# Semi-microscopic level densities

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# RECOMMENDED INPUT PARAMETER LIBRARY

Electronic Starter File (known as Reference Input Parameter Library-1) was developed and made available to users throughout the world in 1997

1994-1997: RIPL-1 starter file: http://www-nds.iaea.org/ripl/ ( RIPL-1 )

Second CRP was initiated on *"Nuclear Model Parameter Testing for Nuclear Data Evaluation (Reference Input Parameter Library: Phase II)"*, and completed in 2002. Revision, extension and validation of the original RIPL-1 Starter File to produce a consistent RIPL-2 library of recommended input parameters.

1998-2002: RIPL-2 database: http://www-nds.iaea.org/RIPL-2/ ( RIPL-2 )

Third CRP started in 2003: *"Nuclear model parameters for energy and non-energy applications (RIPL – Phase III)"*. Upgrade and extension of the RIPL database was undertaken. **RELEASED, January 2009.**

2003-2008: RIPL-3 database: http://www-nds.iaea.org/RIPL-3/ ( RIPL-3 )



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# Level Densities: Talk Outline

## Formalism

- Energy-conserving (micro-canonical) formalism of level densities of non-magic nuclei

## Some applications

- Level densities at low excitation energies ( $E < Bn$ ) measured by the Oslo method

  *A. Schiller et al., Nucl. Instrum. Meth. A **447** (2000) 498*

- Neutron resonance spacings ($E \sim Bn$ ) measured by the n_TOF collaboration ( CERN )

  *F. Gunsing et al., Nucl. Instrum. Meth. B **261** (2007) 925* )

## Results

- Sm chain and U-234 preliminary results



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# Level Densities: Formalism

- $\omega(E,M,\pi)$: density of states of angular momentum projection $M$ and parity $\pi$ at excitation energy $E$.

- $\rho(E,J,\pi)$ : density of levels of angular momentum $J$ and parity $\pi$ at excitation energy $E$.

- In the angular-momentum conserving (spherical) formalism used in this work :

  $\rho(E,J,\pi) = \omega(E, M = J,\pi) - \omega(E, M = J+1,\pi)$ if spherical

  $\rho(E,J,\pi) \approx \omega(E, M = J,\pi) - \omega(E, M = J+1,\pi)$ if deformed

- $\rho(E)$ : total level density at excitation energy $E$

$$\rho(E) = \sum_{J,M} \rho(E,J,\pi)$$



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# Level Densities: Assumptions

Basic assumption for even-even nuclei :

- Adiabatic decoupling of collective (boson) and non-collective (fermion) degrees of freedom makes it possible to reproduce the experimental discrete spectrum at low energy and low spin by means of a collective model (**I**nteracting **B**oson **M**odel) and implies

$$\omega(E, M, \pi)$$

$$= \sum_{\pi_i \cdot \pi_c = \pi} \sum_{c=0}^{N_c(\pi_c)} \sum_{i=0}^{N_i(\pi_i)} \sum_{M_i + M_c = M} \delta((E - E_c(M_c, \pi_c) - \varepsilon_i(M_i, \pi_i))$$



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# Level Densities: Intrinsic ⊗ IBM

- Here, index *c* means collective (boson), index *i* means intrinsic ( = non-collective, or fermion. $\omega(E,M,\pi)$ can be written as a convolution of a collective and an intrinsic state density:

$$\omega(E,M,\pi)$$

$$= \sum_{\pi_i\cdot\pi_c=\pi} \sum_{c=0}^{N_c(\pi_c)} \int_0^\infty dE_i \sum_{M_i+M_c=M} \omega_{\text{intr}}(E_i,M_i,\pi_i)\,\omega_{\text{coll}}(E-E_i,M_c,\pi_c) \quad ,$$

$$\omega_{\text{intr}}(E_i,M_i,\pi_i) = \sum_{j,M_j,\pi_j} \delta(E_i-\varepsilon_j(M_j,\pi_j))\,\delta_{\pi_i\pi_j}\,\delta_{M_iM_j} \,,$$

$$\omega_{\text{coll}}(E-E_i,M_c,\pi_c) = \sum_{l,M_l,\pi_l} \delta(E-E_i-E_l(M_l,\pi_l))\,\delta_{\pi_c\pi_l}\,\delta_{M_cM_l} \,.$$



