase ε=0 (A=144) to ε=0.250 (A=154)
leads to increasing strength in the tail region.

Standard in reaction networks (width related to quadrupole softness)

Why a microscopic theory?

- There may be still fragments of the nucleonic orbitals
- Low level density-discreteness
- Ratio M1/E1
- Predictions for neutron-rich nuclei, based on developments of mean field theory



Present (γ, γ) data
$+ (\gamma, p)$ data
$+ (\gamma, n)$ data

Lorentz curve:
$E_0 = 16.8$ MeV
$\Gamma = 4.1$ MeV
$\frac{\pi}{2}\sigma_0\Gamma = 60 \frac{NZ}{A}$ MeV mb

RPA with static deformation

Nilsson or Woods-Saxon potential+ isovector dipole-dipole interaction

$$H = h_{Nilsson/WS}(\beta, \gamma, \ldots) + \frac{m\omega_o^2}{A}\eta(R_\pi - R_\upsilon)^2$$

η = *1.5 - 3* (ratio np/nn of effective force)
$R_{\pi,\nu}$ = center of mass for Z protons or N neutrons, resp.

Solve RPA equation of motion for the E1 and M1 modes

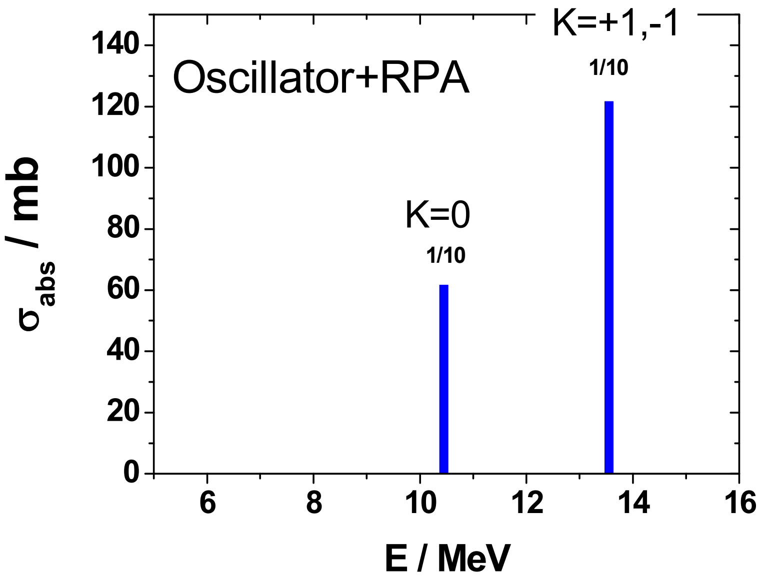
$$\left[H, \Omega_i^+\right] = E_i \Omega_i^+$$

to find the vibrational modes $\boldsymbol{E_i}$ (*i* ≈ *3000-5000*) and their E1 and M1 strengths.

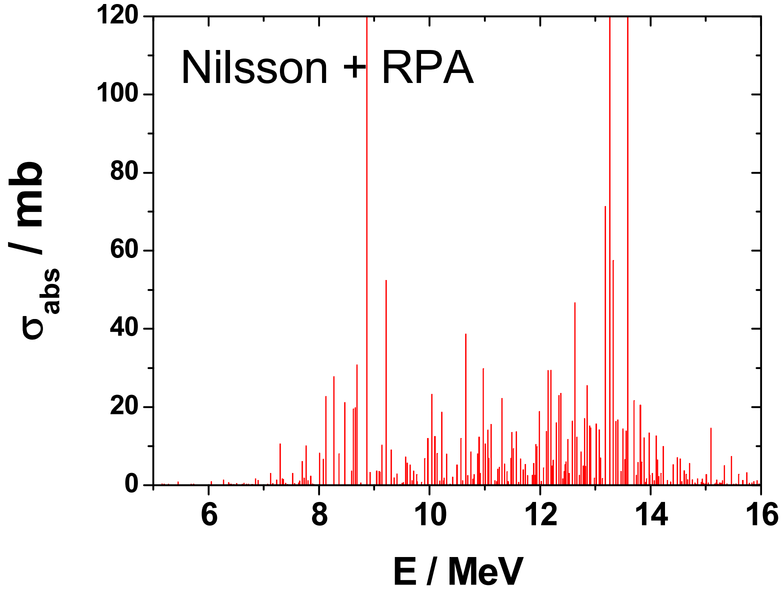
Method: Contour Integration in complex plane.
Generates strength function Lorentzian-smoothed with narrow width

RPA solution for ^{154}Sm

Oscillator: split of strength



Nilsson: Landau fragmentation and split of E1 strength

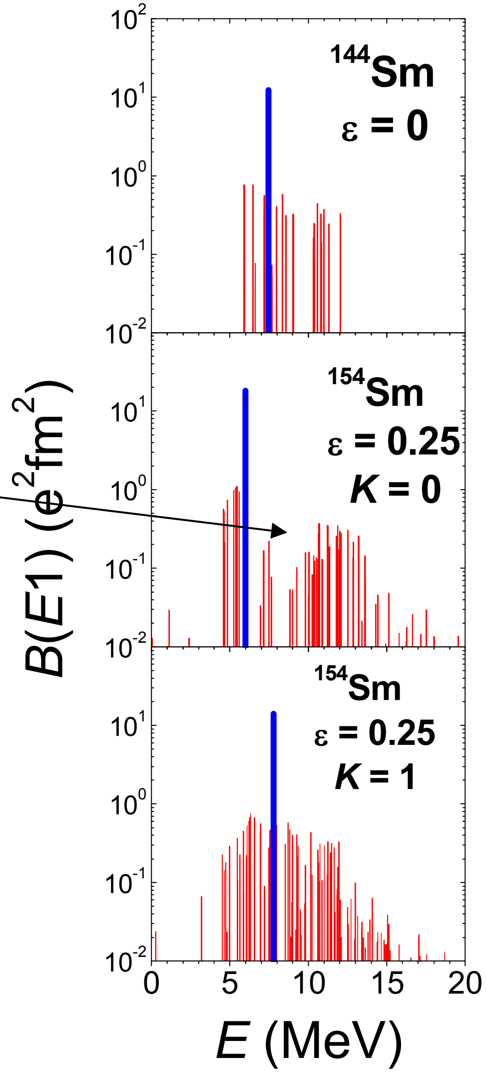


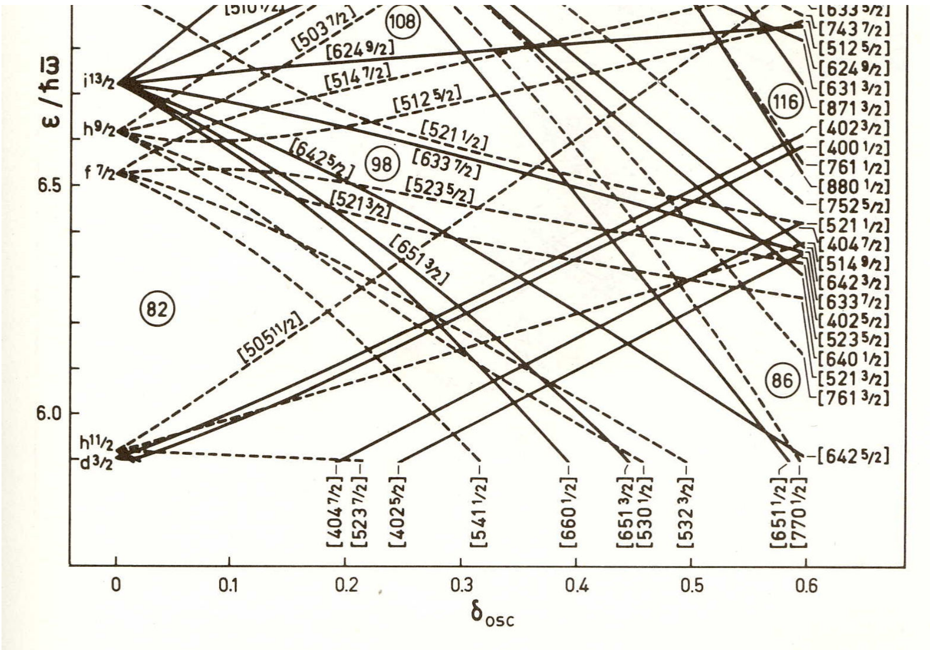
E1 strength the individual particle-hole excitations

B(E1,ph) = <p| *M(E1)*|h> ^{2}
no interaction between the 1p1h excitations

Oscillator
  One line for spherical
  Two lines for axial deformed (K=0 and K=±1)

Nilsson
  Many lines according to the deformation splitting,
    intermediate Structure





Nuclear shape is not rigid

- RPA for fixed shape
- QPM for spherical shape
- Many nuclei have large amplitude shape fluctuations
- Several show shape coexistence

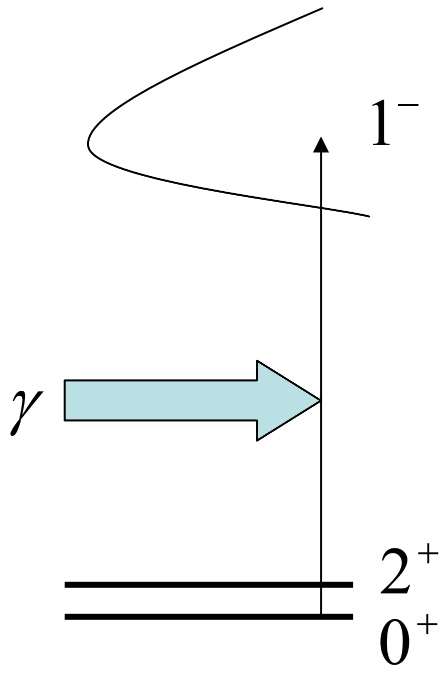
Instantaneous Shape Sampling - ISS

Many nuclei have a soft, fluctuating shape.

The quadrupole dynamics is slow compared to the dipole vibrations:

$$\frac{\Delta t_{quadrupole}}{\Delta t_{dipole}} \sim \frac{E(2^+)}{E(1^-)} \sim \frac{1}{10}$$

The absorbed γ quant sees a snapshot of the fluctuating shape.



$$\sigma_\gamma(E) = \sum_n P(\beta_n, \gamma_n)\sigma_\gamma(E, \beta_n, \gamma_n).$$

Probability for instantaneous shape *n*.

Calculated by RPA for instantaneous shape *n*.

Probability distribution for the shapes

Te quadrupole shape dynamics is described by the IBA1.

$$H(\zeta) = c\left[(1-\zeta)\hat{n}_d - \frac{\zeta}{4N_B}\hat{Q}^{\chi}\cdot\hat{Q}^{\chi} \right], \quad \hat{n}_d = d^{\dagger}\cdot\tilde{d}.$$

$$\hat{Q}^{\chi} = (s^{\dagger}\tilde{d} + d^{\dagger}s) + \chi(d^{\dagger}\tilde{d})^{(2)}$$

E. A. McCutchan *et al.*, Phys. Rev. C **69**, 064306 (2004).

Number of bosons: 10
Diagonalize *H* ⟶ excitation energies, *E2* transition rates
Fitted to experiment ⟶ parameters ⌘ &
Ground state |0>

Diagonalize

$$\hat{q}_2 = [Q^\chi \otimes Q^\chi]_0 \,,$$
$$\hat{q}_3 = [Q^\chi \otimes [Q^\chi \otimes Q^\chi]_2]_0 \,,$$

Localized states |*n*>

$$\beta_n^2 = \sqrt{5}\left(\frac{4\pi e_B}{3ZeR^2}\right)^2 q_{2,n}, \quad \cos 3\gamma_n = \sqrt{\frac{7}{2\sqrt{5}}}\frac{q_{3,n}}{(q_{2,n})^{3/2}},$$

$$\beta_n^2 = \sqrt{5}\left(\frac{4\pi e_B}{3ZeR^2}\right)^2 q_{2,n}, \quad \cos 3\gamma_n = \sqrt{\frac{7}{2\sqrt{5}}}\frac{q_{3,n}}{(q_{2,n})^{3/2}},$$

$$P(\beta_n, \gamma_n) = \left| < 0 \mid n > \right|^2$$

TABLE I: IBA parameters ζ, χ, e_B, and equilibrium deformation parameters β, γ calculated by means of the micro-macro method. In case of shape coexistence, two sets are listed. Their respective proportion in the ground state is given in percentage.

