# Brink-Axel hypothesis 54 years later

## History around 1950

a) Neutron capture gamma rays.
b) Giant dipole resonance.
c) The Brink – Axel hypothesis

**Factorization of gamma spectrum!**

## Applications and discussion:

d) Gamma transitions to a definite final state.
e) E1 emission from highly excited nuclei.
E1 emission below neutron threshold.
f) Lorentzian shape for GDR cross section,
How good is it for low photon energies?

# Neutron capture γ-rays

An active area of research in early 1950s

Kinsey and collaborators at Chalk River

Also at Harwell near Oxford.

Hughes and Harvey: γ-ray data collection.

Hughes on Leave in Oxford in 1954
from Brookhaven.

# Hughes and Harvey (1954)
# Experimental gamma widths ~ A

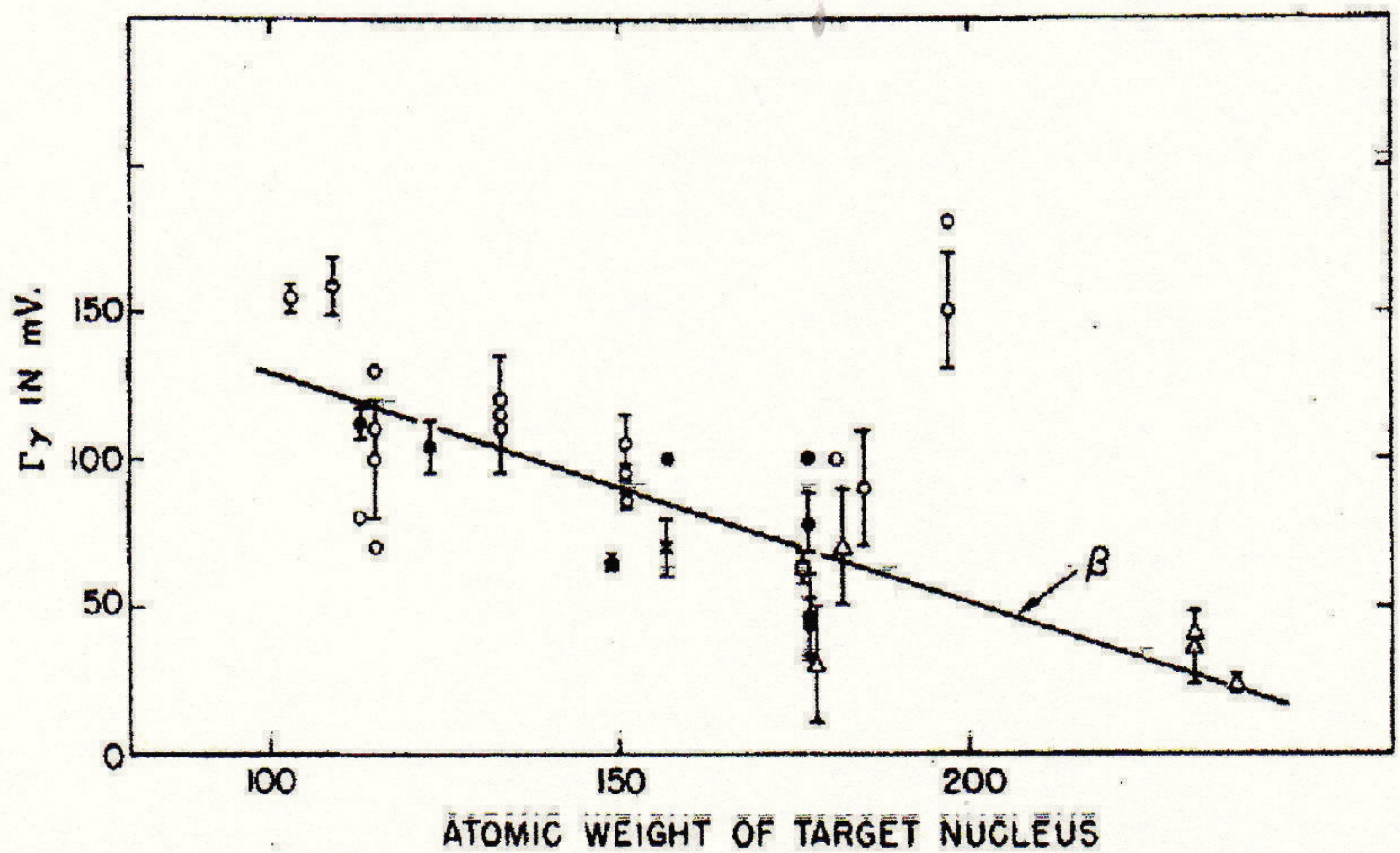



**Fig. 1.** Radiation-widths ($\Gamma_\gamma$) of slow-neutron resonance-levels plotted against atomic weight of the target nucleus. The symbols, ⊙ odd *Z*–odd *N*, ● even *Z*–even *N*, △ even *Z*–odd *N*, ⊡ odd *Z*–even *N*, and × assignment unknown, refer to the compound nucleus. The errors are shown for the values of $\Gamma_\gamma$ that have an accuracy of 20 mV. or better. Almost all the other points have large enough errors to agree with the straight line $\beta$

