Combinatorial Nuclear Level Density Model

Sven Åberg, Lund University, Sweden

2nd Workshop on
Level Density and Gamma Strength
Oslo, May 11 - 15, 2009

Senter for Akseleratorbasert Forskning og Energifysikk

Combinatorial Nuclear Level Density Model

I. Introduction

II. Microscopic method for calculating level density

- Combinatorical intrinsic level density
- Pairing
- Rotational enhancements
- Vibrational enhancements
- Role of residual interaction

III. Result

(a) Data at neutron separation energy
(b) Details of level density (Oslo data)
(c) Observed discrete states
(d) Angular momentum distribution
(e) Parity enhancements (Richter exp.)
(f) Fission barriers

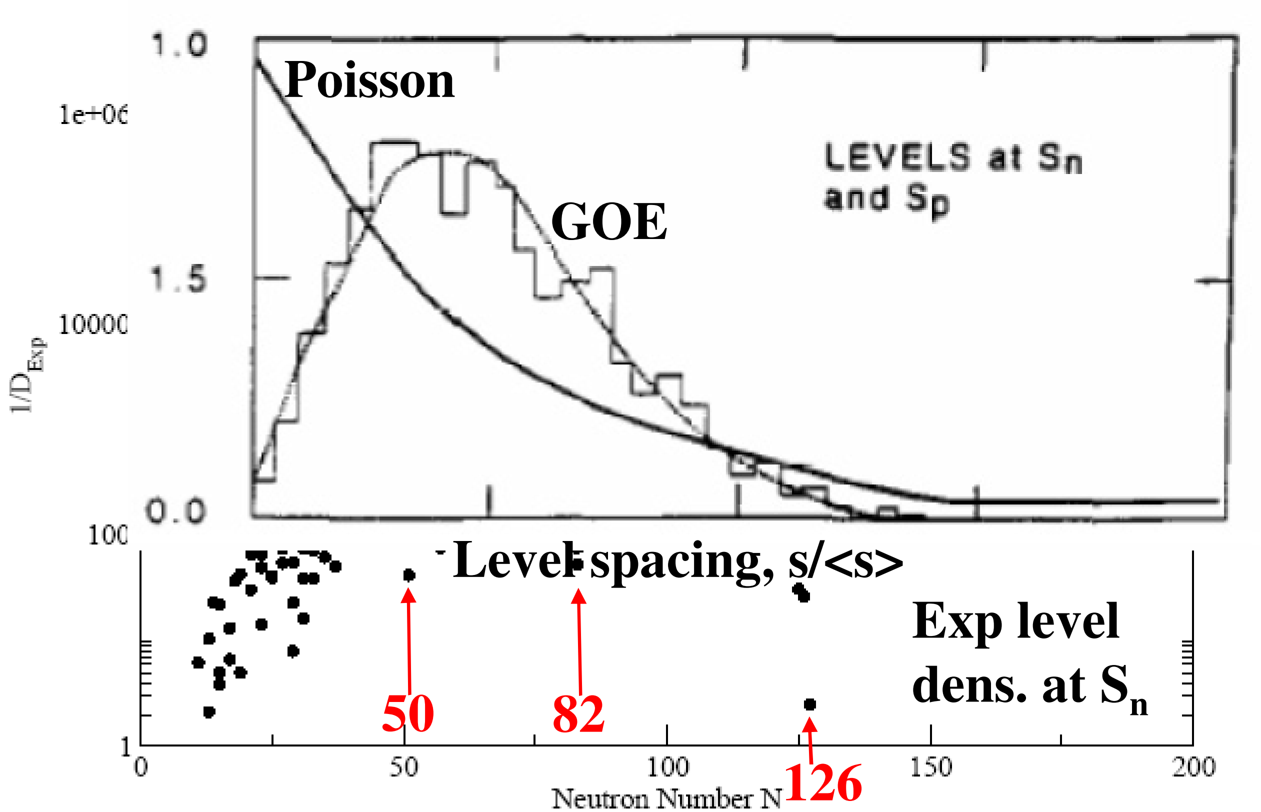
In collaboration with: **H. Uhrenholt, Lund**
T. Ichikawa, RIKEN
P. Möller, Los Alamos

Subm to PRC, arXiv:0901.1087

Neutron resonance region

- Fluctuations of eigen energies and wave functions are described by random matrices – same for all nuclei

- Level density varies from nucleus to nucleus:



- Fully chaotic but shows strong shell effects!

Level density

$$\rho(E_{exc}, I, \pi) = P(E_{exc}, \pi) F(E_{exc}, I) \rho(E_{exc})$$

where *P* and *F* project out parity and angular momentum, resp.