^{A}X	ζ	χ	e_B	%	β	γ
^{88}Sr	0.0	-1.20	0.043	100	0.0	0°
^{90}Zr	0.0	-1.20	0.013	64	0.0	0°
	0.60	-0.31	0.040	36		
^{92}Mo	0.25	-1.32	0.040	100	0.0	0°
^{94}Mo	0.29	-1.20	0.064	100	0.02	60°
^{96}Mo	0.20	-1.32	0.069	100	0.10	60°
^{98}Mo	0.0	-1.20	0.053	60	0.18	37°
	0.59	-0.03	0.106	40		
^{100}Mo	0.0	-1.20	0.053	40	0.21	32°
	0.61	-0.10	0.106	60		

P. E. Garrett *et al.*, Phys. Rev. C **68**, 024312 (2003).
G. Cata *et al.*, Z. Phys. A **335**, 271 (1990).

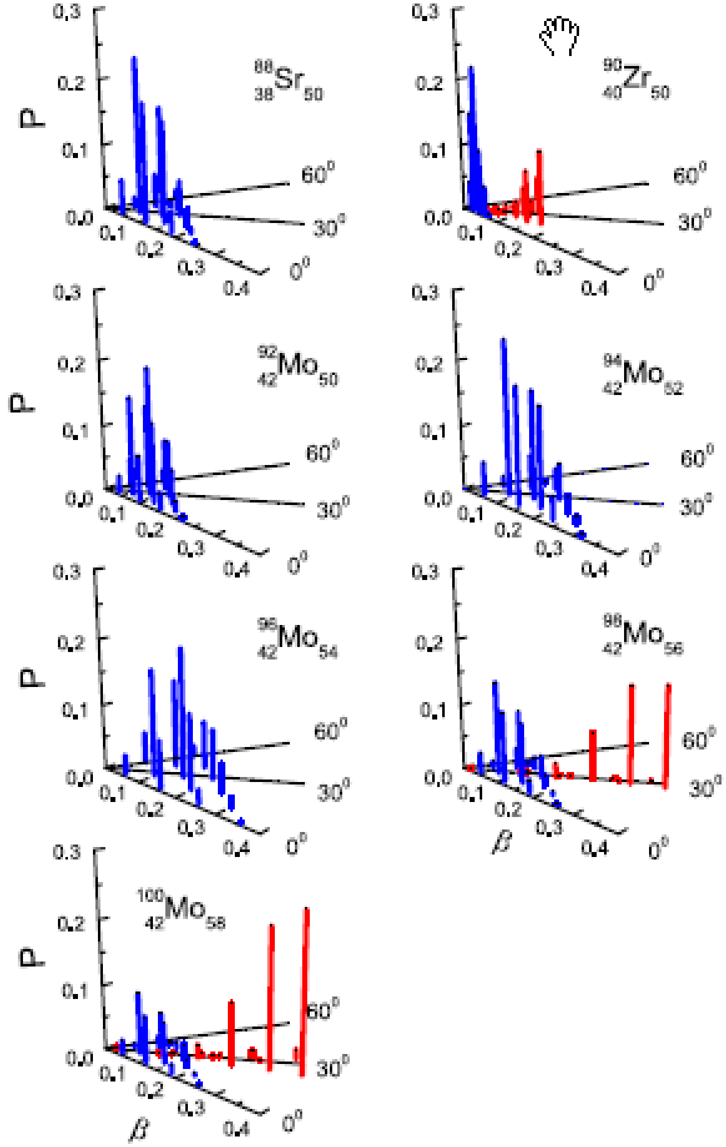


FIG. 1: (Color online) Probability distributions of the instantaneous nuclear shapes over the $\beta - \gamma$ plane. Coexisting distributions are distinguished by their color.

Shape coexistence

Large fluctuations

RPA for the set instantaneous deformations

Woods-Saxon potential+ isovector dipole-dipole interaction

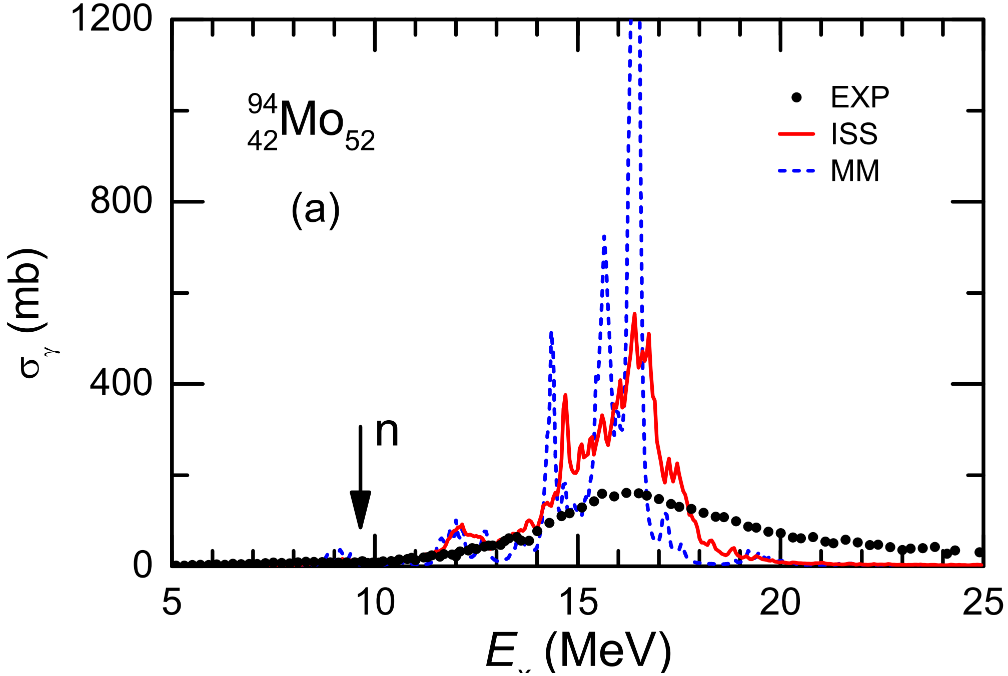
$$H = h_{WS}(\beta_n, \gamma_n) + \frac{m\omega_o^2}{A}\eta(\vec{R}_\pi - \vec{R}_\upsilon)^2 + \frac{\kappa}{2}\vec{\Sigma}^2$$

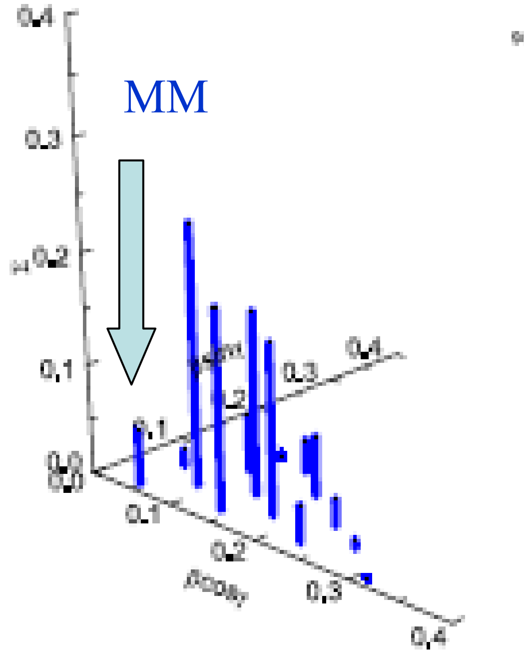
η adjusted to reproduce the GDR position in the region
$R_{\pi,\nu}$ = center of mass for Z protons or N neutrons, resp.
♦ ◘ total spin of all nucleons, & adjusted to reproduce the spin flip resonance
Solve RPA equation of motion

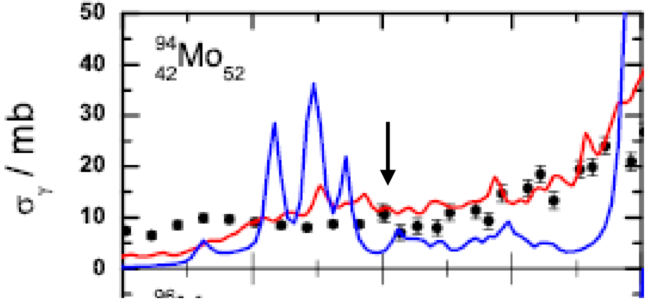
$$\left[H, \Omega_i^+\right] = E_i \Omega_i^+$$

to find the vibrational modes $\boldsymbol{E_i}$ ($i \approx 3000\text{-}5000$) and their E1 strengths.

Method: Contour Integration in complex plane.
Generates $\sigma_\gamma(E, \beta_n, \gamma_n)$ (smoothed with narrow width D) .



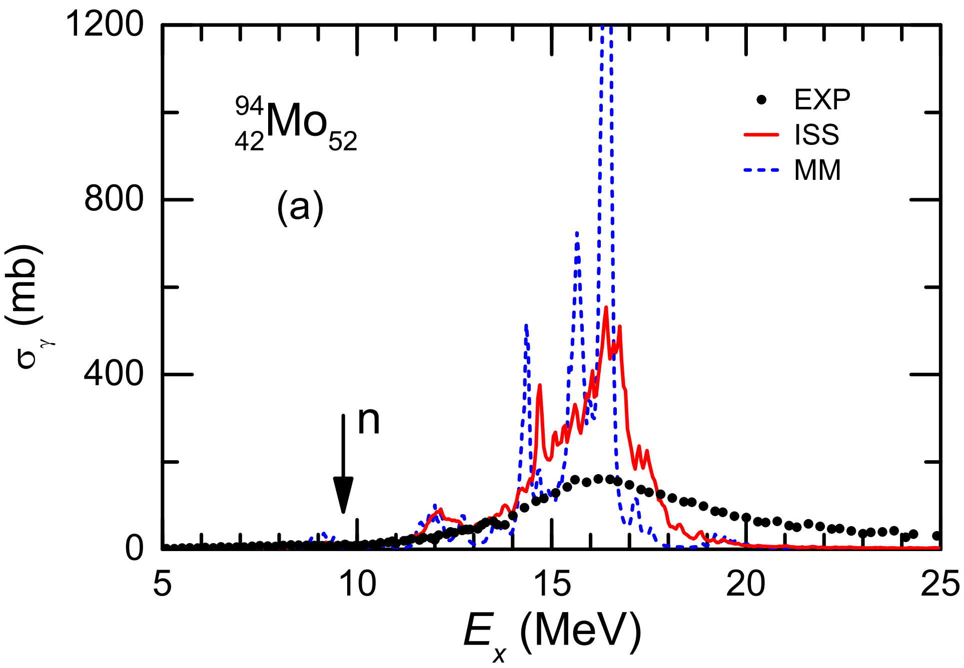




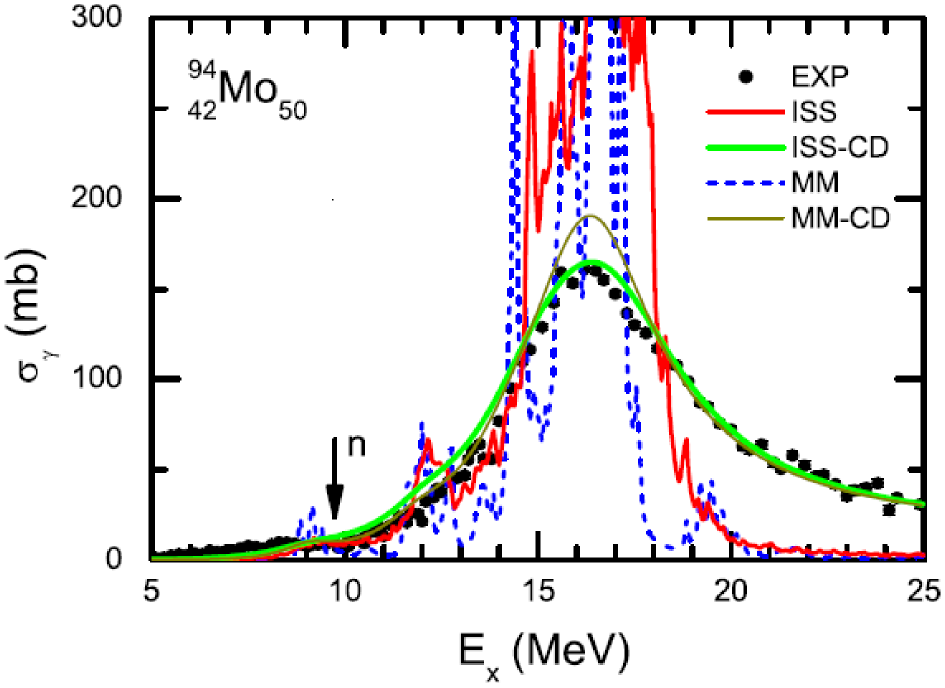
Elbe data + (n,γ_0)

Equilibrium deformation (Micro-Macro - MM)