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# Intrinsic state density

First step: state density of a system of non-interacting fermions computed by means of an exact recursive algorithm

*F. C. Williams Jr,* "An iterative method for the calculation of nuclear level densities",

*Nucl. Phys. A* ***133*** *(1969) 33-49*

as implemented in the TOTSTADE code

*E. Mainegra and R. Capote,* "Nuclear state density calculations: An exact recursive approach"

*Comp. Phys. Comm.* ***150*** *(2003) 43-52*



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# Intrinsic state density: Recursivity

From the generating function of the state density $\omega$ at excitation energy $E$ for a system of $n$ independent fermions distributed among $m$ single-particle levels $\{\varepsilon_m\}$ one easily obtains a recursive relation for $\omega(n,E)$ :

$$Z_m(x,y) = \prod_{i=1}^{m}\left(1 + xy^{\varepsilon_i}\right) = \sum_{n=o}^{m} \omega(n,E) x^n y^E$$

$$Z_m(x,y) = (1 + x\varepsilon_m) Z_{m-1}(x,y)$$

$$\omega_m(n,E) = \omega_{m-1}(n,E) + \omega_{m-1}(n-1, E + \varepsilon_n - \varepsilon_m)$$

The recursive formulae given above are applied to protons and neutrons separately.



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# Intrinsic Level Densities: MC (1)

Second step: the recursive state densities, $\omega_{rec}(n,E)$ , are then used to define scale factors and weight functions of Monte Carlo estimators of state densities of interacting fermions

*N. Cerf, "Combinatorial nuclear level density by a Monte Carlo method", Phys. Rev.* ***C 49*** *(1994) 852-866.*

As implemented in CAIN (E. Mainegra and R Capote , unpublished).

If the residual interaction is the pairing interaction, a configuration $C$ is a set of of BCS occupation numbers, $\{n_k(C)\}$ , of single-particle levels. A trial move from configuration $C_a$ to configuration $C_b$ is evaluated with the Metropolis algorithm, so as to generate a random walk with a limit distribution as required.



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# Intrinsic Level Densities: MC (2)

- The ratio of the weight functions used in the algorithm

$$W(C_b)/\,W(C_a) = \omega_{rec}(E_a)/\,\omega_{rec}(E_b)$$

corrects the sampling for the exponential increase of the state density (importance sampling).

- The cumulative number of states up to a given excitation energy $E$

$$N_{rec}(E_{max}) = \int^{Emax} \omega_{rec}(U)dU$$

determines the scale factor, $S(E)$ , of the Monte Carlo estimator of the BCS state density

$$S_{rec}(E_{max}) = N_{rec}(E_{max})/\Sigma_j\,(1/W(C_j),$$

where the sum runs over the steps of the random walk.


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# Intrinsic Level Densities: MC (3)

The resulting BCS and Fermi-gas state densities for one kind of nucleon and the convolution of neutron and proton state densities are

$$\omega_{\text{intr}}(E,M,\pi)=S_{rec}(E_{\max})\sum_{j=1}\frac{\delta(E-E_{C_j})\delta_{MM_j}\delta_{\pi\pi_j}}{W(E_{C_j})}$$

$$\omega^{\text{BCS}}_{\text{intr}}(E,M,\pi)=S_{rec}(E_{\max})\sum_{j=1}\frac{\delta(E-E_{C_j}+P_{C_j})\delta_{MM_j}\delta_{\pi\pi_j}}{W(E_{C_j})}$$

$$=\int_0^E dQ\sum_{\pi_N\cdot\pi_Z=\pi}\sum_{M_N+M_Z=M}\omega^N_{\text{intr}}(Q,M_N,\pi_N)\,\omega^Z_{\text{intr}}(E-Q,M_Z,\pi_Z)$$


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# Collective state density

General formulae

$$\omega_{\text{coll}}(E,M,\pi)=\sum_{J=0}^{J_{\max}(\pi)}(2J+1)\sum_{c}\delta(E-E_c^{J,\pi})f_{\text{coll}}(M,\pi),$$

$$\sum_{M=-J_{\max}(\pi)}^{+J_{\max}(\pi)}f_{\text{coll}}(M,\pi)=1,$$

$$\omega_{\text{coll}}(E,\pi)=\sum_{J=0}^{J_{\max}(\pi)}(2J+1)\sum_{c}\delta(E-E_c^{J,\pi})$$


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# Collective state density - IBM

- The collective energies, $E_c(J,\pi)$ , are eigenvalues of an Interacting Boson Model Hamiltonian, where excitations of positive parity are produced by the interaction of *s* ( $L^\pi = 0^+$ ) and *d* bosons ( $L^\pi = 2^+$ ), whose total number, $N_b$ , is equal to the number of collective particle pairs, or hole pairs, whichever the smaller, in valence shells (=> non-magic nuclei).