In (backshifted) Fermi gas model:

$$P(E_{exc}, \pi) = 0.5$$

$$F(E_{exc}, I) = \frac{I + 0.5}{\sigma^2} \exp\left(-\frac{(I + 0.5)^2}{2\sigma^2} \right)$$

$$\rho(E_{exc}) = \frac{\sqrt{\pi}}{12 a^{1/4} U^{5/4}} \exp\left(2\sqrt{aU}\right)$$

where

$$U = E_{exc} - E_{shift}$$

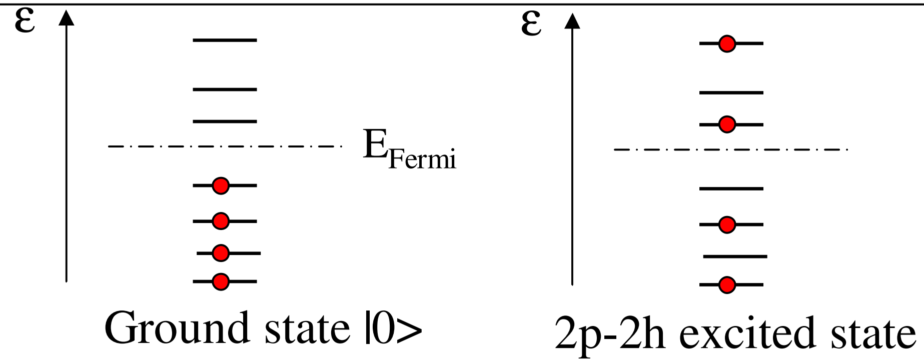
Backshift parameter, E_{shift}, level density parameter, *a*, and spin cutoff parameter, σ, are typically fitted to data (often dep. on E_{exc})

We want to have:
Microscopic model for level density to *calculate* level density, *P* and *F*.
Obtain: Structure in ρ, and parity enhancement.

II. Microscopic method for calculation of level density

(a) Intrinsic excitations - combinatorics

Mean field: folded Yukawa potential with parameters (including deformations) from Möller et al.



All np-nh states with n ≤ 9 included (for protons as well as for neutrons)

Count all states and keep track of seniority (v=2n), total parity and K-quantum number for each state

Energy: $E_\mu(v, K, \pi)$

Level density composed by v-qp exitations

2 quasi-particle excitation: seniority v=2
v quasi-particle excitation: seniority v

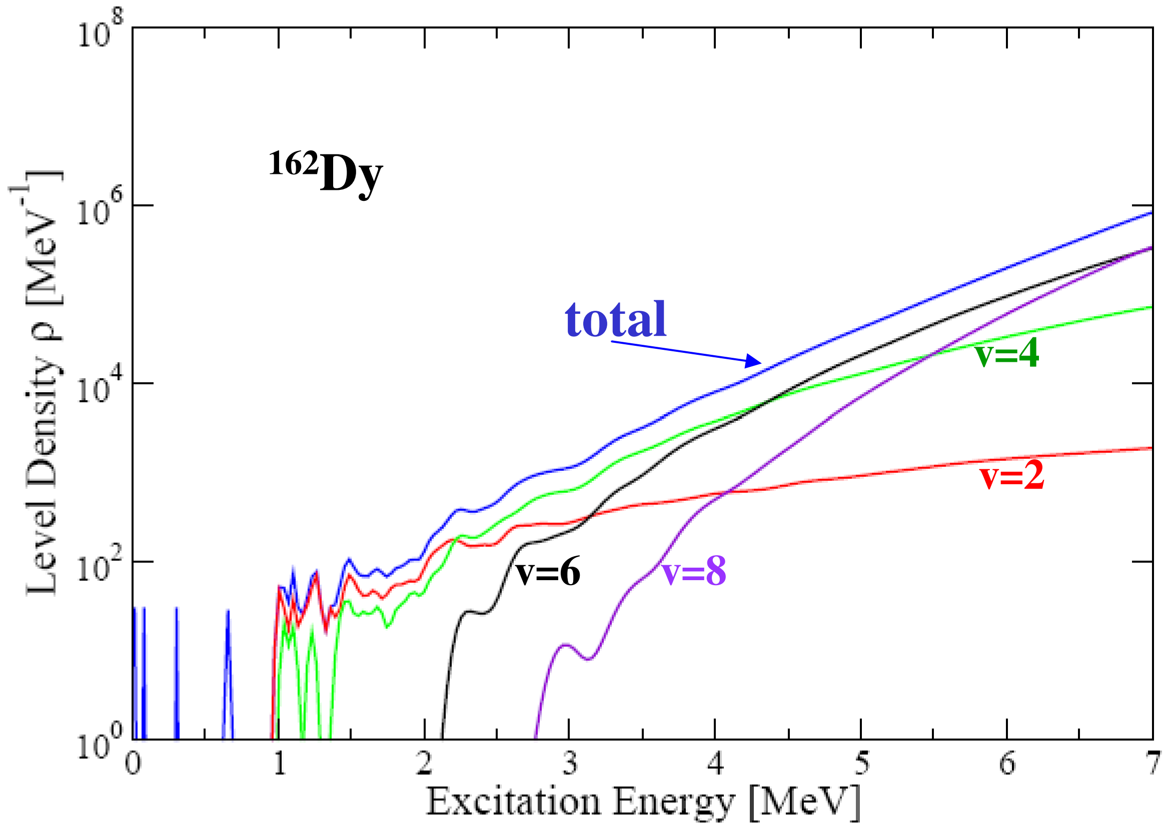
In Fermi-gas model:

$$\rho_{v-qp}(E_{exc}) \propto E_{exc}^{v-1}$$

Total level density:

$$\rho_{tot}(E_{exc}) = \sum_{v=0,2,4....} \rho_{v-qp}(E_{exc})$$

$$\propto \exp(2\sqrt{aE_{exc}})$$



Pairing

For EACH state, μ:

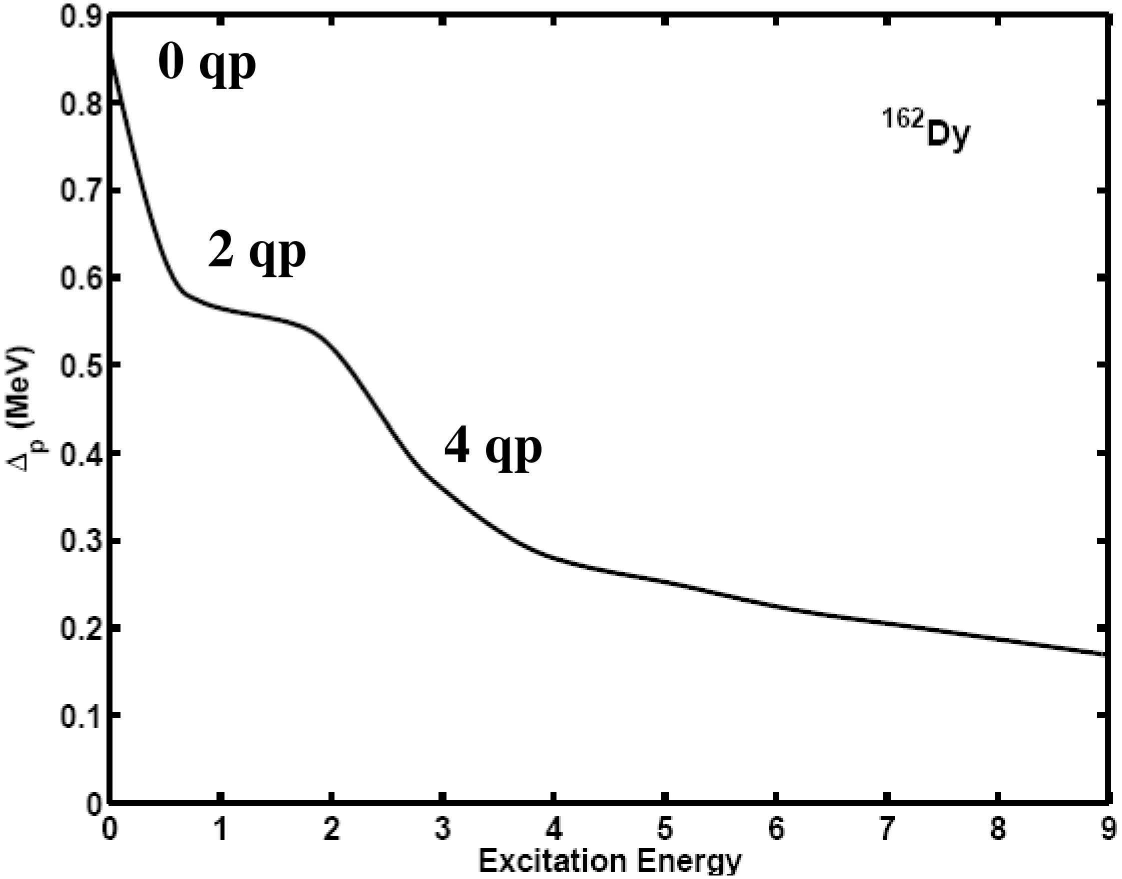
Solving BCS equations provides:

Energy, E_μ,corrected for pairing (blocking accounted for)