Instantaneous Shape Sampling - ISS



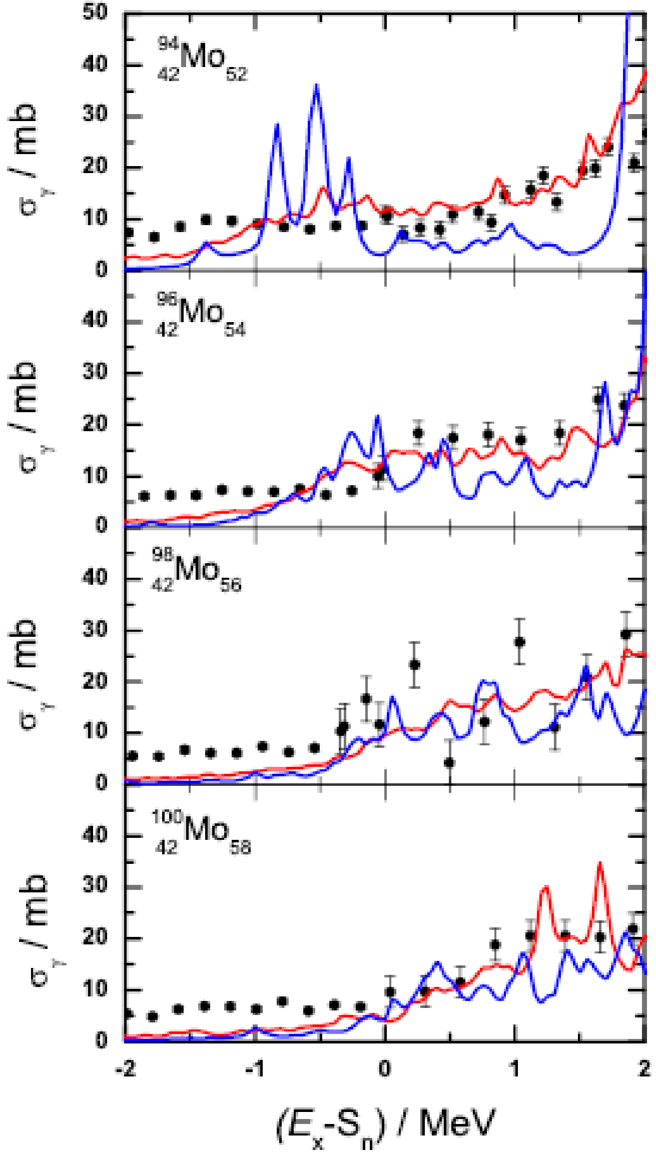
Landau fragmentation
+ Shape fluctuations
give too weak damping in GDR region

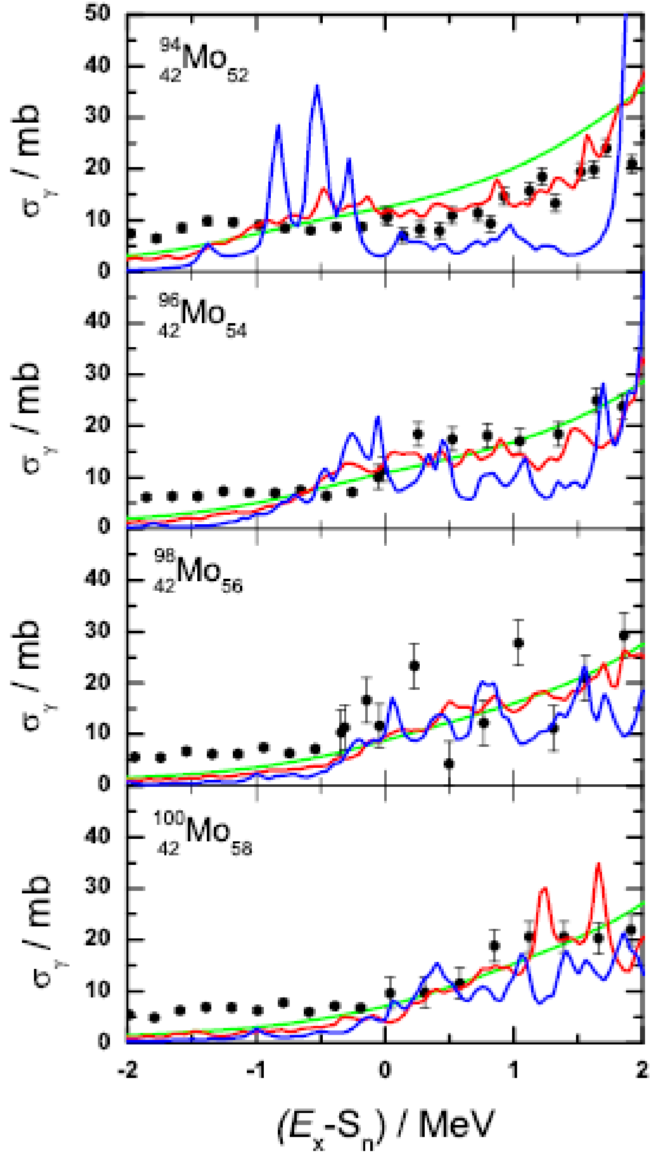


There are other degrees of freedom that cause "Collisional damping"
Taken into account phenomenologically

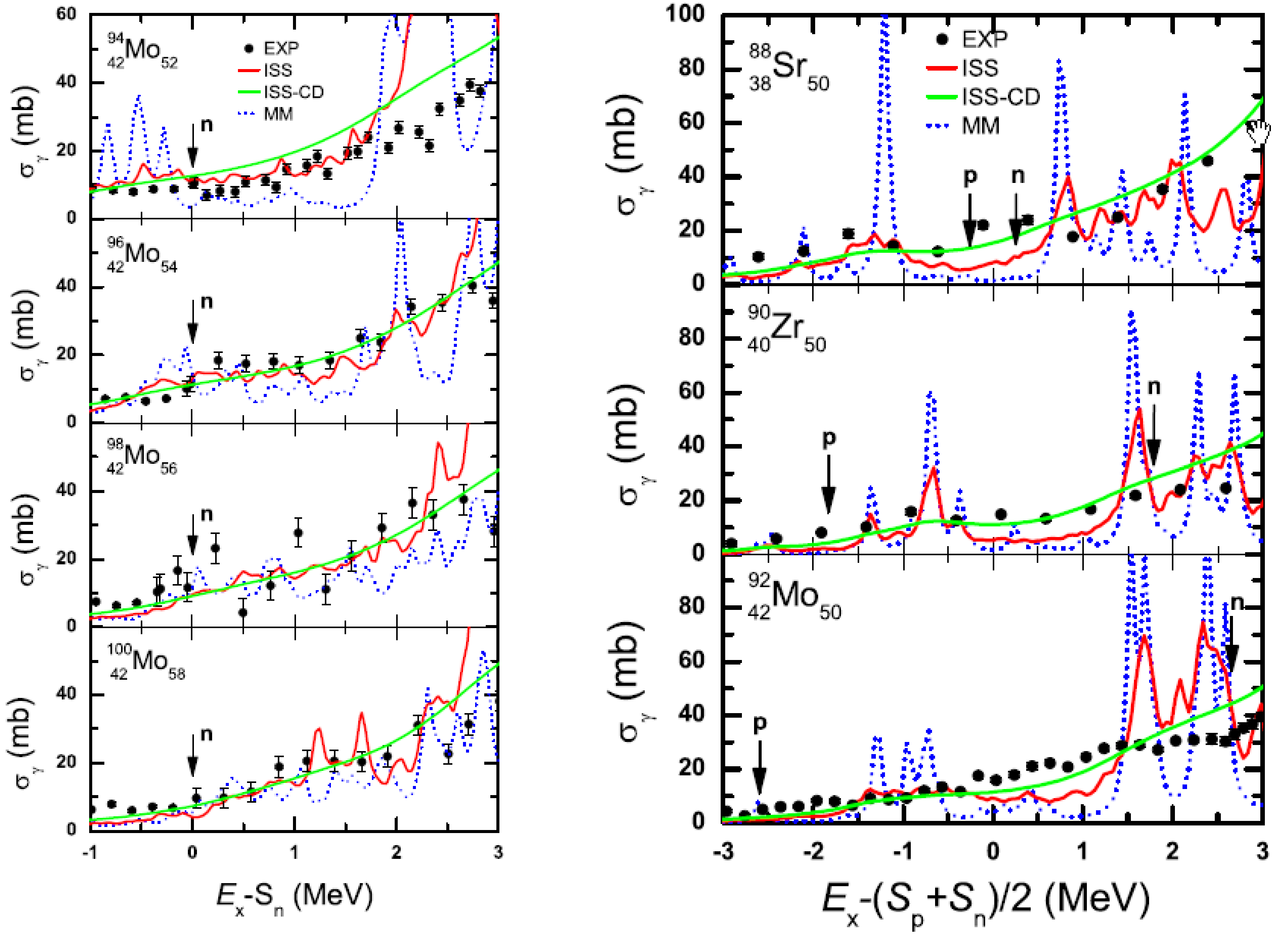
$$\sigma_{ISS-CD}(E) = \int dE' \sigma_{ISS}(E') \frac{\Gamma(E')}{2\pi\left((E-E')^2 + (\Gamma(E)/2)^2\right)}$$

$$\Gamma(E) = \alpha E^2, \quad \alpha = 0.014 MeV^{-1} (N > 50),$$
$$= 0.0105 MeV^{-1} (N = 50)$$





Collision damping does not shift dipole strength into the threshold region
It only smoothes the cross section

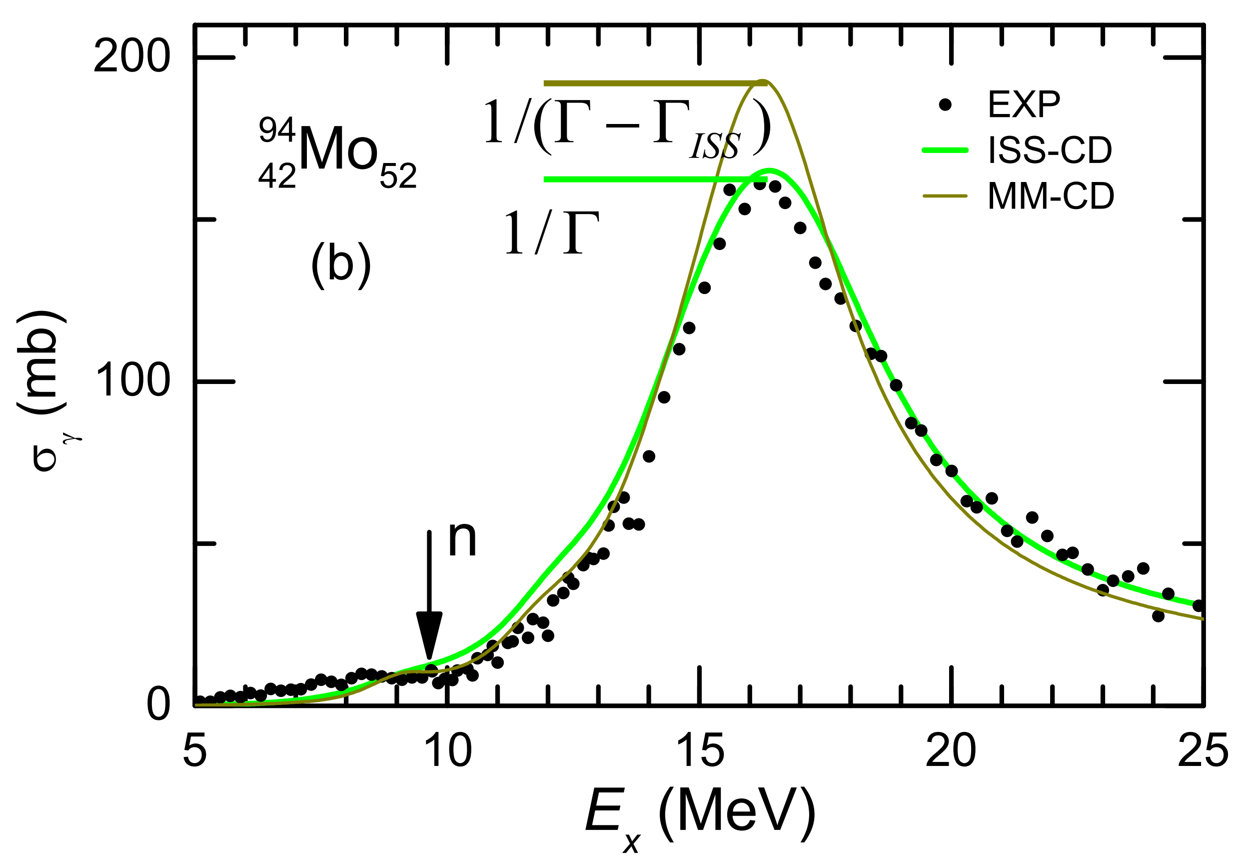


$^{94}_{42}Mo_{52}$
$^{96}_{42}Mo_{54}$
$^{98}_{42}Mo_{56}$
$^{100}_{42}Mo_{58}$
EXP
ISS
ISS-CD
MM
n
σ_γ (mb)
E_x-S_n (MeV)
$^{88}_{38}Sr_{50}$
$^{90}_{40}Zr_{50}$
$^{92}_{42}Mo_{50}$
p
E_x-(S_p+S_n)/2 (MeV)

Composition of the width near the GDR peak

If there are several mechanisms, each generating a Lorentz spread, the result is again a Lorentzian with the total width being the sum of the individual widths.

N	Γ_{LD+ISS}	Γ_{CD}	$\Gamma_{TOTAL}[MeV]$
50			
52	1.7	4.0	5.7
54	2.7	3.6	6.3
56	2.5	3.5	6.0
58	4.0	3.9	7.9



$$\Gamma_{LD} = 1.1 MeV$$
$$\Gamma_{ISS} = 0.6 MeV$$
$$\Gamma_{CD} = 4.0 MeV$$
$$\Gamma = 5.7 MeV$$

Dipole Strength near the threshold

- Landau Damping + Dynamic Deformation can explain the strength
- These doorways are mixed with more complex states-> smoothing (collsional damping)
- Some properties of the individual ph-excitations survive (isospin dependence)
- About 7% M1 (need to be compared with experiment)
- Indication for “pygmy collectivity” so far???