# Giant Dipole Resonance

Discovered in Uranium by Baldwin and Klaiber in 1947

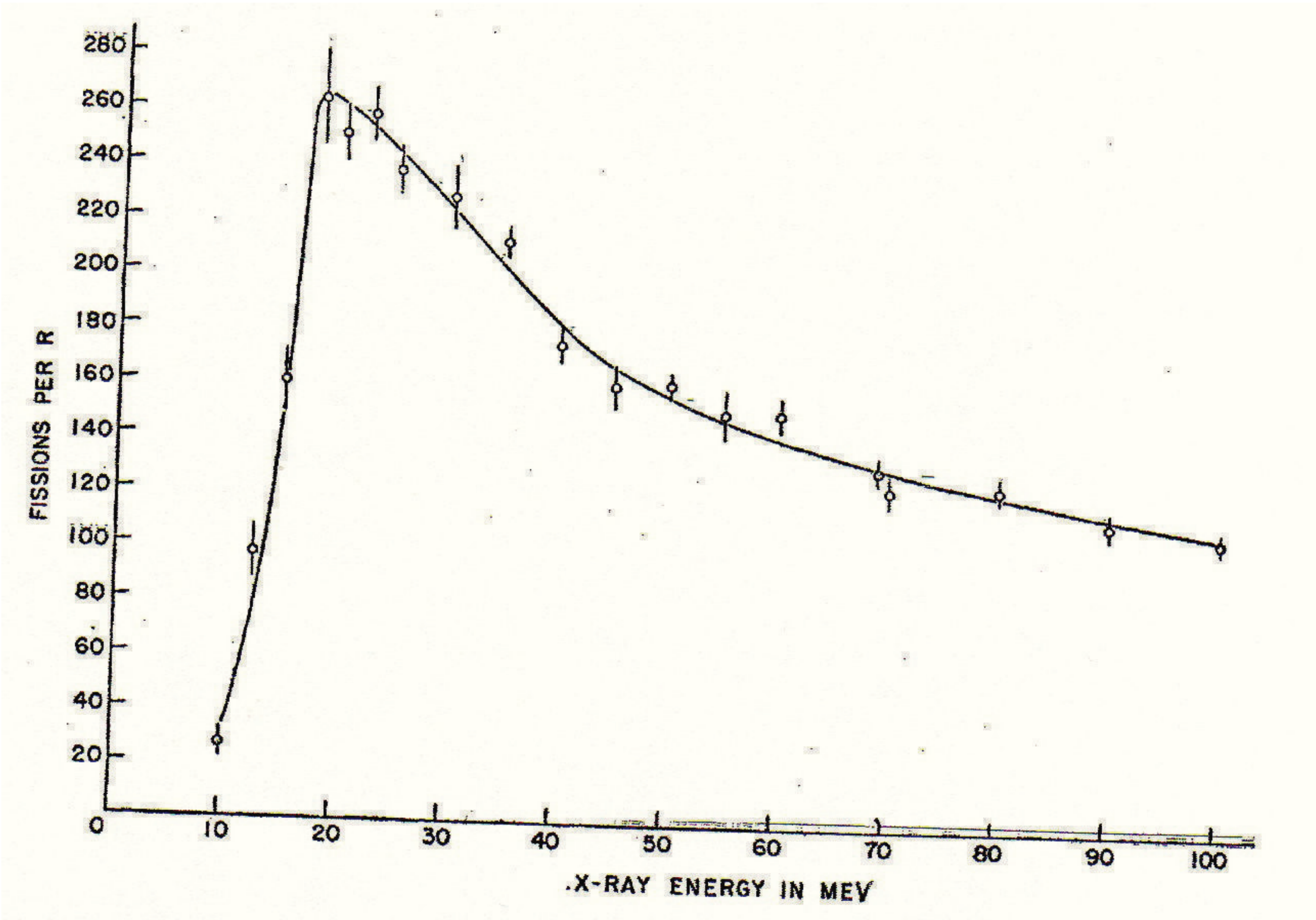
Gamma rays from Betatron at GEC research labs.

in γ-fission and (γ, n) reactions.

Theory: Goldhaber and Teller (1948)

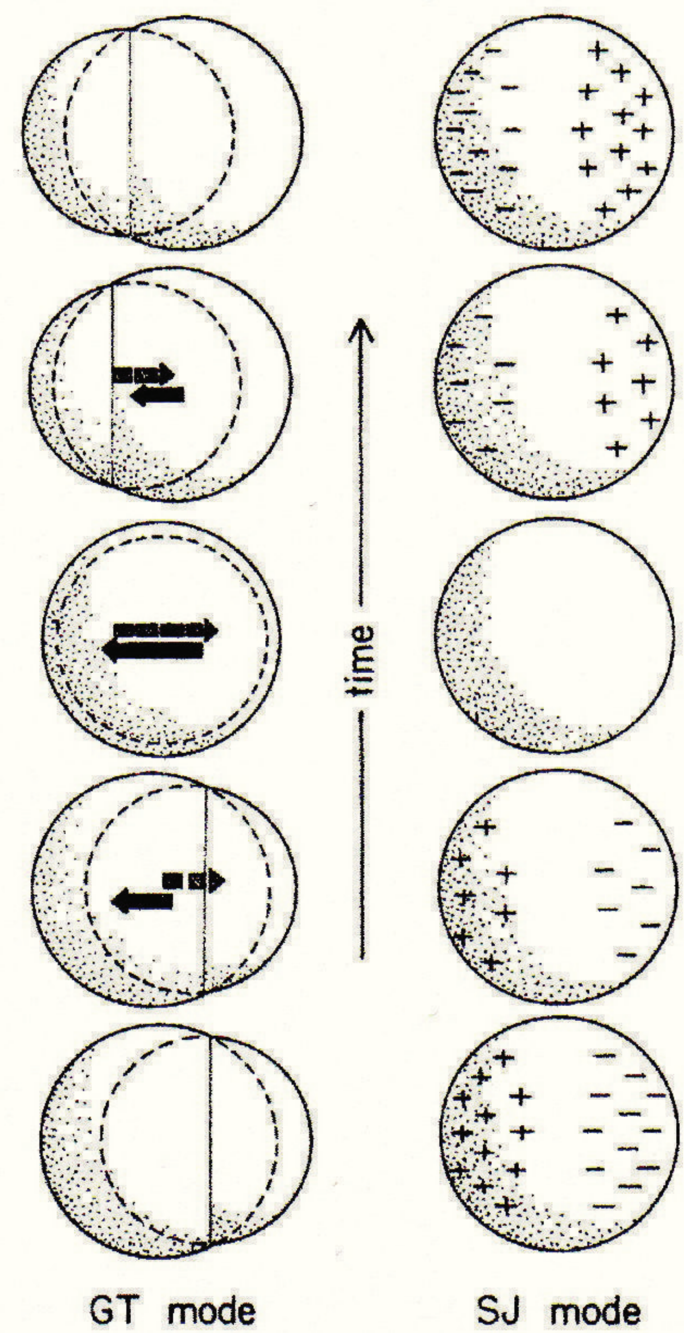
Steinwedel and Jensen (1950)

# Photofission of Uranium
# Baldwin and Klaiber (1948)



# GDR Models

- Left
  - Goldhaber & Teller (1948)
- Right
  - Steinwedel & Jensen (1950)



# Steinwedel and Jensen (1950) Hydrodynamical Theory

Quantitative theory: Related dipole resonance energy to the nuclear summetry energy and gave an explicit dependence on N, Z, and A.

$$E_0 = 60\left(4\mathrm{NZ}/A^2\right)^{(1/2)} A^{-(1/3)}\, MeV$$

An explicit formula for the integrated cross section which satisfied the dipole sum rule.

$$\left(\frac{\hbar}{2mc^2}\right)\int \sigma\, d\omega = 2\pi^2\left(\frac{e^2}{2mc^2}\right)^2\left(\frac{NZm}{AM}\right)\left(\frac{\hbar c}{e^2}\right)$$

# Axel-Brink hypothesis

David Brink thesis Oxford May 1955.

Peter Axel, Phys. Rev. **126** (1961) 671.

The GDR is a collective state involving vibration of the protons against the neutrons and it should not be too sensitive to the detailed structure of the initial state.

If it were possible to perform the photo effect on an excited state the cross section for absorption of a photon with energy *E* would have the same energy dependence as from the ground state.

# Predicted γ-widths of neutron resonances

Relate absorption to emission by detailed balance.

Assume a Lorenzian peak with position $E_m$ and total width Γ for the gamma absorption.