Pairing gaps, Δ_n and Δ_p

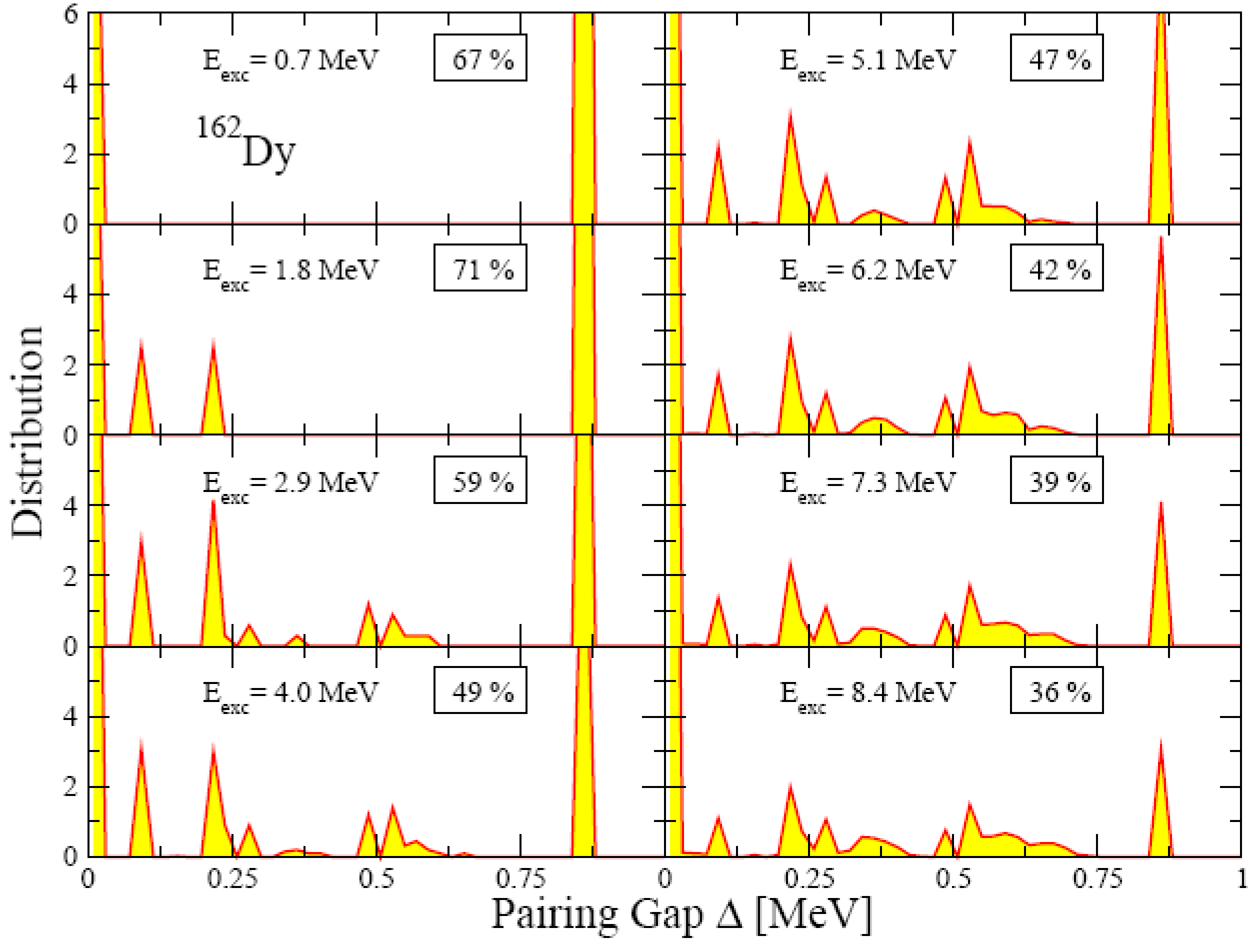
Energy: $E_\mu(v, K, \pi, \Delta_n, \Delta_p)$

Mean proton pairing gap vs excitation energy:



Pairing remains at high excitation energies!

Distribution of proton pair gaps



No pairing phase transition!

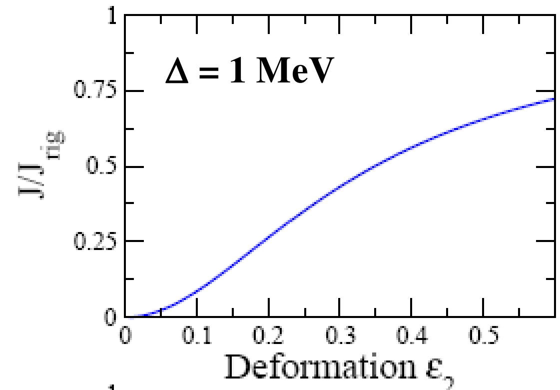
Rotational enhancement

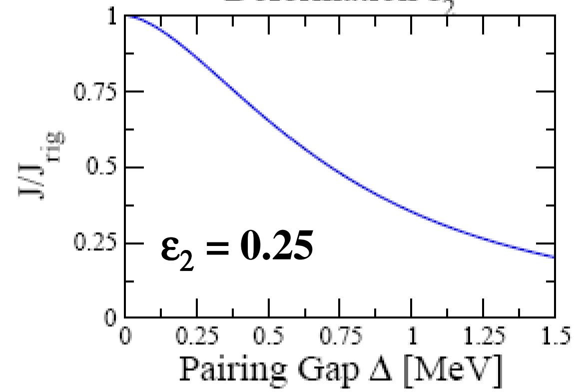
Each state with given K-quantum number is taken as a band-head for a rotational band:

$E(K,I) = E(K) + \hbar^2/2J\ (\varepsilon, \Delta_n, \Delta_p)\ [I(I+1)-K^2]$

where moment of inertia, J, depends on deformation and pairing gaps of that state [1]

Energy: $E_\mu(v, I, K, \pi, \Delta_n, \Delta_p)$





Double counting of rotational states? — **^{162}Dy**

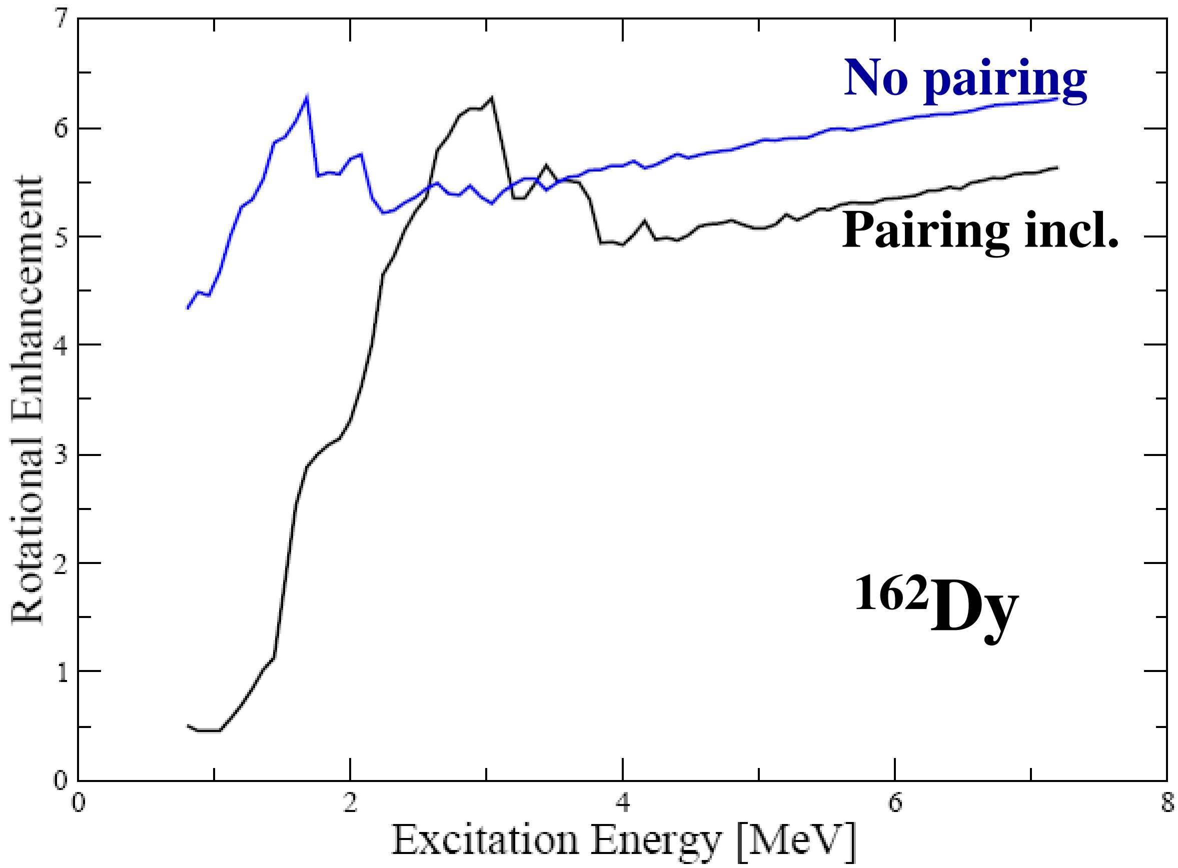
Rotational 2$^+$ energy: $\frac{\hbar^2}{2\Im} 2(2+1) \approx 90 A^{-5/3} MeV$ — **80 keV**

Energy of j_x matrix el.: $\varepsilon \cdot \hbar\omega \approx \varepsilon \cdot 41 A^{-1/3} MeV$ — **1900 keV**

Not important for E_{exc} below ~50 MeV

[1] R. Bengtsson and S. Åberg, Phys. Lett. B172 (1986) 277.