Ongoing

- Other nuclei to test the ISS
- More sophsticated RPA (Skyrme, Kvasil)
- Properties of the ph-states in the low-energy range
- Inclusion of two-phonon states
- More predictive description of quadrupole dynamics (IBA+PES, GCM)

More stringent condition for validity of ISS

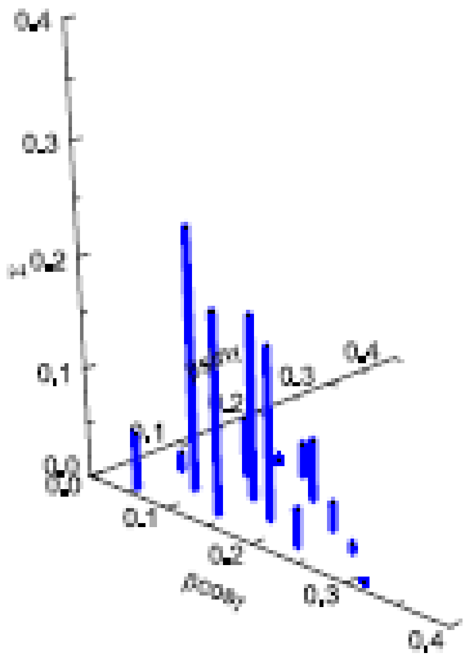
$$\frac{d\omega_1}{d\beta}\beta_0 > \omega_2$$

$$10MeV \times 0.2 > 0.6MeV$$

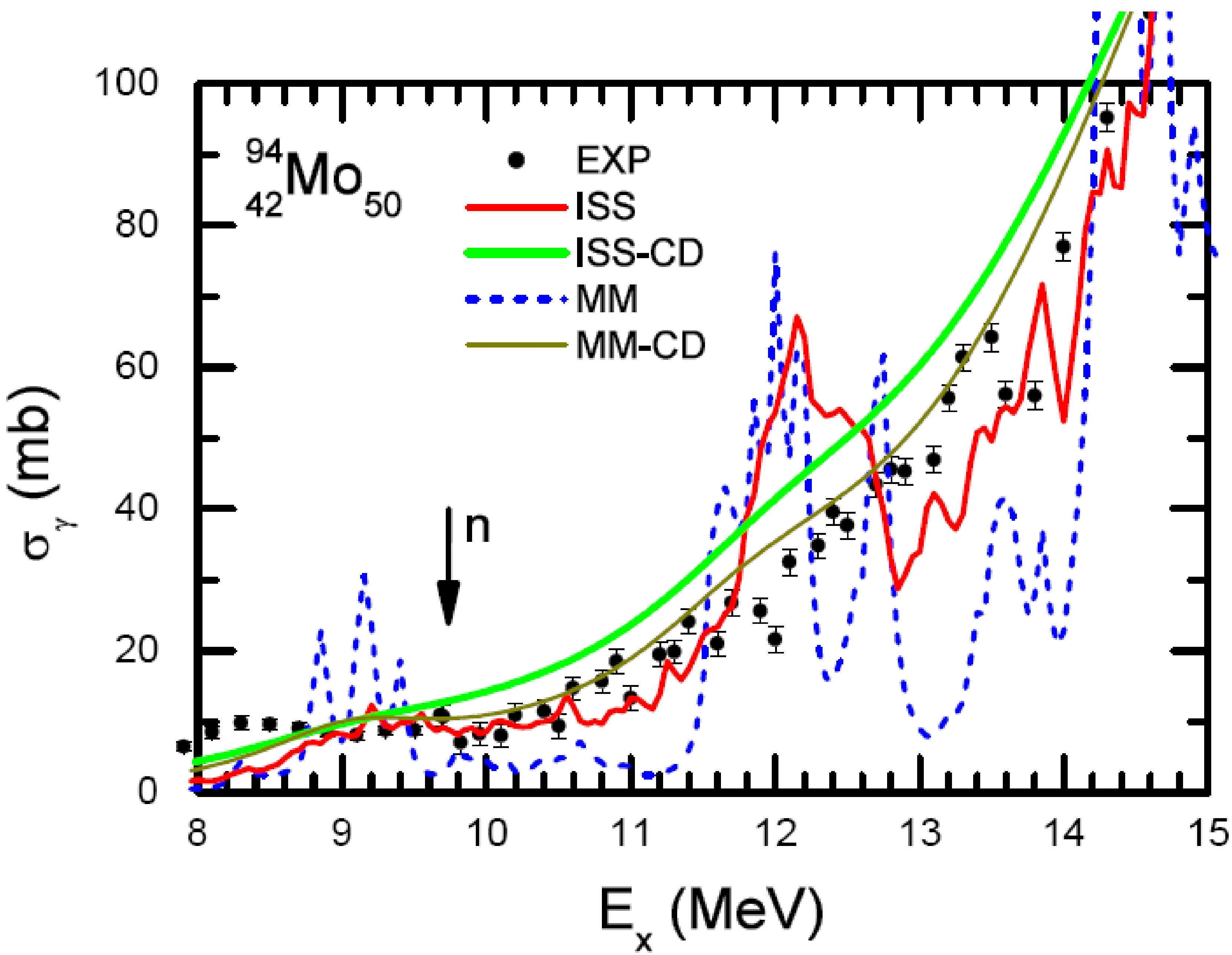
OK

Scatter around 10 MeV incoherently

better

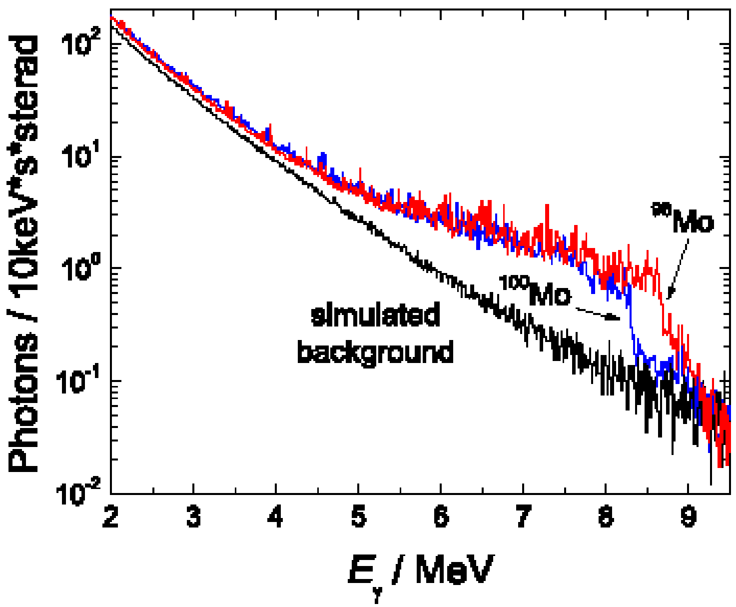


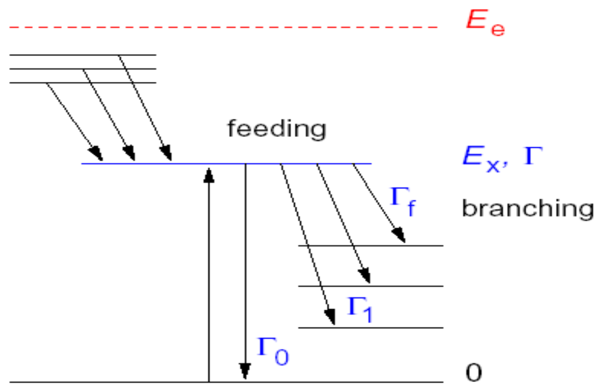
Displacement of the dipole states by the deformation fluctuations must be larger than the energy of the collective quadruple excitation.



$^{94}_{42}Mo_{50}$
EXP
ISS
ISS-CD
MM
MM-CD
n
100
80
60
40
20
0
8
9
10
11
12
13
14
15
σ_γ (mb)
E_x (MeV)

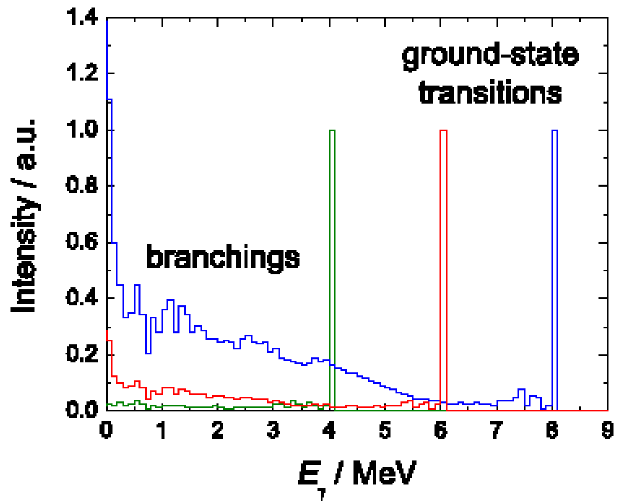
Problem of photo excitation by Bremsstrahlung



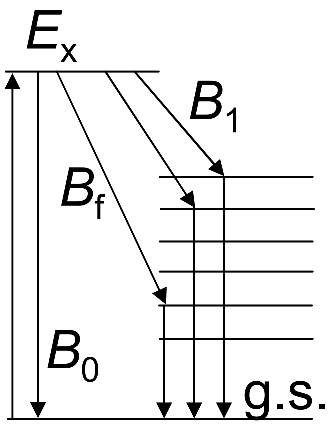


Correction of feeding and branching

The scatter spectrum measured by Bremsstrahlung contains all deexcitations , i.e. ground state transitions together with feeding and branching transitions.
Cascade simulation for feeding and branching need to be performed.



Monte Carlo code for simulations of γ cascades



$$B_f = \frac{I_f}{\sum_{i=0}^{N} I_i} = \frac{\Gamma_f}{\Gamma}$$

- Level scheme for J=0,1,2 using level densities from the systematics given in Phys. Rev. C 72, 044311 (2005) and Wigner distribution for the nearest-neighbour spacings.

1. Creation of a nuclear realisation:

- Partial decay widths using strength functions from RIPL2 systematics and Porter-Thomas distribution of the partial widths.

2. Monte Carlo method:

- Excitation of a level with J=1 according to σ_γ.
- Deexcitation according to B_f.

same strategy as „DICEBOX“ code:
F. Bečvář, Nucl. Instr. and Meth. A 417, 434 (1998)

Low-lying dipole strength-Theory

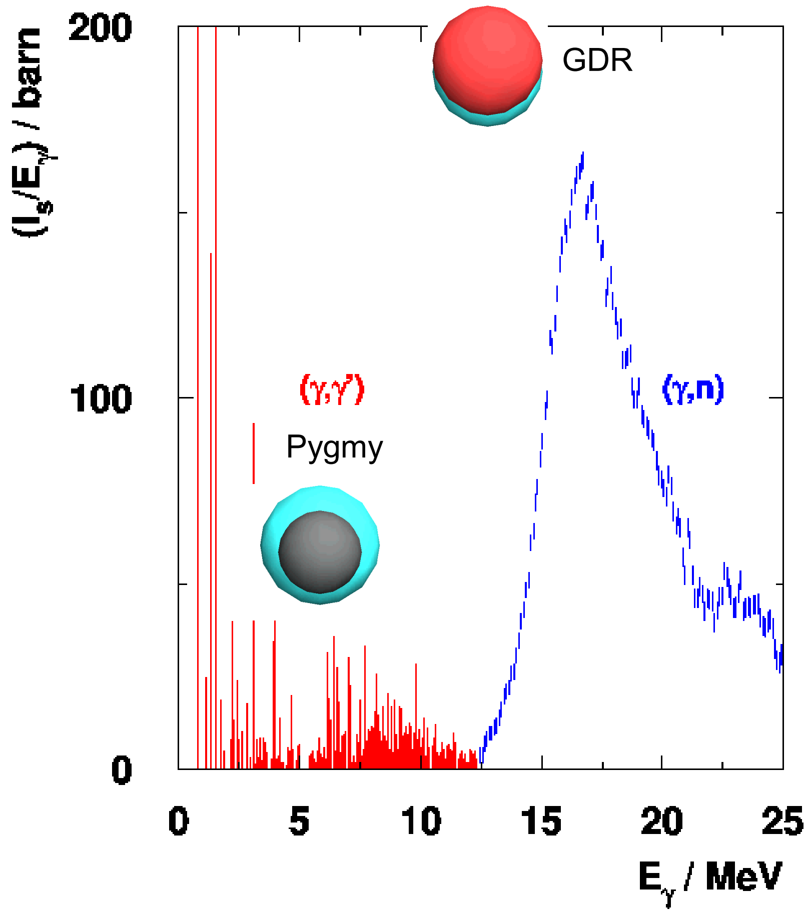
Pygmy resonance:

Possible existence a low lying “extra strength” - few percents of the TRK sum - due to a soft oscillation of a core against a neutron skin.

Pygmy Effect ~ N-Z (neutron excess).