- Excitations of negative parity are produced by replacing one of the positive parity bosons with an *f* boson ($L^\pi = 3^-$)

- For each choice of $J^\pi$ the IBM Hamiltonian is fully diagonalised by means of a modified version of the OCTUPOLE code ( *D. Kusnezov, unpublished* ), with parameters adjusted on the experimental discrete spectra.


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# Interactive Boson Model

Multipole expansion of the *s-d-f* Hamiltonian

$$\hat{H}_{sdf} = \varepsilon_d \hat{n}_d + \varepsilon_f \hat{n}_f + \alpha \hat{P}_{sdf}^{+} \cdot \hat{P}_{sdf} + \beta \hat{Q}_{sdf} \cdot \hat{Q}_{sdf} + \gamma \hat{L}_{df} \cdot \hat{L}_{df}$$

$$+ \delta \hat{Q}_f \cdot \hat{Q}_{sd} + \zeta \hat{L}_d \cdot \hat{L}_f ,$$

$$\hat{P}_{sdf}^{+} = -s^+ \cdot s^+ + d^+ \cdot d^+ + f^+ \cdot f^+ ,$$

$$\hat{Q}_{sd} = \left[ s^+ \times \tilde{d} + d^+ \times \tilde{s} \right]^{(2)} - \frac{\sqrt{7}}{2} \left[ d^+ \times \tilde{d} \right]^{(2)} ,$$

$$\hat{Q}_f = -\frac{3\sqrt{42}}{10} \left[ f^+ \times \tilde{f} \right]^{(2)} , (\hat{Q}_{sdf} = \hat{Q}_{sd} + \hat{Q}_f )$$

$$\hat{L}_d = \sqrt{10} \left[ d^+ \times \tilde{d} \right]^{(1)} ,$$

$$\hat{L}_f = 2\sqrt{7} \left[ f^+ \times \tilde{f} \right]^{(1)} , (\hat{L}_{df} = \hat{L}_d + \hat{L}_f )$$