Rotational enhancement



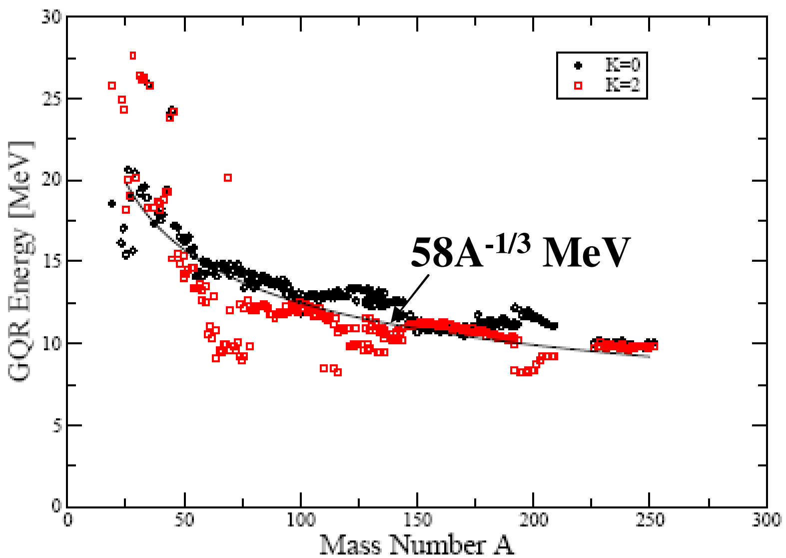
Vibrational enhancement

Add QQ-interaction corresponding to
Y_{20} (K=0) and Y_{22} (K=2), double-stretched [1]

Solve Quasi-Particle Tamm-Dancoff
for EACH state, *i*:

$$\frac{1}{\chi_{2K}} = \sum_{\mu\nu} \frac{\left|\langle \mu \left| \bar{Q}_{2K} \right| \nu \rangle\right|^2 (U_\mu V_\nu + V_\mu U_\nu)^2}{(E_\mu^{\mathrm{qp},i} + E_\nu^{\mathrm{qp},i}) - (\hbar\omega)_j^i}$$

Isoscalar giant quadrupole resonances well described with QPTD:



[1] H. Sakamoto and T. Kishimoto, Nucl Phys A501 (1989) 205
S. Åberg, Phys. Lett. B157 (1985) 9.

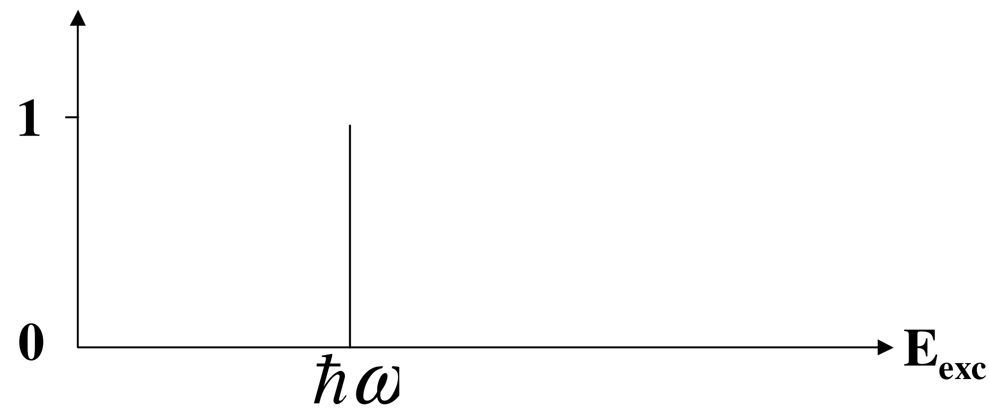
Exact correction for double-counting of states

For *each* phonon state QPDA solution gives:

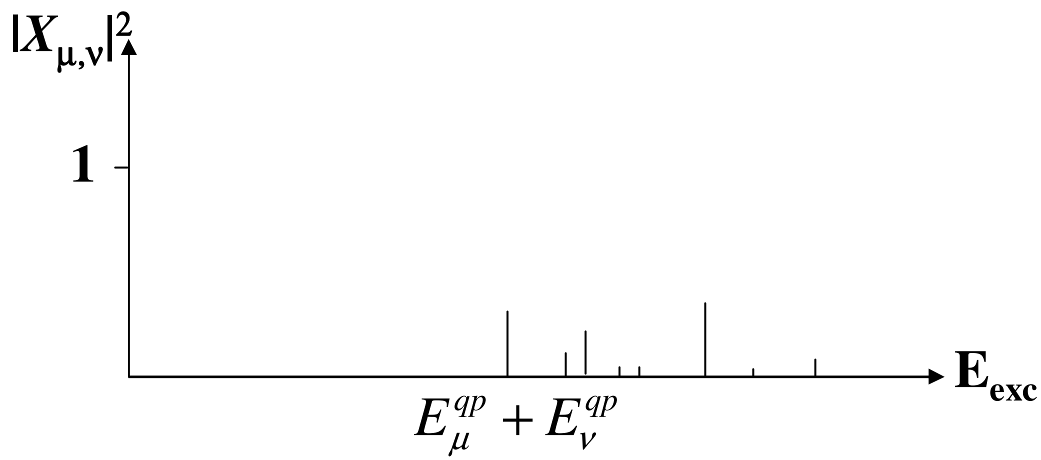
Phonon energy: $\hbar\omega$

Phonon w.f.: $\mathcal{O}^\dagger = \sum_{\mu,\nu} X_{\mu,\nu} a_\mu^\dagger a_\nu$