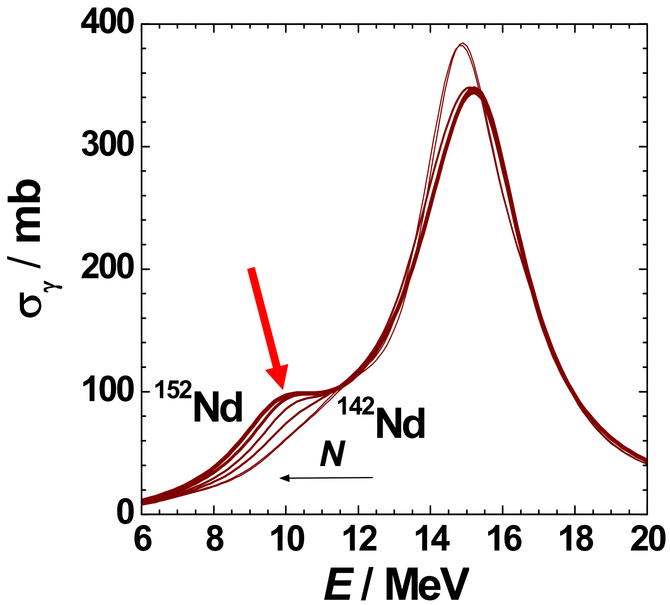
Our point to be investigated:
Nuclear deformation generates extra E1 strength in the tail region

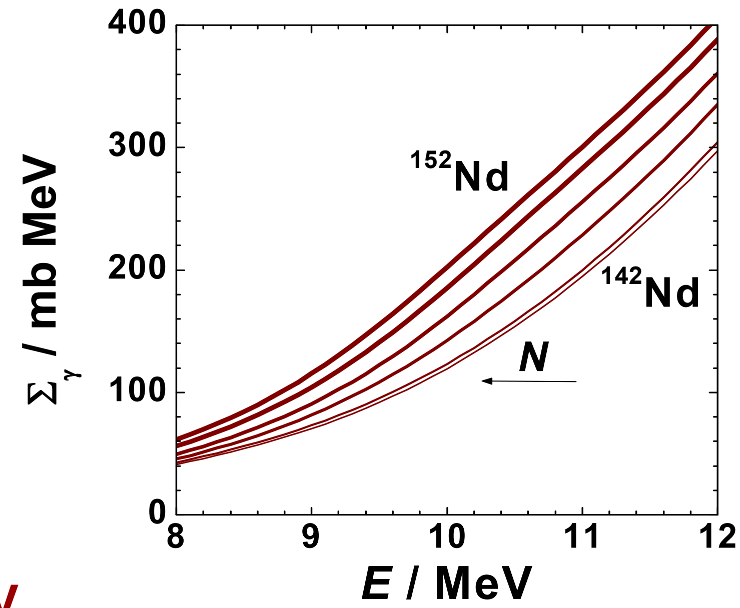
This is a shell effect
i.e. NOT directly correlated to N-Z !



Nd series A=142-152

Nilsson + RPA

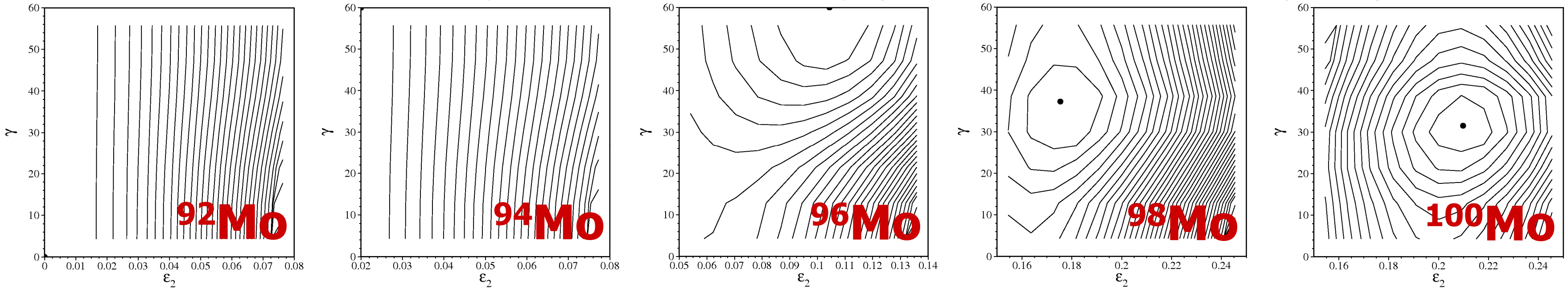




Lorentzian widths **$\Gamma_{K=\pm1}$= $\Gamma_{K=0}$ = 2.5 MeV**.
Reduced Γ due to splitting of Nilsson levels
Shape parameters ε from Moeller&Nix
Deformation increase ε=0 (A=142) to ε=0.255 (A=152)
→ increasing E1 strength in the tail region.

Triaxial shapes: Mo series A=92-104

Shapes: calculated by Nilsson-Strutinsky (Rossendorf TAC program)



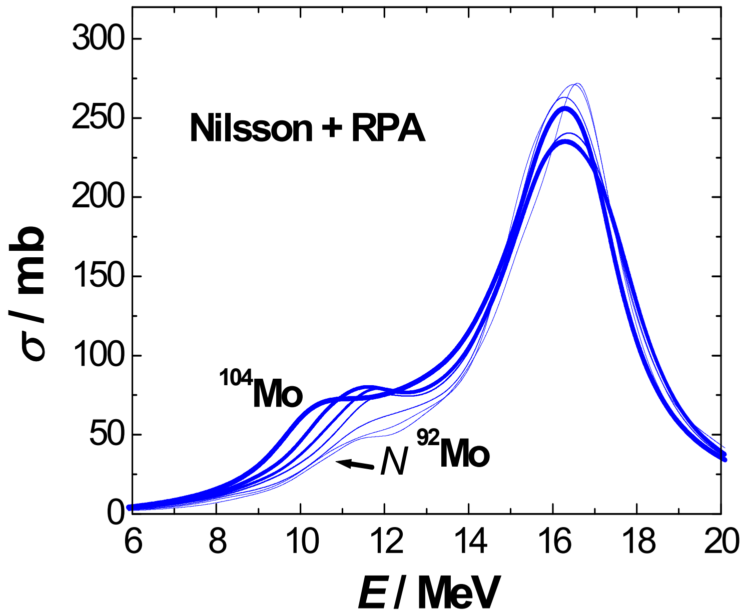
A	92	94	96	98	100	102	104	
ε	0.	0.02	0.10	0.18	0.21	0.24	0.25	TAC
γ /deg	-	60	60	37	32	25	16	
ε	0.03	0.05	0.18	0.20	0.22	0.24	0.25	FRLD
γ /deg	-	-	32	28	25	25	16	

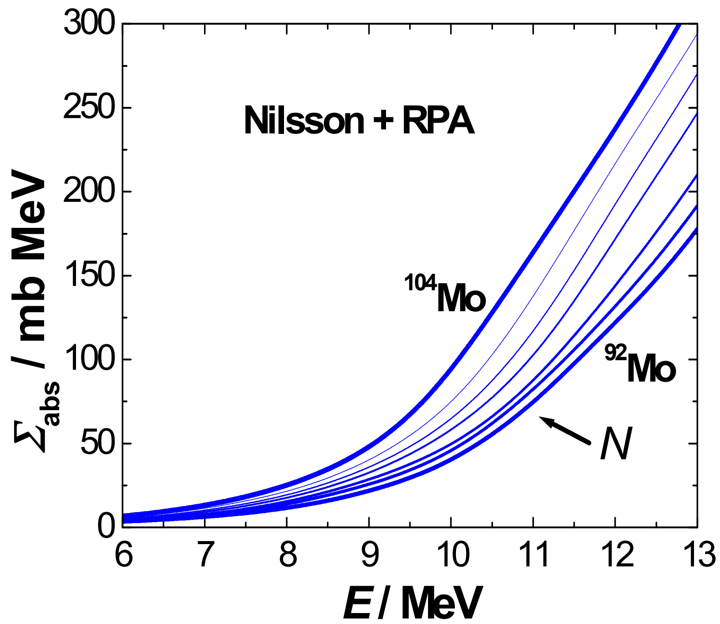
Experimental information for A=92-100 from

a) RIPL2 data for (γ,x) cross section for E above S_x and
b) (γ,γ´) data for E below S_x from experiments with Bremsstrahlung at e-linac ELBE at Dresden-Rossendorf

92-104 Mo with Nilsson + RPA

Deformation effect:
Increase of E1 strength at lower-energies

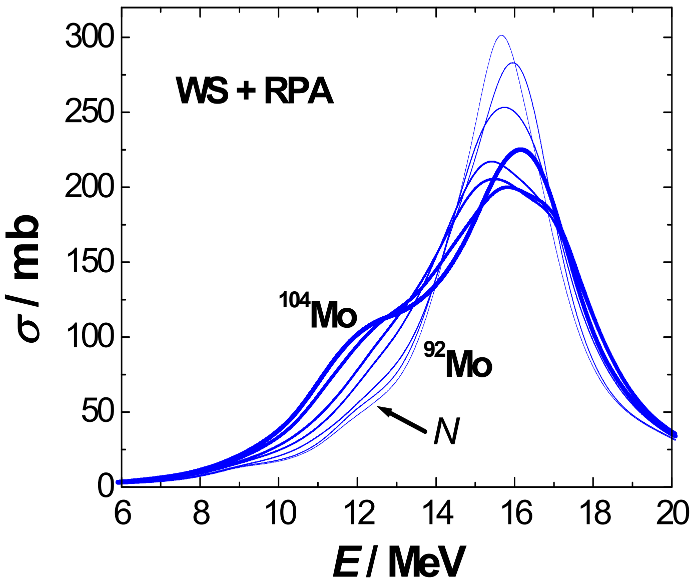


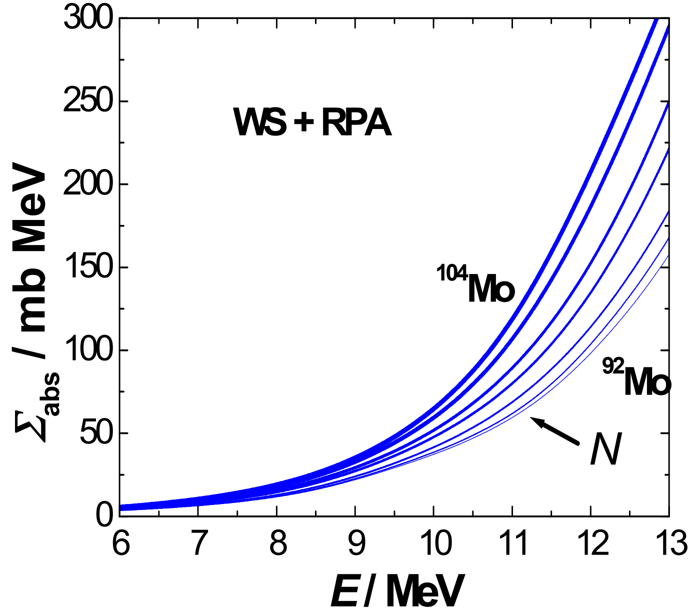


A	92	94	96	98	100	102	104
ε	0.	0.02	0.10	0.18	0.21	0.24	0.25
γ /deg	-	60	60	37	32	25	16

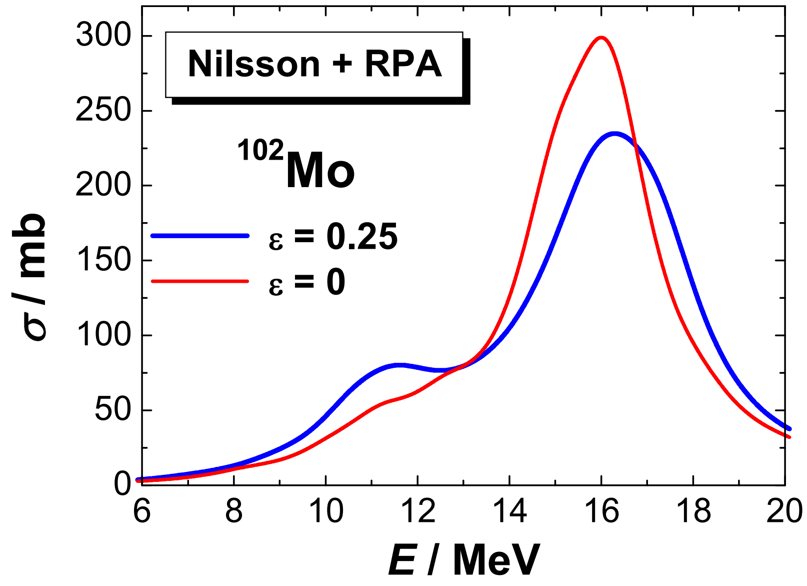
92-104 Mo with Woods-Saxon + RPA

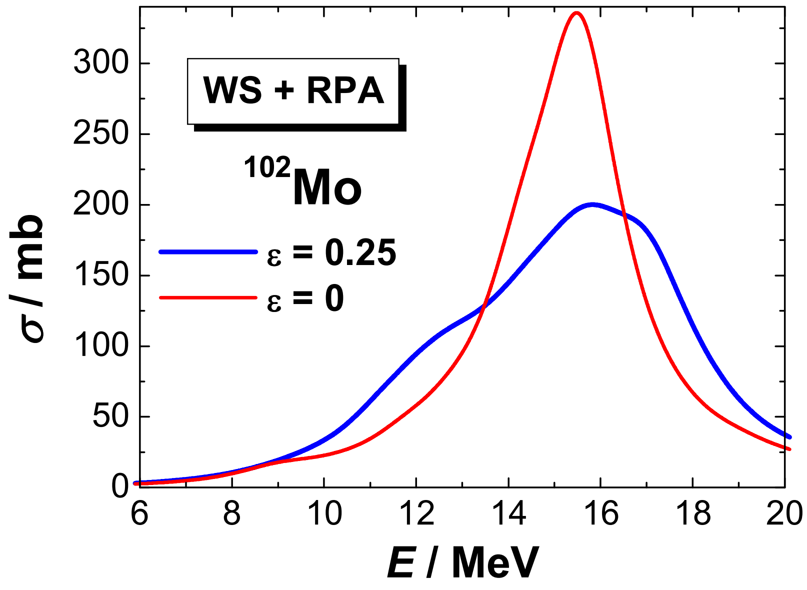
WS+RPA shows qualitatively the same increase of E1 strength at lower-energies with deformation