Gamma width for emitting a photon with energy $E_\gamma$ from a state with energy $E_0$ and spin *I.*

$$\Gamma_{\gamma,I}(E_0, E_\gamma)\rho_I(E_0) = \frac{4}{3\pi}\frac{NZ}{A}\frac{e^2}{\hbar c}\frac{1}{Mc^2}\frac{\Gamma E_\gamma^4}{(E_\gamma^2 - E_m^2)^2 + \Gamma^2 e^2}$$

# Sum over final states

Assume density of states with spin I:

$$\rho_I(E) \propto (2\mathrm{I}+1)\,\rho_0(E)$$

Then the total width is

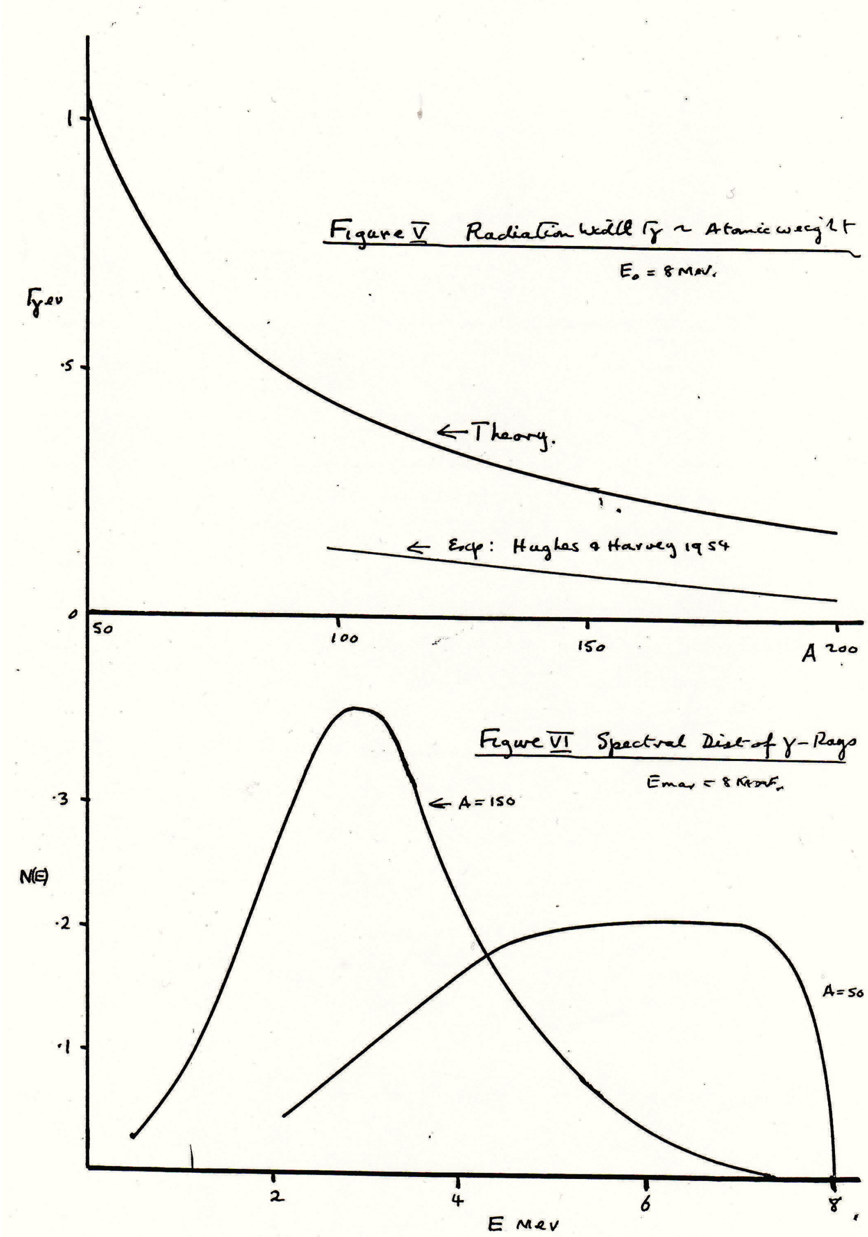
$$\Gamma_\gamma(E_0) = \int_0^{E_0} dE_\gamma\, C\, \frac{\rho(E_0 - E_\gamma)}{\rho_0(E_0)}\, \frac{\Gamma E_\gamma^4}{(E_\gamma^2 - E_m^2)^2 + \Gamma^2 E_\gamma^2}$$

with $C = \dfrac{4}{3\pi}\,\dfrac{NZ}{A}\,\dfrac{e^2}{\hbar c}\,\dfrac{1}{Mc^2}$

No dependence on initial spin *I*.

# Widths and Spectra

- Top
- Radiative widths of
- of resonances
- against A
- Bottom
- Spectral distribution
- Of gamma rays
- For A=50 and A=150



## Factorization; Density of initial states; Gamma absorption cross section from final state. Have replaced Lorentzian form for the absorption cross section by a general form independent of $E_0$.

$$\Gamma_{\gamma,I}(E_0, E_\gamma) = \frac{1}{\rho_I(E_0)} \frac{E_\gamma^2}{(\pi \hbar c)^2} \sigma_{abs}(E_\gamma)$$

Gamma spectrum for a continuum of final states.

$$\frac{d\Gamma_\gamma}{dE_\gamma} = \frac{E^2}{(\pi \hbar c)^2} \sigma_{abs}(E_\gamma) \frac{\rho_0(E_0 - E_\gamma)}{\rho_0(E_0)}$$

Assumes $\rho_I(E) \propto (2\mathrm{I}+1)\rho_0(E)$

# How good is this formula?

$$\Gamma_{\gamma,I}(E_{0,} E_{\gamma})\rho_I(E_0) = \frac{4}{3\pi}\frac{NZ}{A}\frac{e^2}{\hbar c}\frac{1}{Mc^2}\frac{\Gamma E_\gamma^4}{(E_\gamma^2 - E_m{}^2)^2 + \Gamma^2 e^2}$$

Example from Ramen et al Oak Ridge 1981

1) Neutron capture in $(5/2)^-$ states of $^{173}Yb$ to form $2^-$ and $3^-$ resonances .

2) E1 gamma rays to $2^+$ and $4^+$ states in ground state rotational band of $^{174}Yb$.

3) 13 discrete 2-quasi particle excited states with spin-parity $1^+$ to $4^+$.

4) Closely spaced states at higher energy.

# Raman et al results

Measured strength function for gamma widths of discrete final states in $^{174}$Yb.

GDR parameters from measured GDR in $^{173}$Lu.