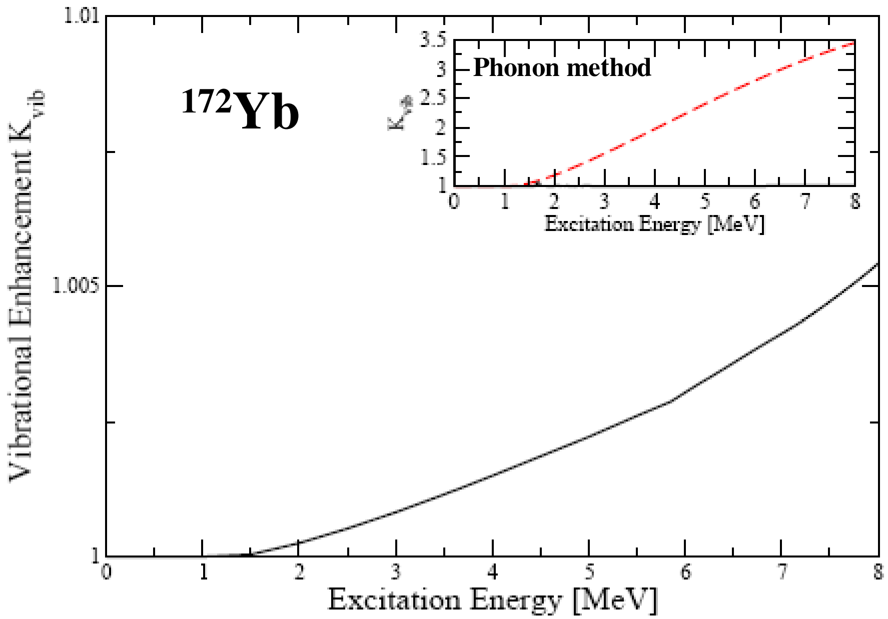
***Include* phonon state in level density:**