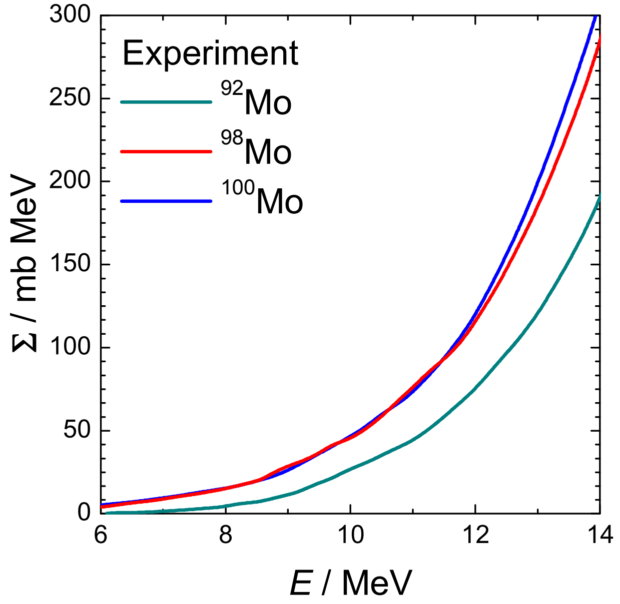
102 Mo spherical versus deformed shape

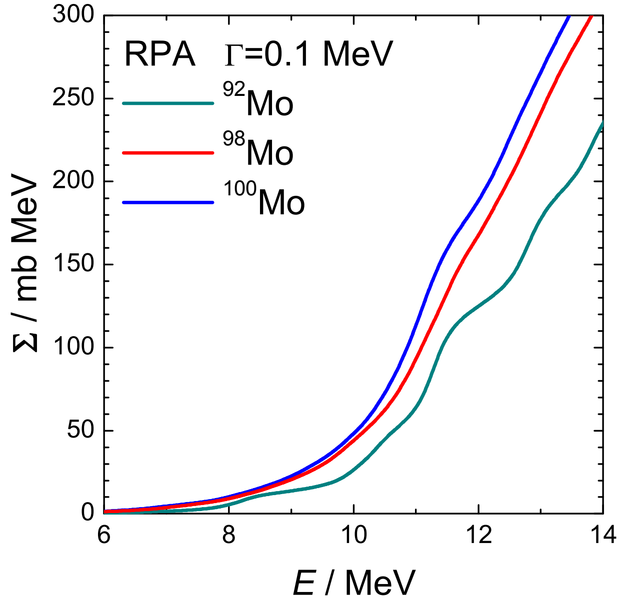




Compared to Nilsson the Woods-Saxon yields a „smoother“ and broader distribution

Comparison: Experiment versus Nilsson+RPA

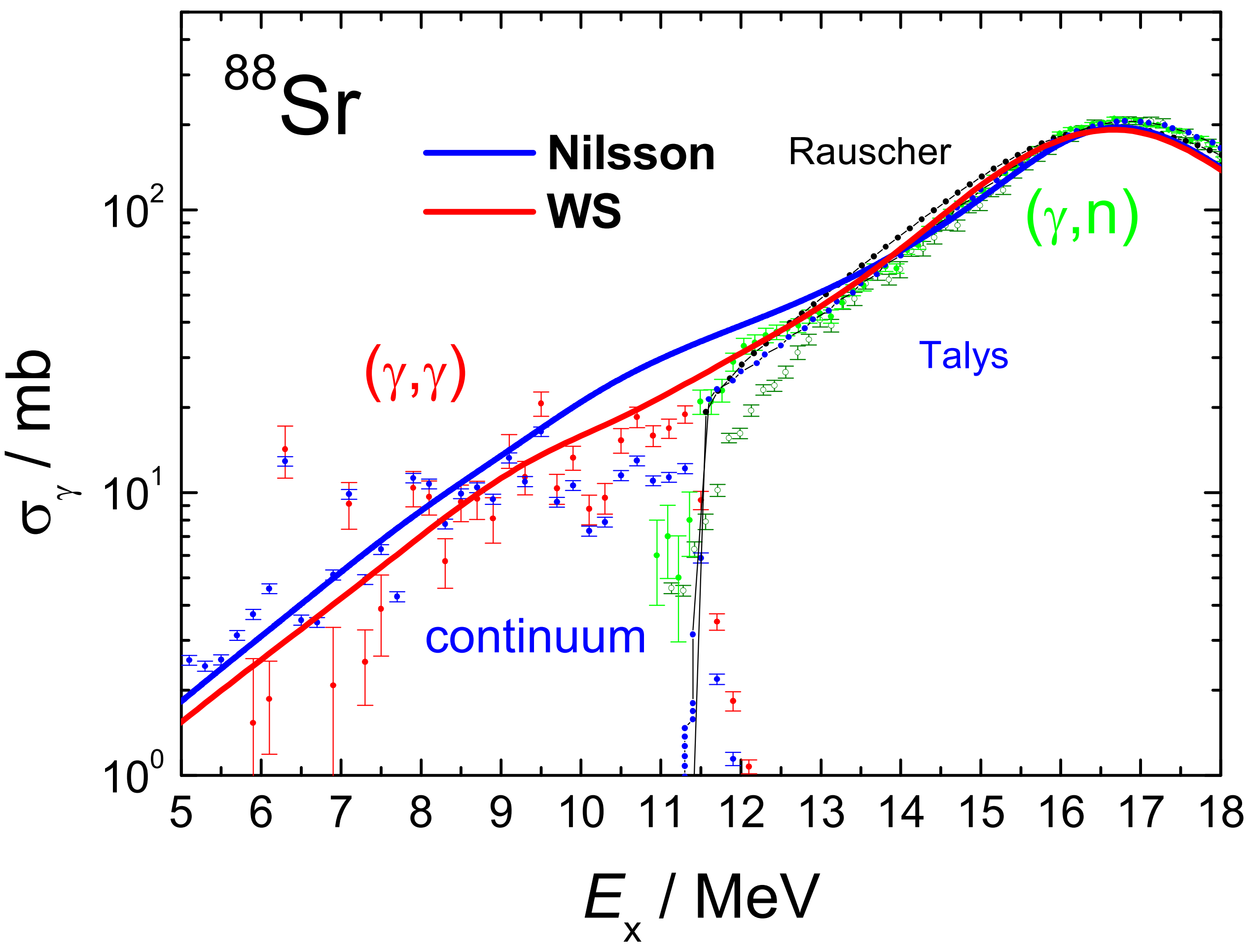




$$\Sigma_{\exp}(E) = \sum_{i>4\,MeV}^{E} \left(\sigma_{\gamma} \cdot \Delta E\right)$$

The dipole strength increases with the deformation.

The spherical 88Sr: Nilsson vs. Woods-Saxon



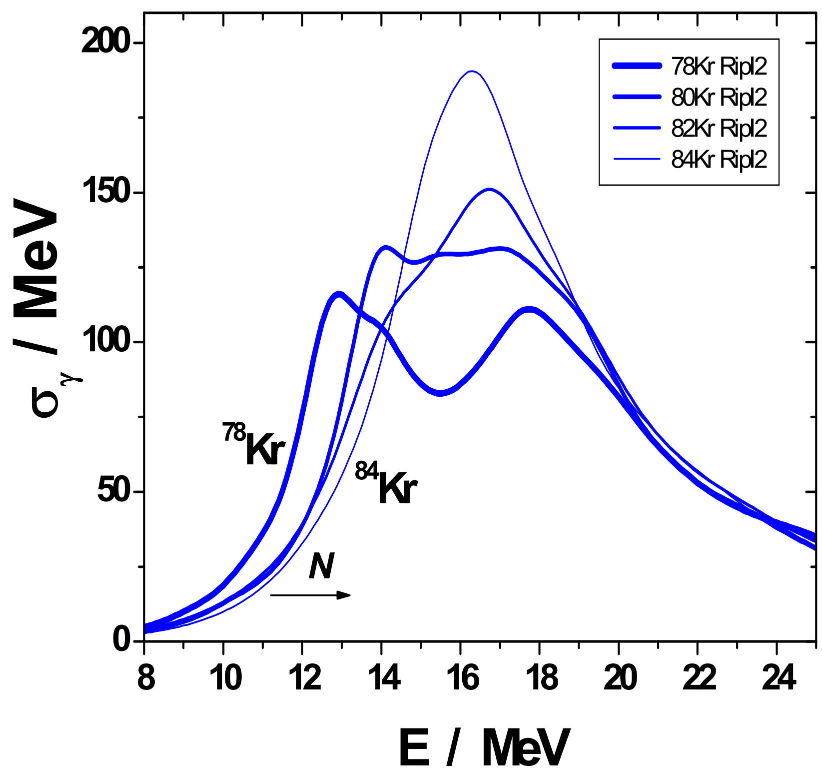
Kr series A=78-84

Deformation decrease from A=78 to 86 due to N=50 shell closure
ε= - 0.225 (oblate) in ^{78}Kr
ε= 0. (spherical) in ^{86}Kr

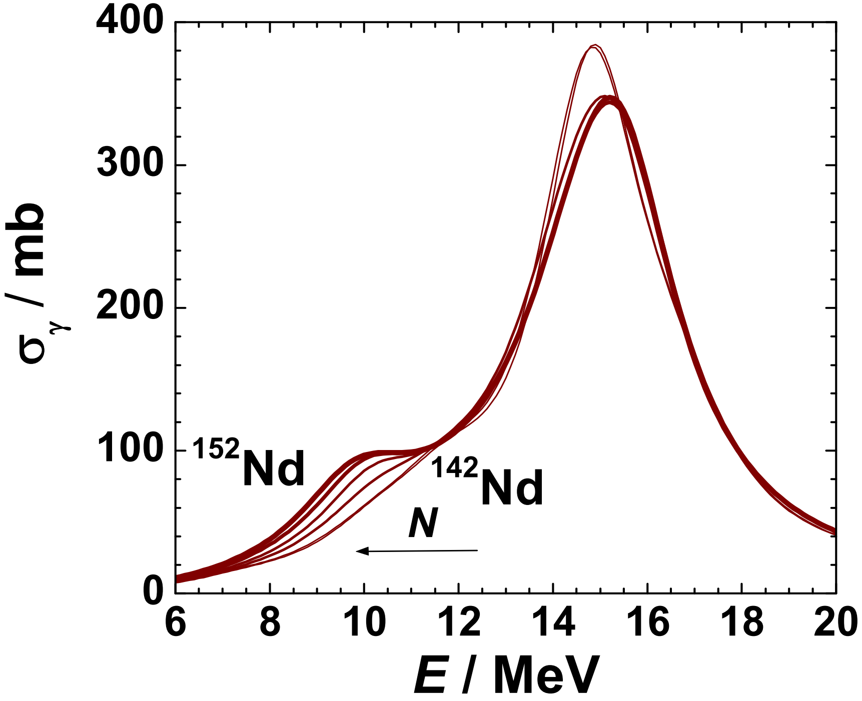
Prediction:
low energy E1 strength is supposed to decrease with N-Z
According to Pygmy interpretation the E1 strength should increase

Experiment ?