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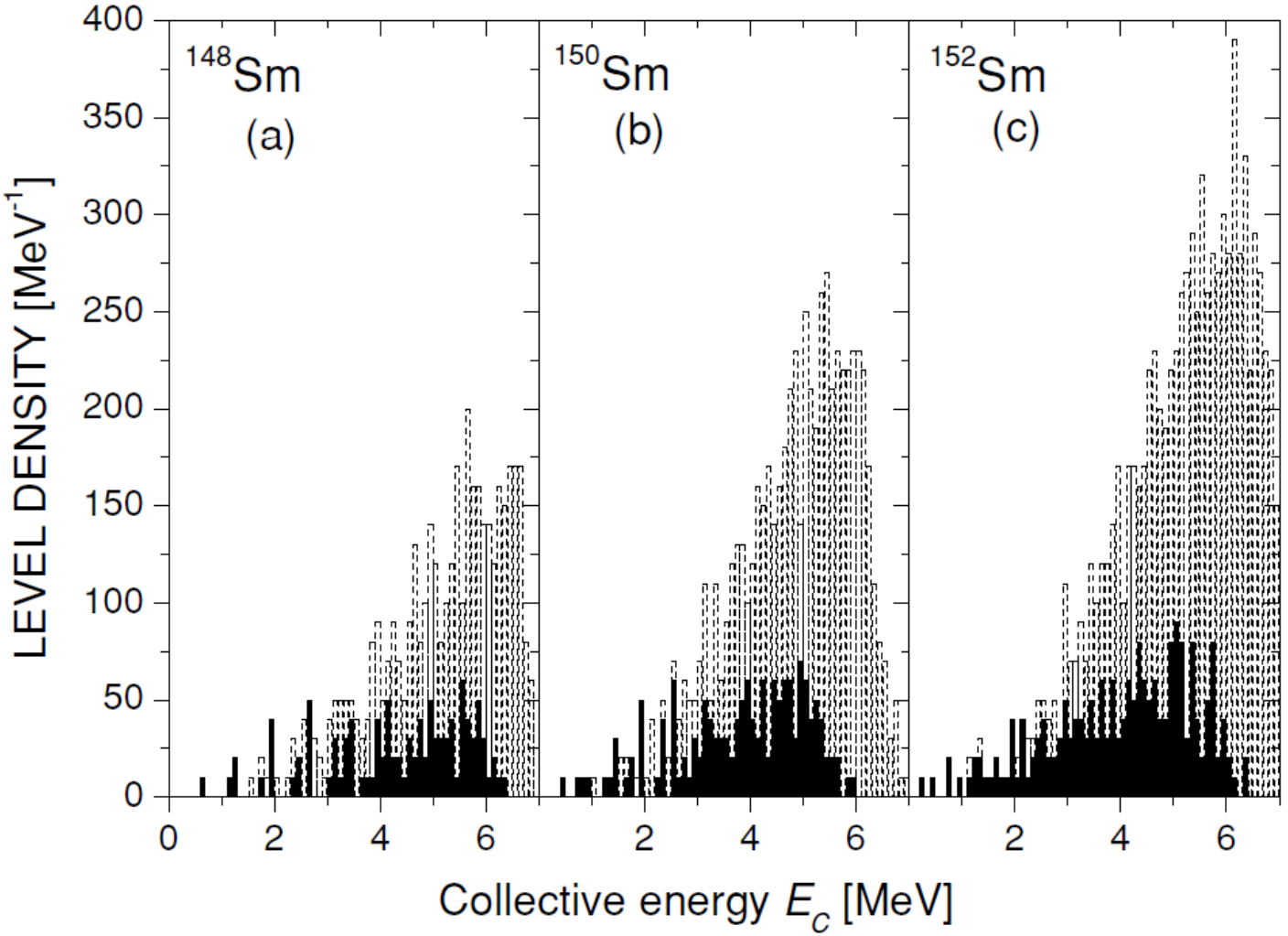
# IBM collective levels: Parity dependence





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# LEVEL DENSITIES – Sm nuclei

Level densities of transitional samarium nuclei

*R. Capote, A. Ventura , F. Cannata, J.M. Quesada*
*Phys. Rev. C **71** (2005) 064320*

Total level densities ($E \leq B_n$) and *s*-wave neutron resonance spacings for compound nuclei $^{148\text{-}149\text{-}150\text{-}152}Sm$ have been calculated and compared with available experimental data:

- total level densities of $^{148\text{-}149}Sm$ measured by the Oslo group

  *S. Siem et al., Phys. Rev. C **65** (2002) 044318*

- the *s*-wave neutron resonance spacing in $^{151}Sm(n,\gamma)$ measured by the n_TOF collaboration (*Phys. Rev. Lett. **93** (2004) 161103*).