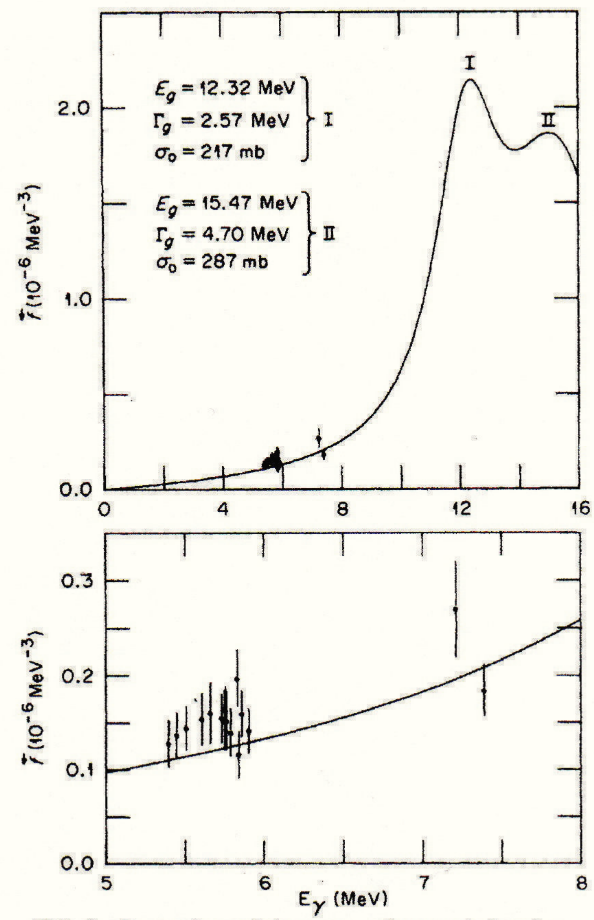



FIG. 2. Comparison of the measured strength function with the prediction (solid curve) of the Axel-Brink giant dipole resonance model.

# Thermodynamics: Density of states ratio related to entropy and temperature

$$\frac{\rho(E_0 - E_\gamma)}{\rho(E_0)} \approx exp\left[S(E_0 - E_\gamma) - S(E_0)\right] \approx exp\left(-E_\gamma \frac{dS}{dE_0}\right) = exp\left(\frac{-E_\gamma}{T}\right)$$

$$\frac{d\Gamma_\gamma}{dE_\gamma} = \frac{E^2}{(\pi\hbar c)^2}\sigma_{abs}(E_\gamma)\frac{\rho(E_0 - E_\gamma)}{\rho(E_0)}$$

Gamma spectrum in terms of the nuclear temperature

$$\frac{d\Gamma_\gamma}{dE_\gamma} = \frac{E_\gamma^2}{(\pi\hbar c)^2}\sigma_{abs}(E_\gamma) exp\left(\frac{-E_\gamma}{T}\right)$$

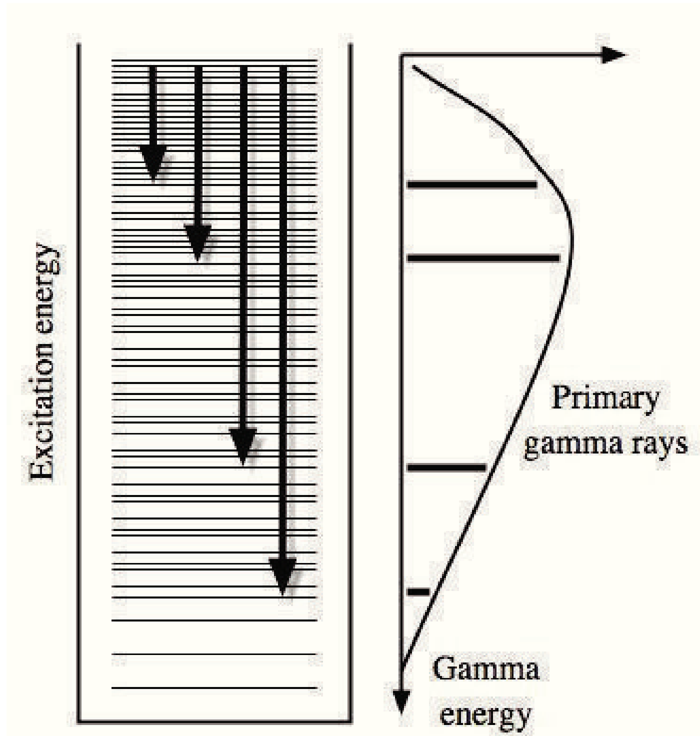
Gamma ray strength function: $f(E_\gamma) = \frac{1}{3\pi^2\hbar^2c^2}\frac{\sigma_{abs}}{E_\gamma}$

# Oslo method for level densities and strength functions

Uses reactions like ($^3$He, α) reactions to excite states below the neutron threshold and measures primary γ-rays.

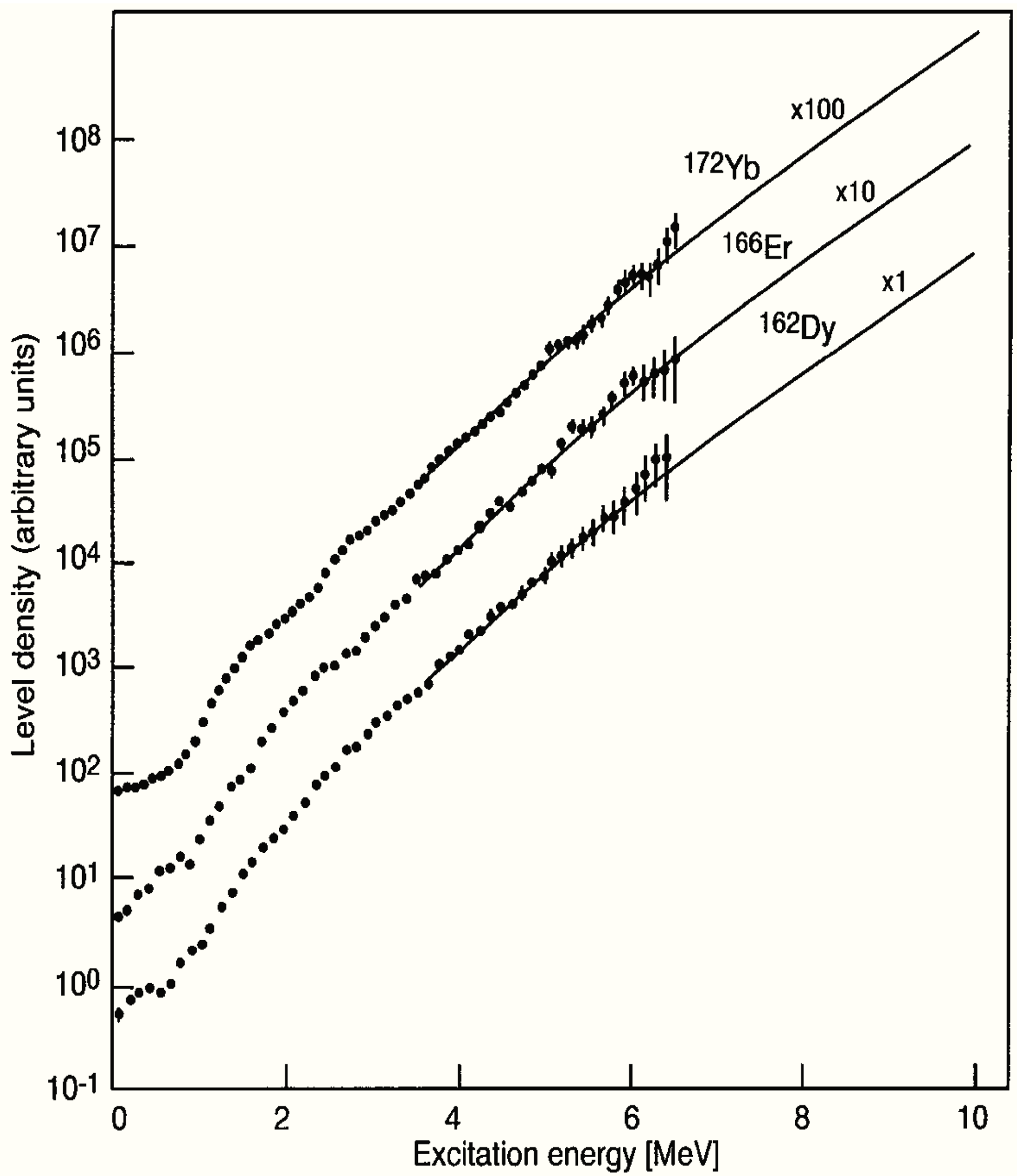
Examples: gamma spectra in $^{162}$Dy, $^{166}$Er and $^{172}$Yb.
Pb isotopes.