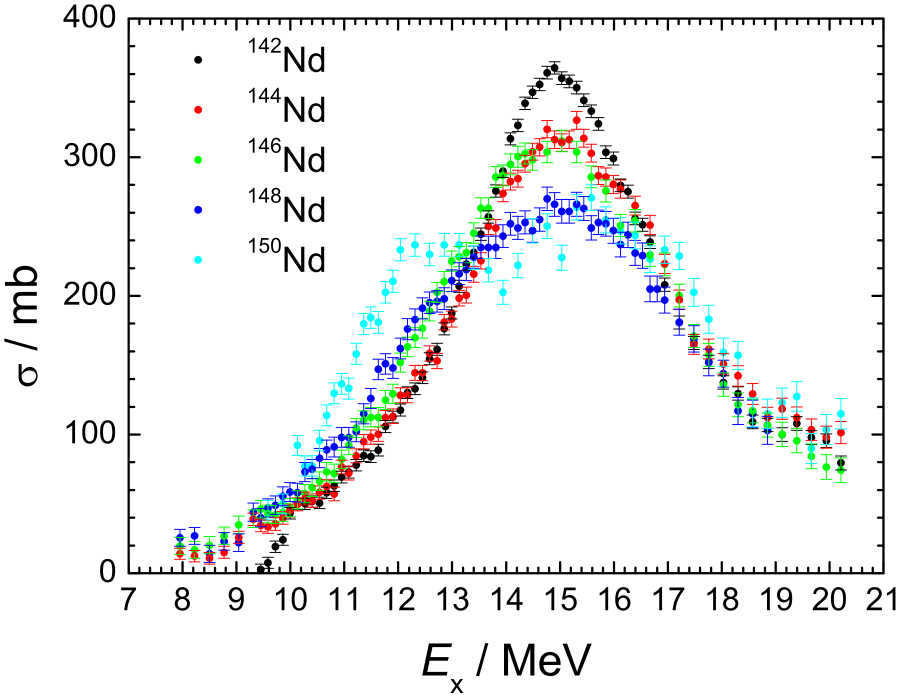
Shell effects can counteract the effect of the neutron excess



Nd: RPA-exp



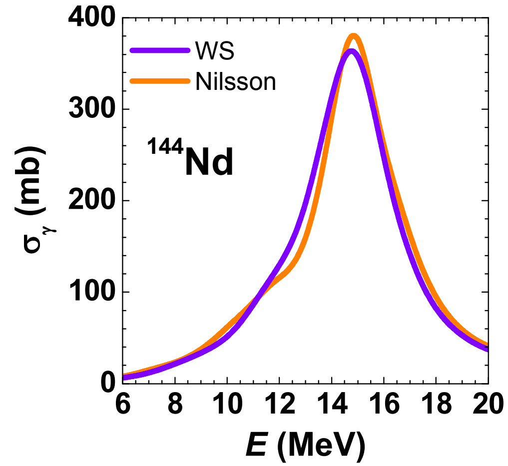
σγ / mb
E / MeV
152Nd
142Nd
N

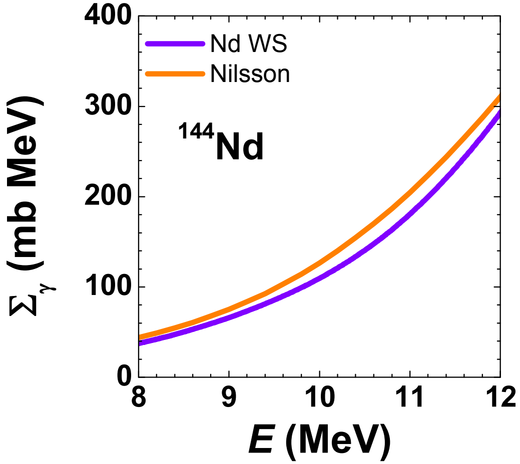


σ / mb
Ex / MeV
142Nd
144Nd
146Nd
148Nd
150Nd

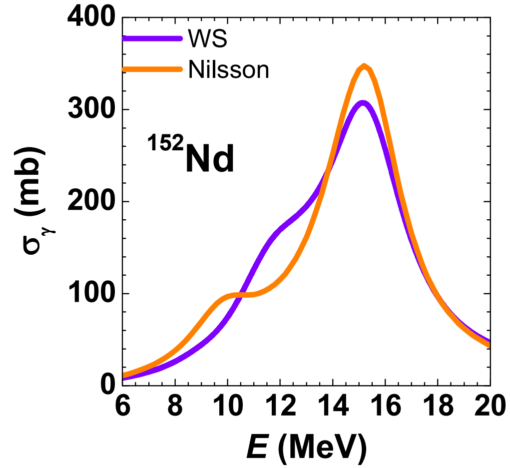
Nilsson vs. Woods-Saxon

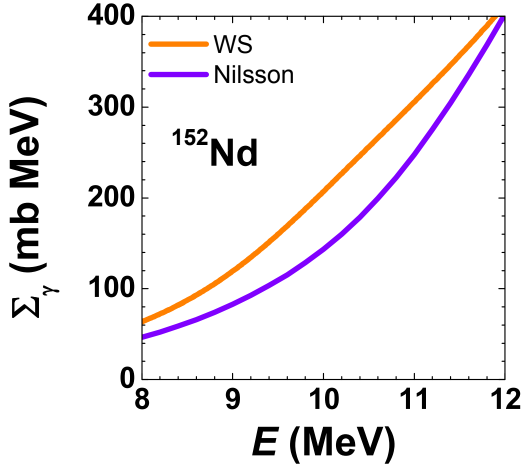
spherical

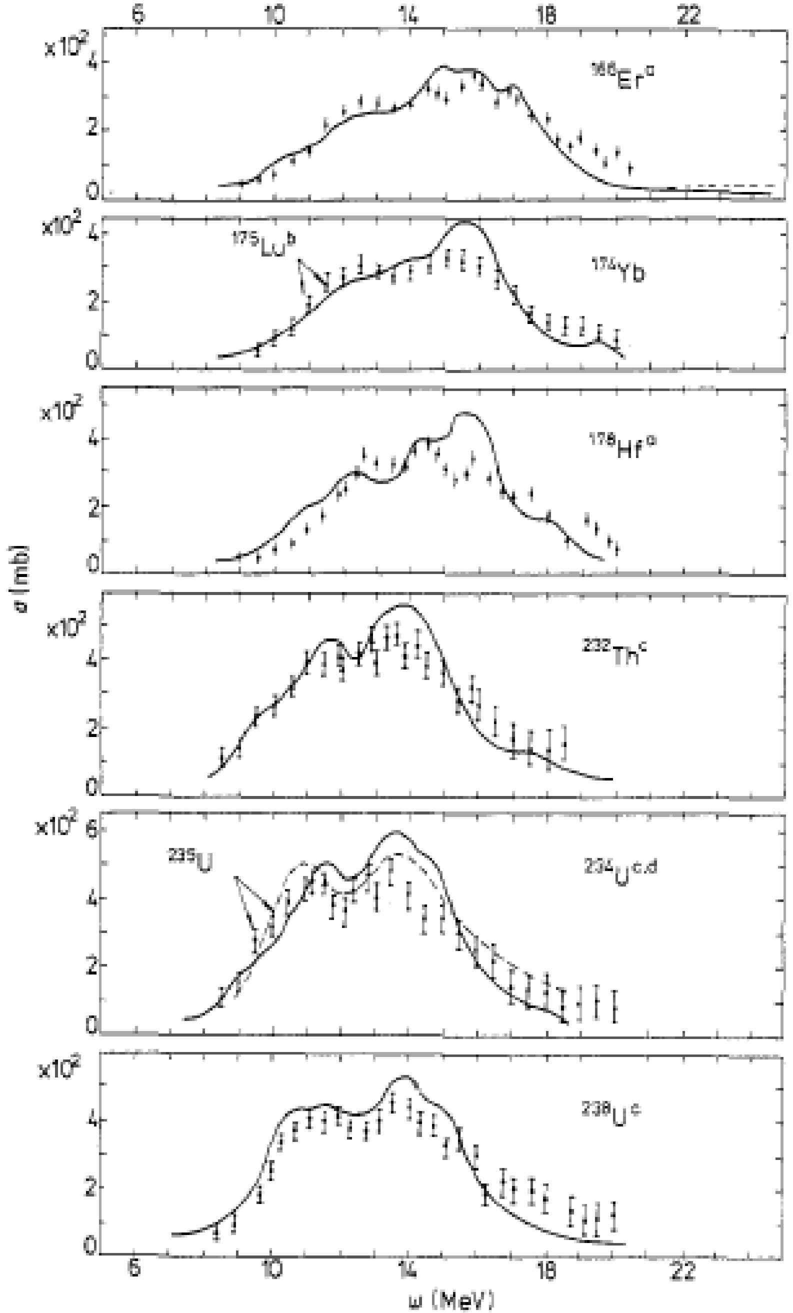




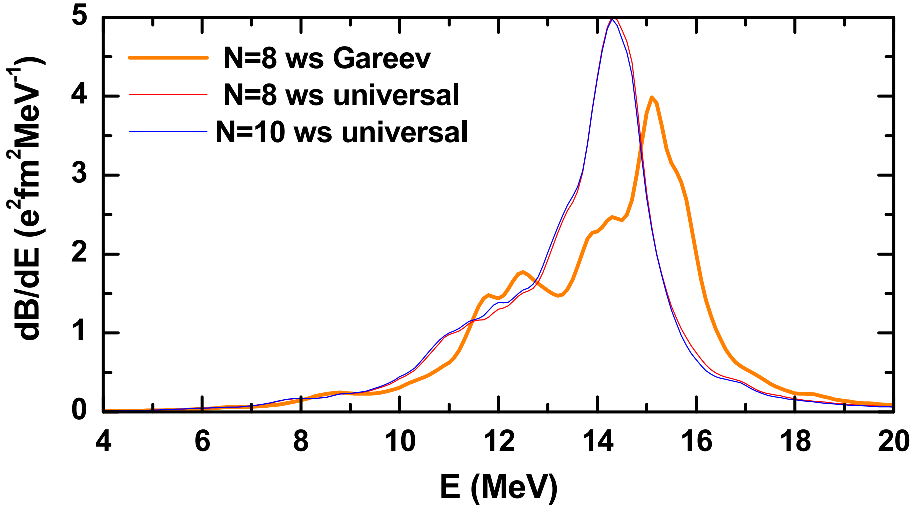
deformed







166Er Γ = 0.6 MeV ws(vesko) + rpa



Akulinichev, Malov, J.Phys. G. 3, 625 (1977)

Future developments

- Find a good deformed potential
- Dynamic deformation
- More realistic residual interaction

Dynamic deformation

The collective quadrupole modes are slow compared with the dipole mode in the region of interest.

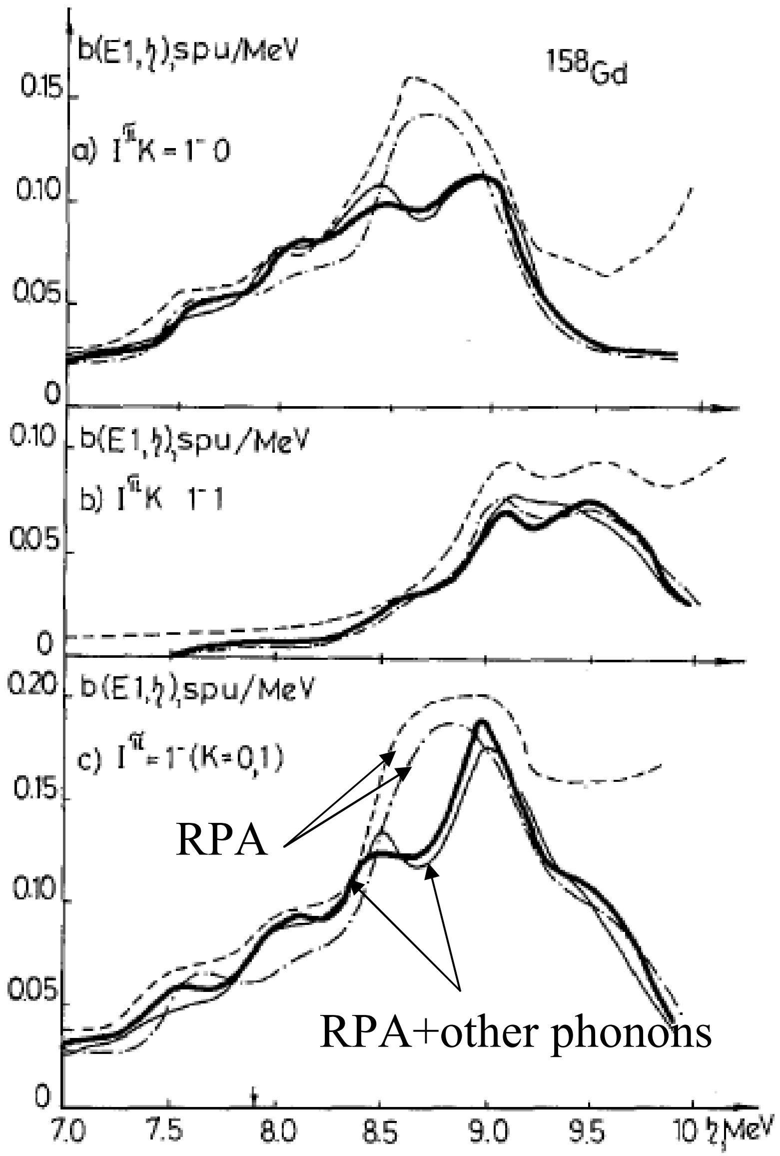
$$\hbar\omega_2 \sim 0.5MeV << \hbar\omega_1 \sim 8MeV$$

Adiabatic approximation:

1) Find dipole response for given ♌📪♑
2) Average over probability P(♌📪♑✆ in ground state

P(♌📪♑):