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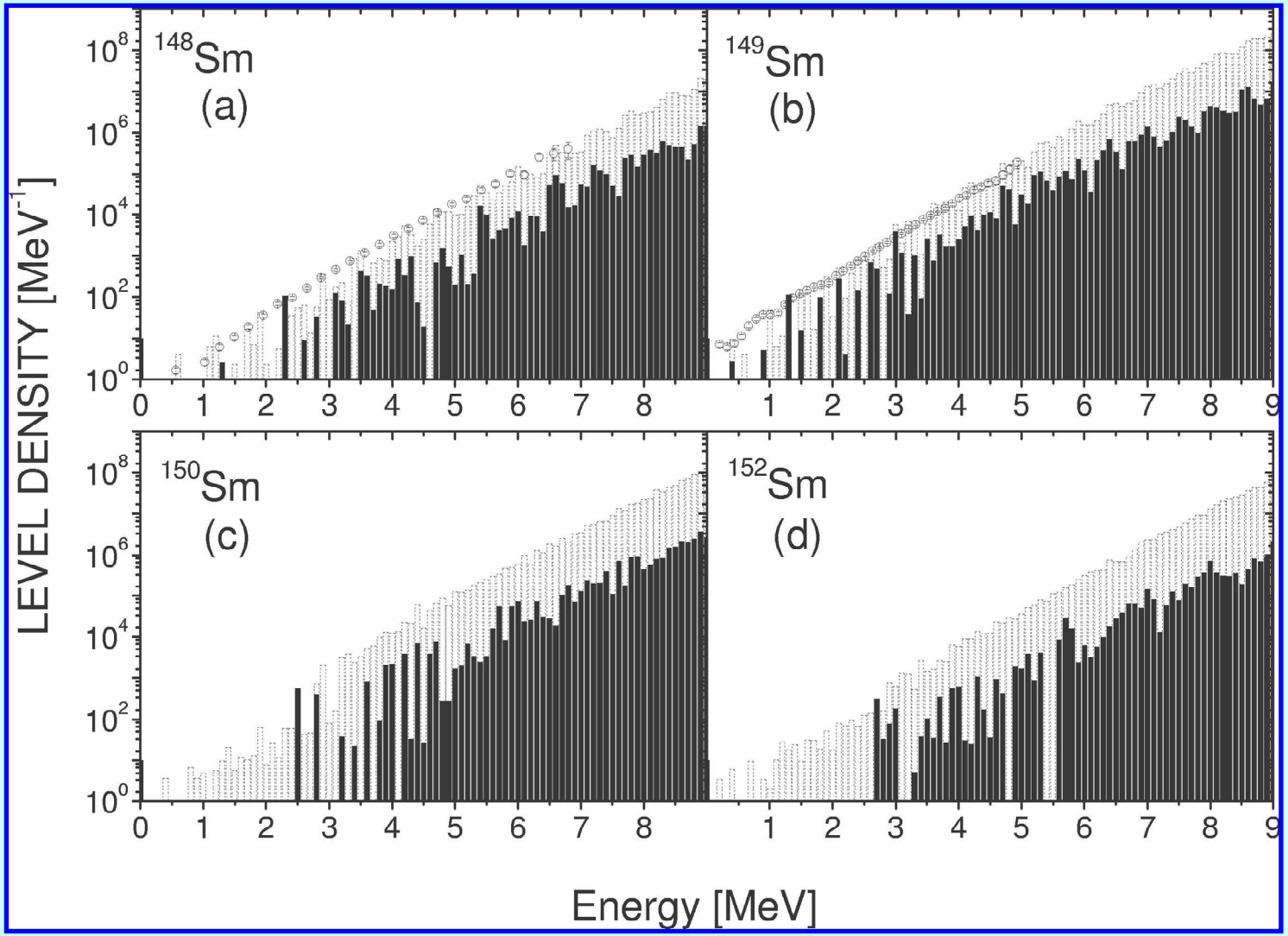
# LEVEL DENSITIES – Sm nuclei





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# LEVEL DENSITIES – Sm nuclei

| Comp. nucleus | $B_n$ (MeV) | Target spin | $D_0$ - exp. (eV) | $D_0$ - calc. (eV) |
|---|---|---|---|---|
| Sm-148 | 8.141 | $7/2^-$ | 5.1±0.5 | 5.4±0.3 |
| Sm-149 | 5.871 | $0^+$ | 100.0±20. | 53.0±2.0 |
| Sm-150 | 7.985 | $7/2^-$ | 2.1±0.3 | 0.94±0.03 |
| Sm-152 | 8.257 | $5/2^-$ | 1.04±0.15<br>1.48±0.04 | 1.2±0.1 |