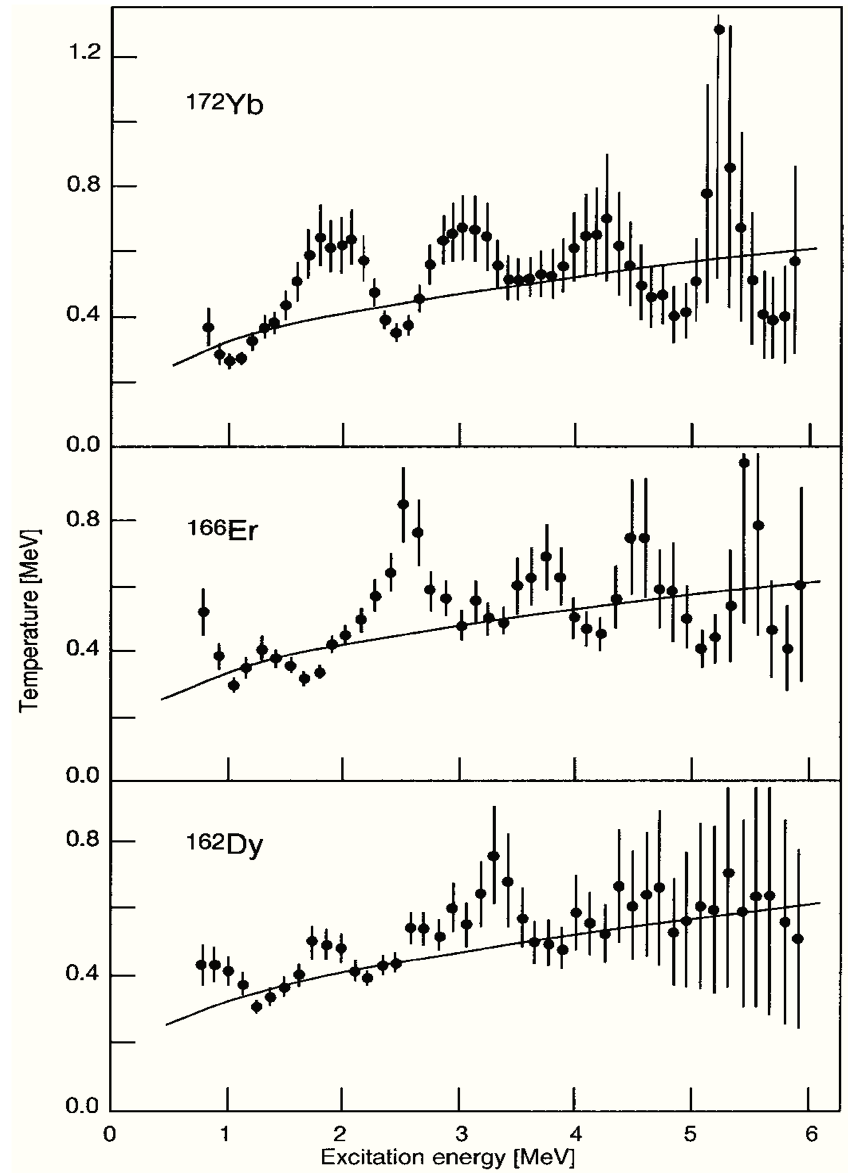
Assumes factorization
Finds density of states
and
Strength function
by measuring γ-rays
at different initial
excitation energies.



# Density of states and temperature $^{172}Yb$, $^{166}Er$ and $^{162}Dy$

Phys.Rev.Lett. 83 (1999) 3150



## Measured strength functions

for $^{205}$Pb and $^{206}$Pb
by Oslo method

Points above the neutron threshold in $^{206}$Pb are taken from n-gamma data. This fixes the normalization.