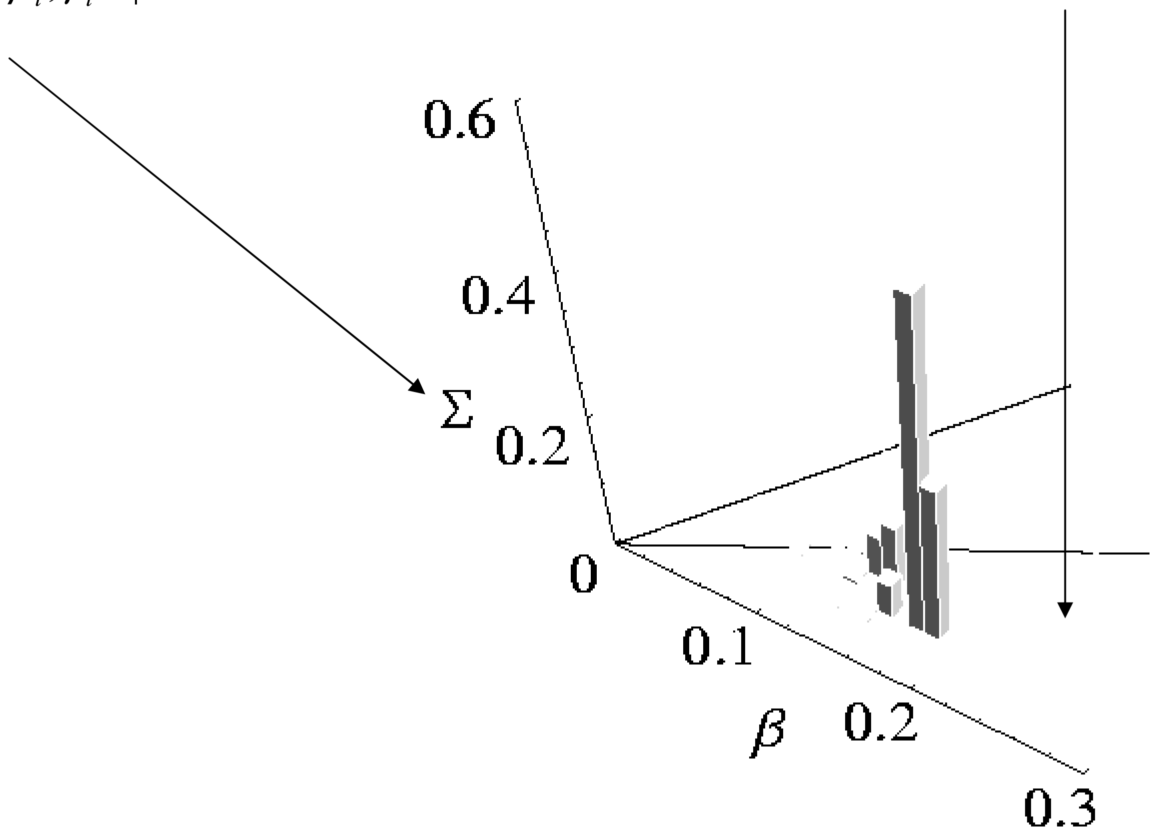
1) IBA systematics
2) Micro approach



Malov, Meliev, Soloviev,
Z. Phys. A320, 521 (1985)

Generating P(♌📪♑) from IBA:

1) Find the IBA parameters
2) Diagonalize H -> |gs>
3) Diagonalize $[Q \times Q]_0 \propto \beta^2, \quad [Q \times Q \times Q]_0 \propto \beta^3 \cos 3\gamma$ -> set of $|\beta_i, \gamma_i>$
4) $P(\beta_i, \gamma_i) = |< gs | \beta_i, \gamma_i >|^2$



Improvements of the RPA planned

Simple dipole-Dipole interaction replaced by Migdal´s interaction

$$F = C_0[f + f'\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + (g + g'\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2]\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2),$$

Treatment: Expansion of the delta-interaction in separable form

$$\delta(x - x') = \sum_{n=0,1,2,\ldots} H_n\left(\frac{x}{\Delta x}\right) e^{-\left(\frac{x}{\Delta x}\right)^2} H_n\left(\frac{x'}{\Delta x}\right) e^{-\left(\frac{x'}{\Delta x}\right)^2}$$

Generate strength function keeping few seperable terms by same method as for the dipole-dipole force.

Summary

Astrophysical network calculations need more precise photo cross sections. This is a challenge for both high resolution experiments and reliable theoretical predictions.

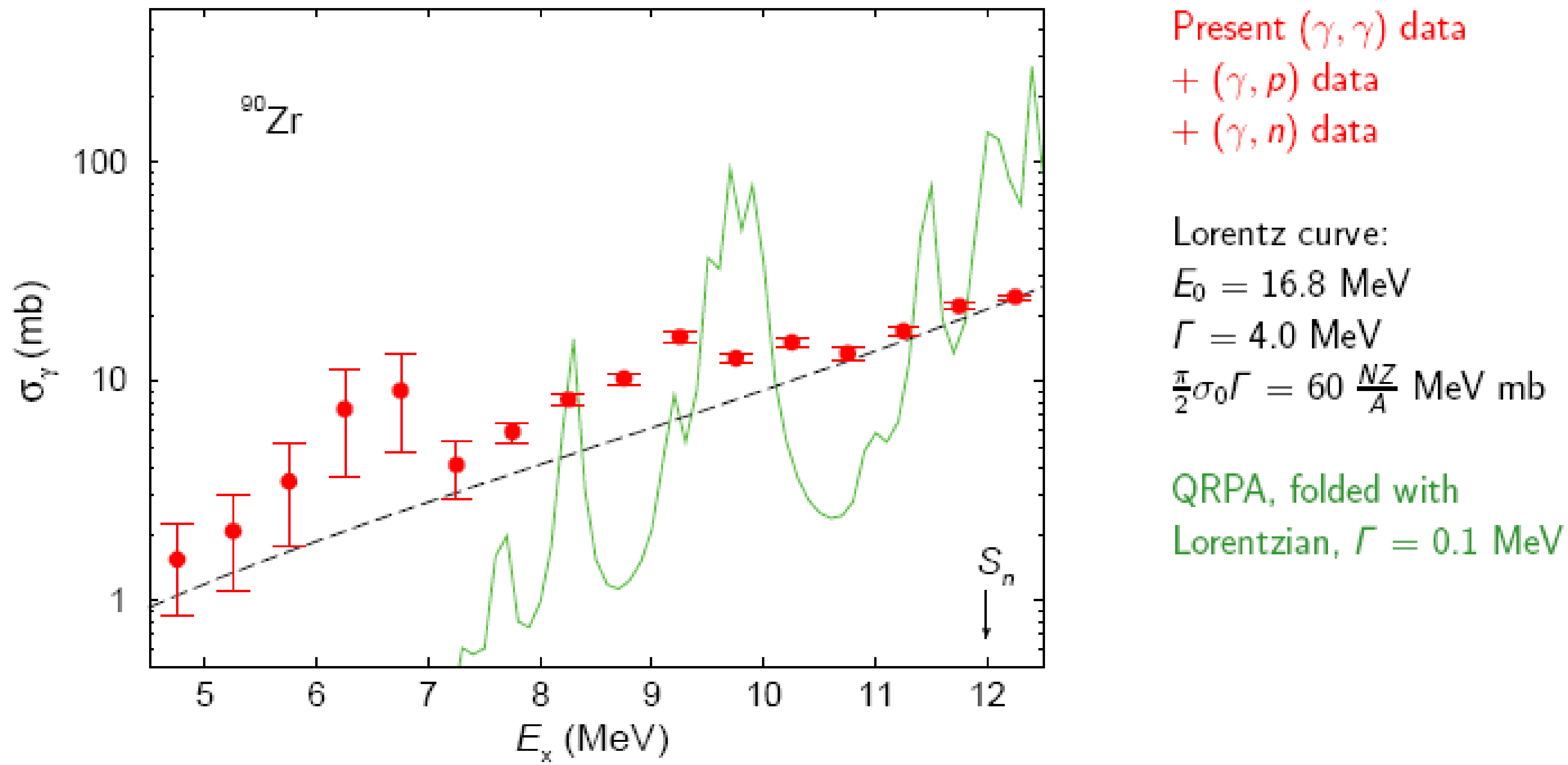
A new reconstruction analysis of the (γ,γ´) spectra provides for the first time photo cross sections which connect these data smoothly to the measured (γ,x) data.

The deformation is an important structure effect for producing low energy E1 strength. RPA with seperable interaction is a fast method for deformed nuclei, suitable for network calculations.

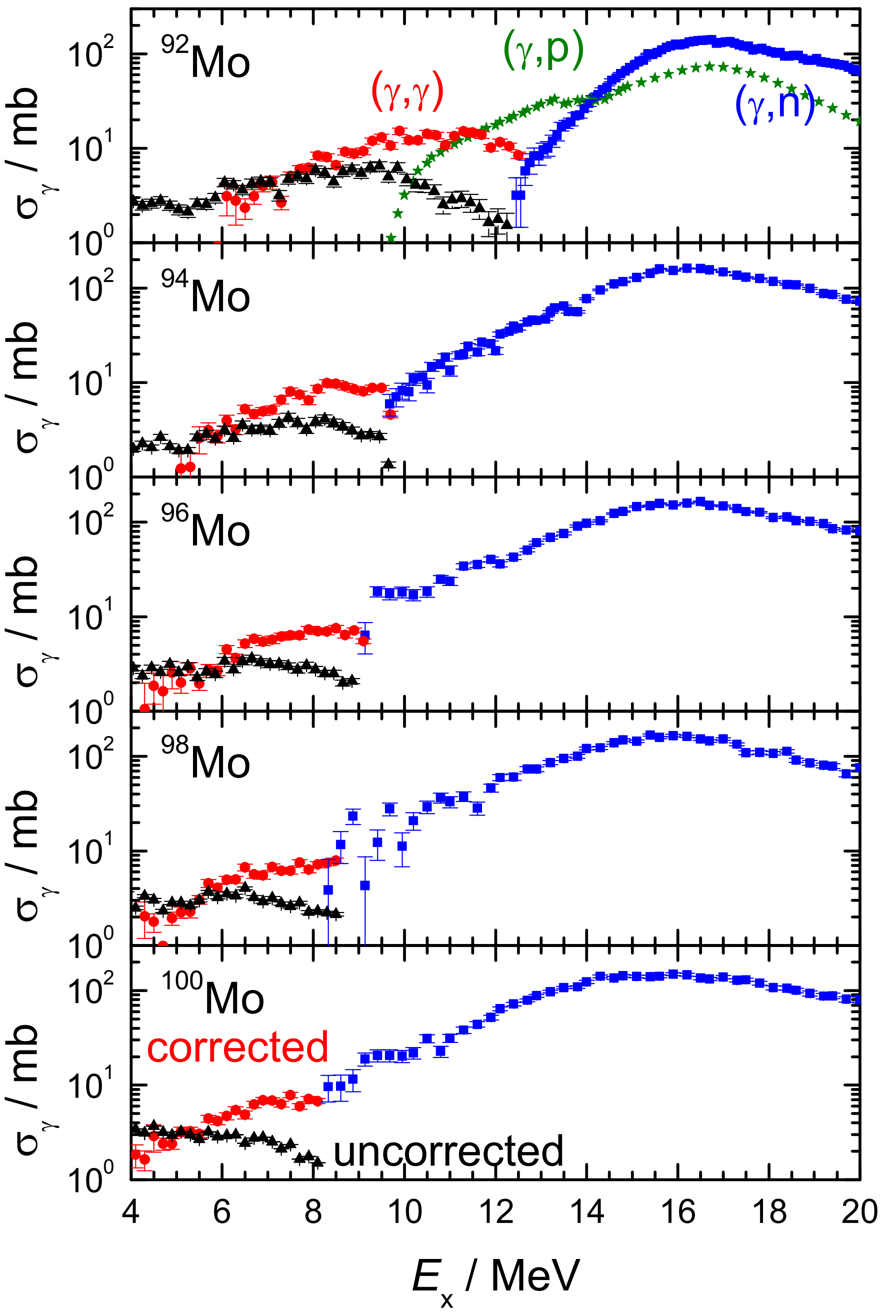
First application to mass 100 nuclides gives promising results for the for low-lying E1 strength.

Problems with the E1 strength function of well deformed rare earth nuclei

Extension to transitional nuclei by means of adiabatic approximaton



90Zr
100
10
1
σγ (mb)
5
6
7
8
9
10
11
12
Ex (MeV)
Sn
Present (γ, γ) data
+ (γ, p) data
+ (γ, n) data
Lorentz curve:
$E_0 = 16.8$ MeV
$\Gamma = 4.0$ MeV
$\frac{\pi}{2}\sigma_0\Gamma = 60 \frac{NZ}{A}$ MeV mb
QRPA, folded with
Lorentzian, $\Gamma = 0.1$ MeV



^{92}Mo
(γ,γ)
(γ,p)
(γ,n)
^{94}Mo
^{96}Mo
^{98}Mo
^{100}Mo
corrected
uncorrected
σ_γ / mb
E_x / MeV

Details of nucleosynthesis