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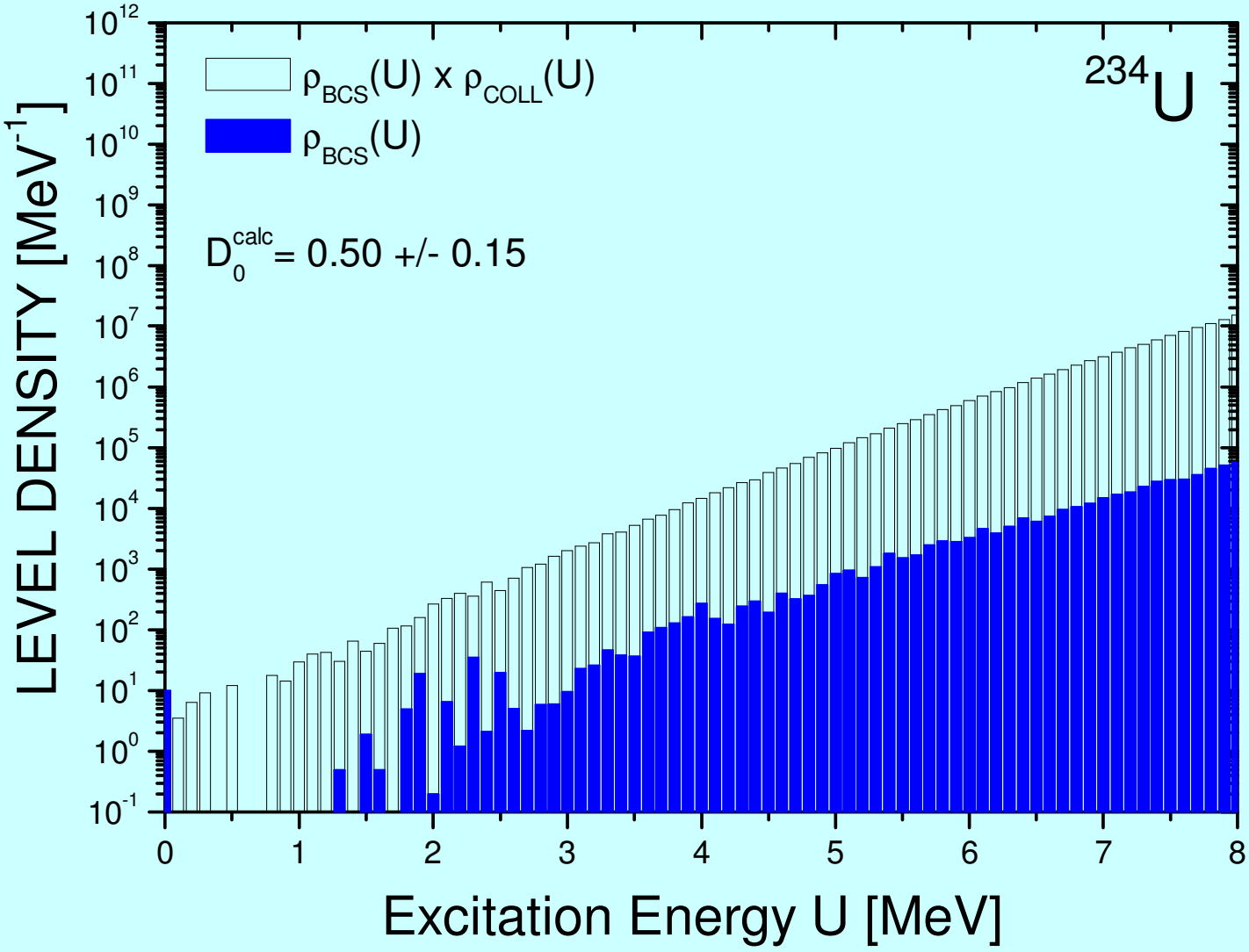
# LEVEL DENSITIES – U-234







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# LEVEL DENSITIES: OUTLOOK

- Odd-mass and odd-odd nuclei at low energy

  Collective state densities should be computed within the framework of the interacting boson-fermion models.

- All nuclei at high energy

  Collective effects should decrease with increasing energy. Bosons are replaced by broken fermion pairs.


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# LEVEL DENSITIES – to do

In adiabatic decoupling approximation, the total state density can be obtained by summing over convolutions of $2k$-quasiparticle state densities with $(N_b - k)$-boson state densities ( $k = 0,1,..,N_b$ ).

$$\omega(E,M,\pi) =$$

$$\sum_{k=0}^{N_b} \sum_{\pi_c \cdot \pi_i = \pi} \sum_{c=0}^{N_c^{(N-k)}(\pi_c)} \int_0^{\infty} dE_i \sum_{M_i + M_c = M} \omega_{\text{intr}}^{(2k)}(E_i, M_i, \pi_i) \omega_{\text{coll}}^{(N_b-k)}(E - E_i, M_c, \pi_c),$$

$$\omega_{\text{coll}}^{(N_b-k)}(E - E_i, M_c, \pi_c) = \sum_{l, M_l, \pi_l} \delta(E - E_i - E_l^{(N_b-k)}(M_l, \pi_l)) \delta_{\pi_c \pi_l} \delta_{M_c M_l}$$


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# SUMMARY

- Intrinsic LD : "Exact" calc. within Indep.Part.M
- Collective LD : IBM
- Enhancement beyond $K_{vib}(U)K_{rot}(U)$

- *Global calculations not possible (IBM), but could be used as benchmark*


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