Phys.Rev. **C** 79, 024316

$$f(E_\gamma) = \frac{1}{3\pi^2\hbar^2c^2}\frac{\sigma_{abs}}{E_\gamma}$$

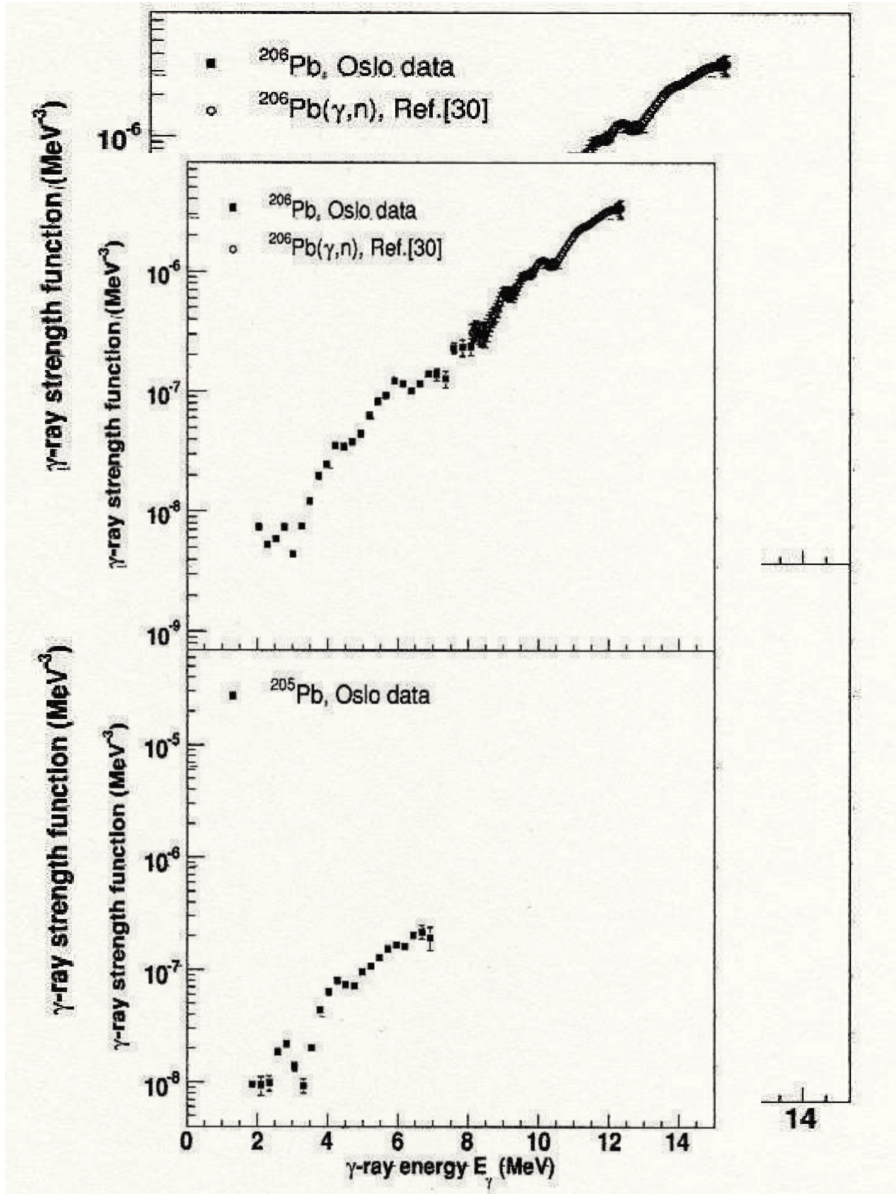



FIG. 14. Normalized γ-ray strength functions for $^{205,206}$Pb as a function of γ-ray energy. The Oslo data (filled squares) of $^{206}$Pb are normalized to photonuclear data (open circles) taken from Ref. [30].

# Photoabsorption Cross-sections

ELBE Lineac Rossendorf (2008)

Mo isotopes: 92Mo to 100Mo

Full circles: photon scattering

Open circles: (γ, n) reactions
Solid circles: (γ, γ) scattering

Solid line: sum of Gaussians

For 92Mo and 94Mo one Gaussian is quite good.

For 98Mo and 100Mo three Gaussians are needed maybe because of deformation.

J. Phys. G: Nucl. Part. Phys. **35** (2008) 014035 A Wagner *et al*

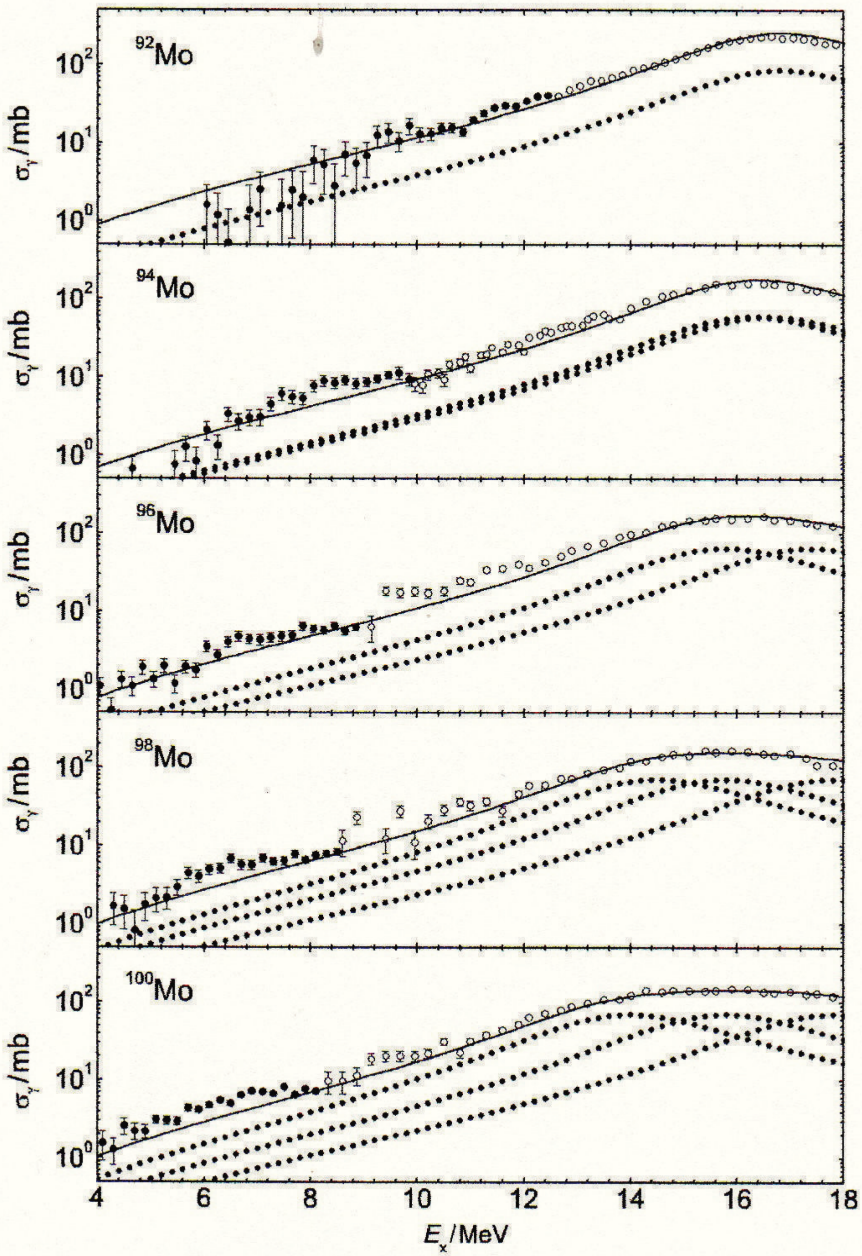



**Figure 4.** Photoabsorption cross sections for all stable even–even molybdenum isotopes derived from photon scattering (filled symbols) and $(\gamma, n)$ reactions (open symbols) [17]. The solid lines indicate the parametrization using the sum of up to three Lorentzians (dashed lines).

GDR in highly excited nuclei; eg. $^{122}$Te
A ~ 122, E* ~150 MeV

Newton et al (1981)
Phys.Rev.Lett. **46** 1384

The influence of the GDR shows up in the bump in the gamma spectrum between 10 and 20 MeV.

Themal excitation of GDR in a hot nucleus.



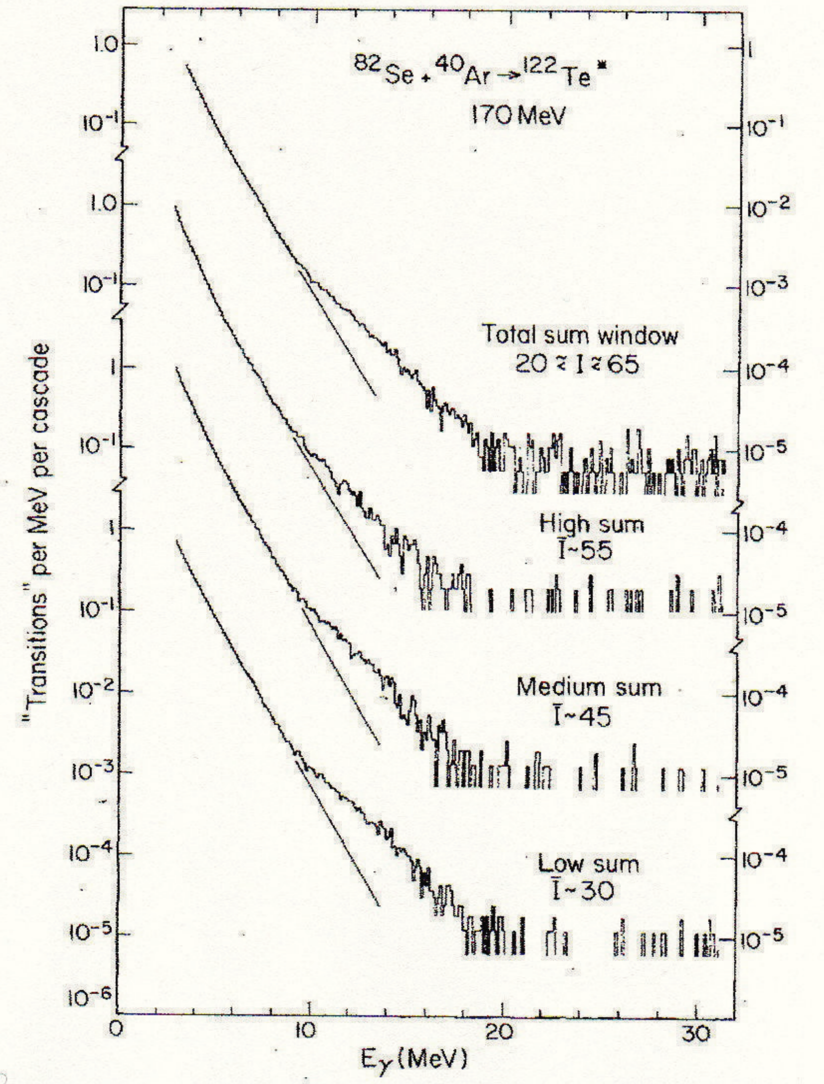



FIG. 1. NaI spectra corresponding to $E_s$ = 10–40 MeV and three windows within this range for the $^{82}$Se + $^{40}$Ar system. The sloping lines show exponential extrapolations of the lower $E_\gamma$ parts of the spectra. The shapes of the true $\gamma$-ray spectra are not expected to differ greatly from these, and hence the ordinate in "transitions per MeV" should be approximately correct.

# Jacobi shape transition in $^{46}$Ti.

Big experiment; groups from Italy, France, Poland, Denmark Sweden.

Euroball (2004)
Eur. Phys. J. A20, 165

Lublin-Strasbourg Drop Model (LSD) developed by Pomorsky and Dudek

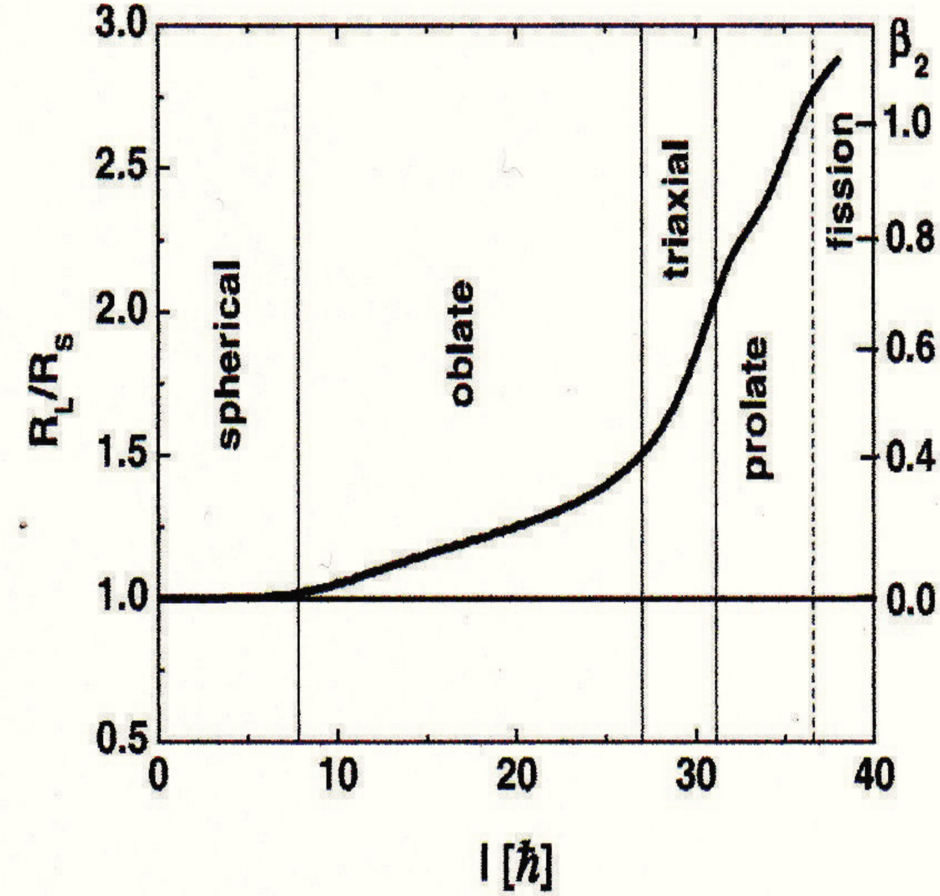



**Fig. 1.** Long-to-short axis ratio and the $\beta_2$ parameter for the equilibrium deformation as a function of spin, obtained from the LSD model calculations for $^{46}$Ti.

# GDR in excited $^{46}$Ti

$^{46}$Ti produced in $^{18}$O + $^{28}$Si high excitation energy.

Broad bump indicates triaxial shape predicted by LSD theory for L=30 hbar.

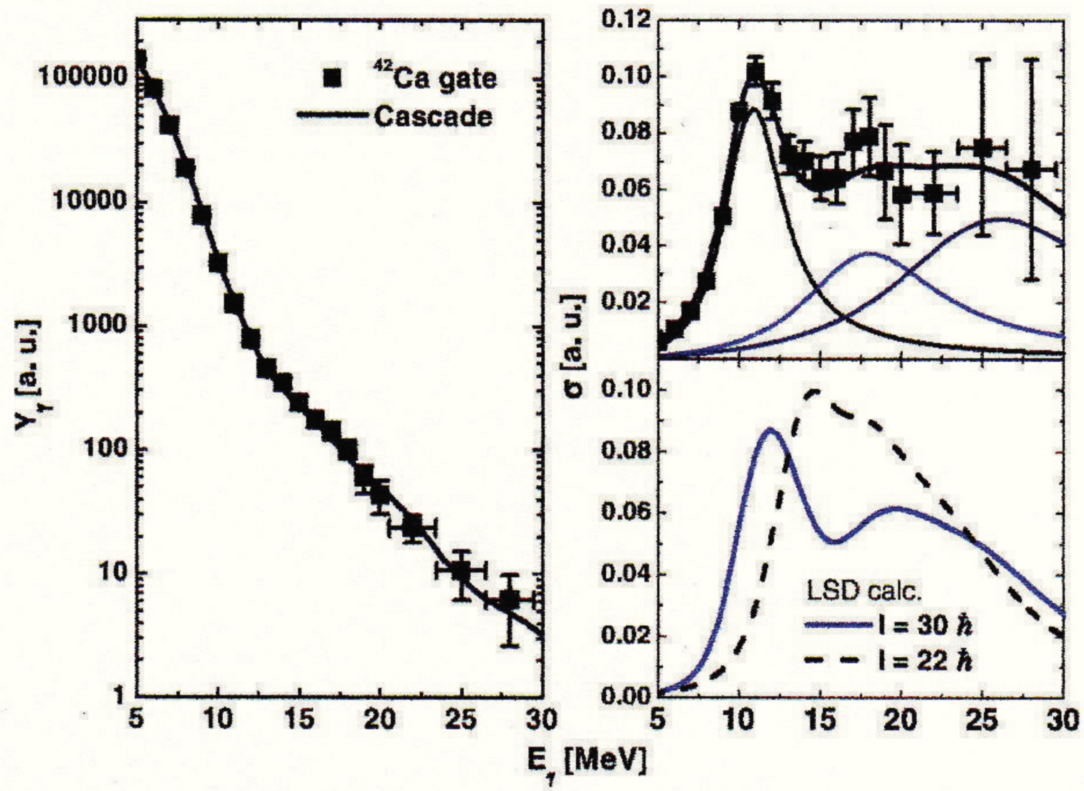



**Fig. 2.** Left: spectrum of the $\gamma$-rays from the decay of the GDR built in hot $^{46}$Ti in coincidence with the discrete transitions in the residual nucleus $^{42}$Ca, together with the Cascade calculations assuming 3-Lorentzian GDR strength function with $E_{\rm GDR}$ = 10.8, 18 and 26 MeV. Upper right: experimentally obtained GDR absorption cross-section and the GDR strength function used in Cascade calculations. Bottom right: thermal shape fluctuation predictions based on potential energies from the LSD model calculations for $I = 22\hbar$ and $I = 30\hbar$.

# Pre-equilibrium Dipole emission

Dipole moment as a function of time calculated using semi-classical transport theory.

Square modulus of Fourier transform gives a resonance peak.

Baran et al Phys. Rev. Lett. 87, 182501

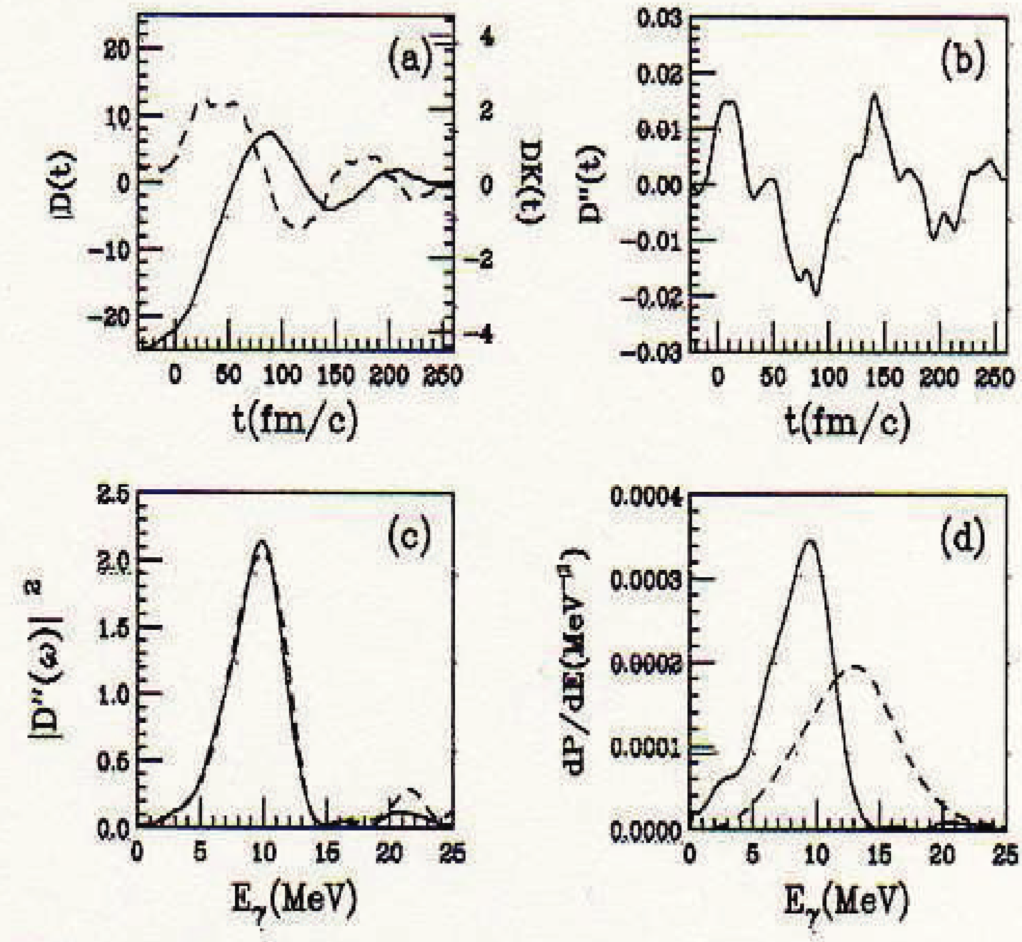



FIG. 1. System Ca + Mo at 4*A* MeV: (a) time evolution of $D(t)$ (solid line, in fm units), and $DK(t)$ (dashed line, in $fm^{-1}$). (b) The same for the acceleration $D''(t)$ (in $c^2/fm$). (c) Power spectrum $|D''(\omega)|^2$ (in $c^2$ units). (d) Bremsstrahlung spectrum (solid line) and the first step statistical spectrum (dashed line).

# Summary

1) GDR on excited states is a useful concept.

2) Factorization into strength function and
density of states factor.

3) Measurement of strength functions
and density of states/nuclear temperature.

4) GDR in highly excited nuclei. Can get quite
detailed information.

5) Pre-equilibrium GDR.