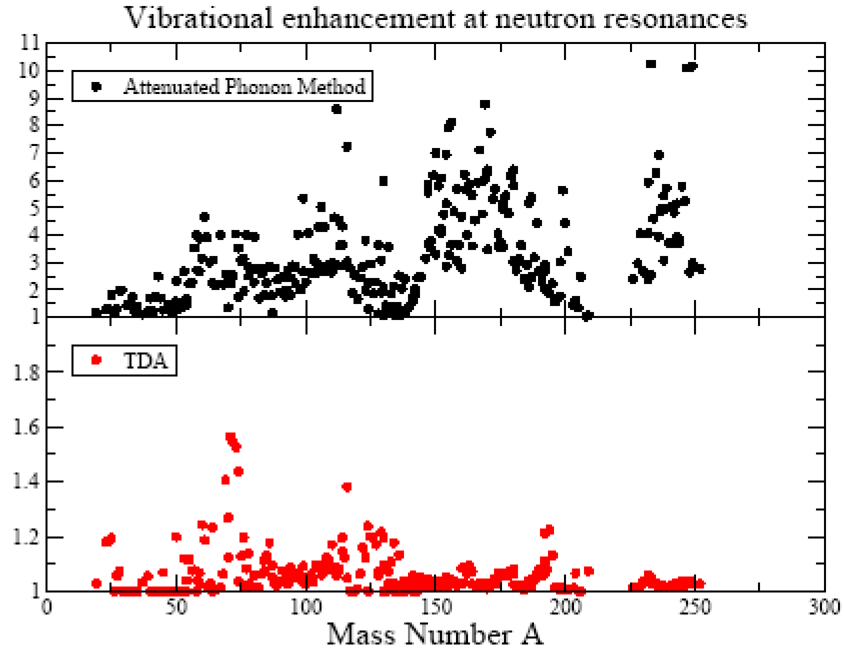


***Exclude* fraction of level density corresponding to QPTD amplitudes:**



Vibrational enhancement





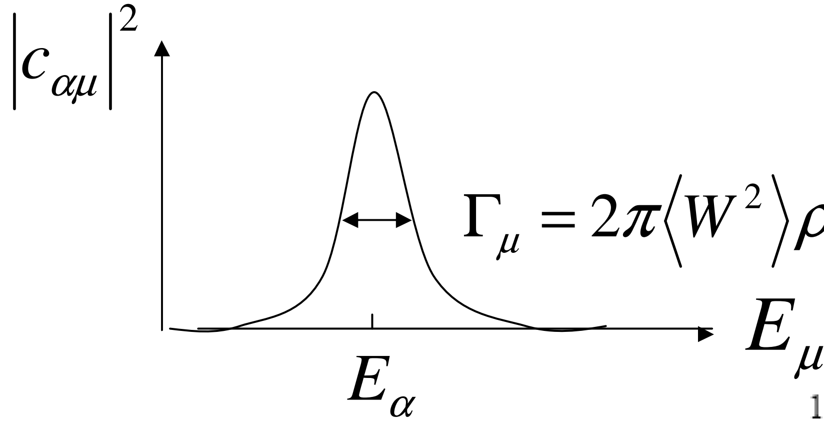
Gives VERY small vibrational enhancement!

*** Phonon energies are not much different from qp-energies**
*** Small collectivity**
*** Phonons can hardly be repeated**

Microscopic foundations for phonon method??

Role of residual interaction on level densities

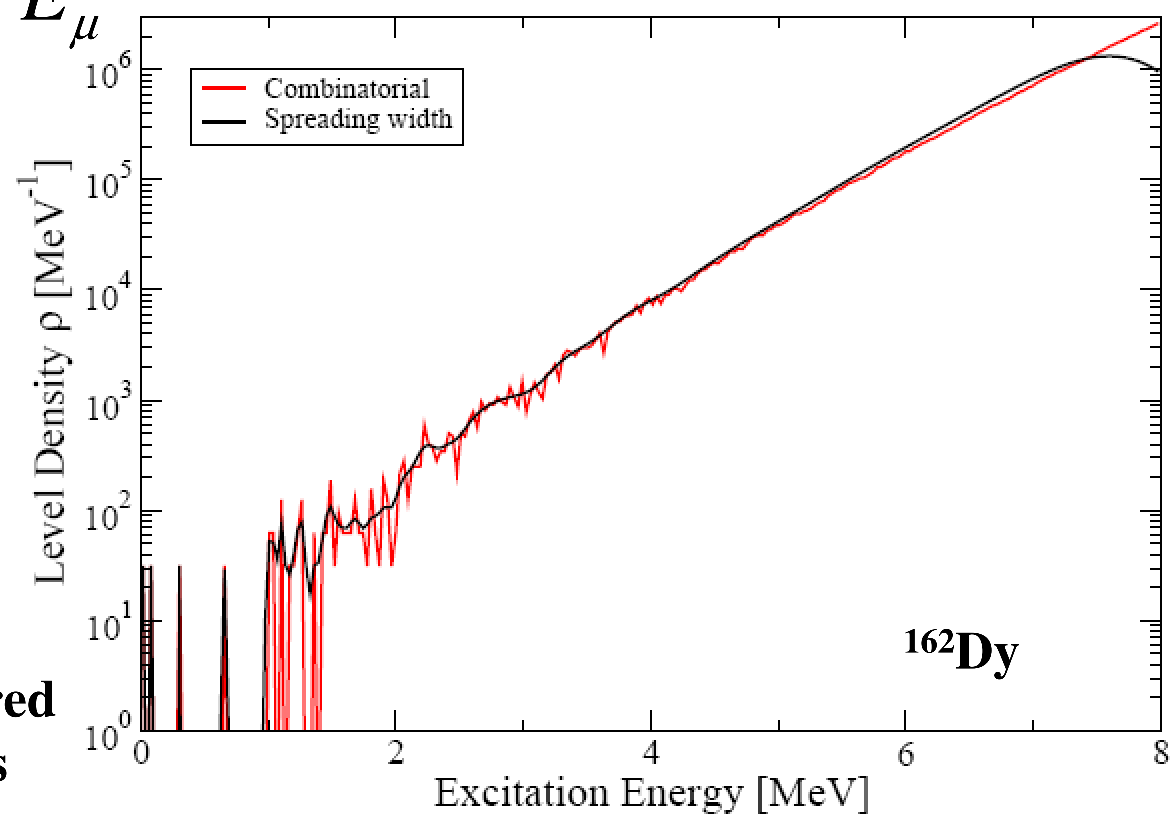
The residual 2-body interaction (W) implies a broadening of many-body states:



$$\Gamma_\mu = 2\pi \langle W^2 \rangle \rho_{2p-2h} \approx 0.039 \left(\frac{160}{A} \right)^{1/2} E_\alpha^{3/2} MeV$$ [1]

Each many-body state is smeared out by the width:

$$\Gamma_\mu (E_{exc})$$



Level density structure smeared out at high excitation energies

[1] B. Lauritzen, Th. Døssing and R.A. Broglia, Nucl Phys A457 (1986) 61.

III. Result

For all nuclei is calculated:

Level density of fixed angular momentum and parity: $\rho(E, I, \pi)$

Total level density:

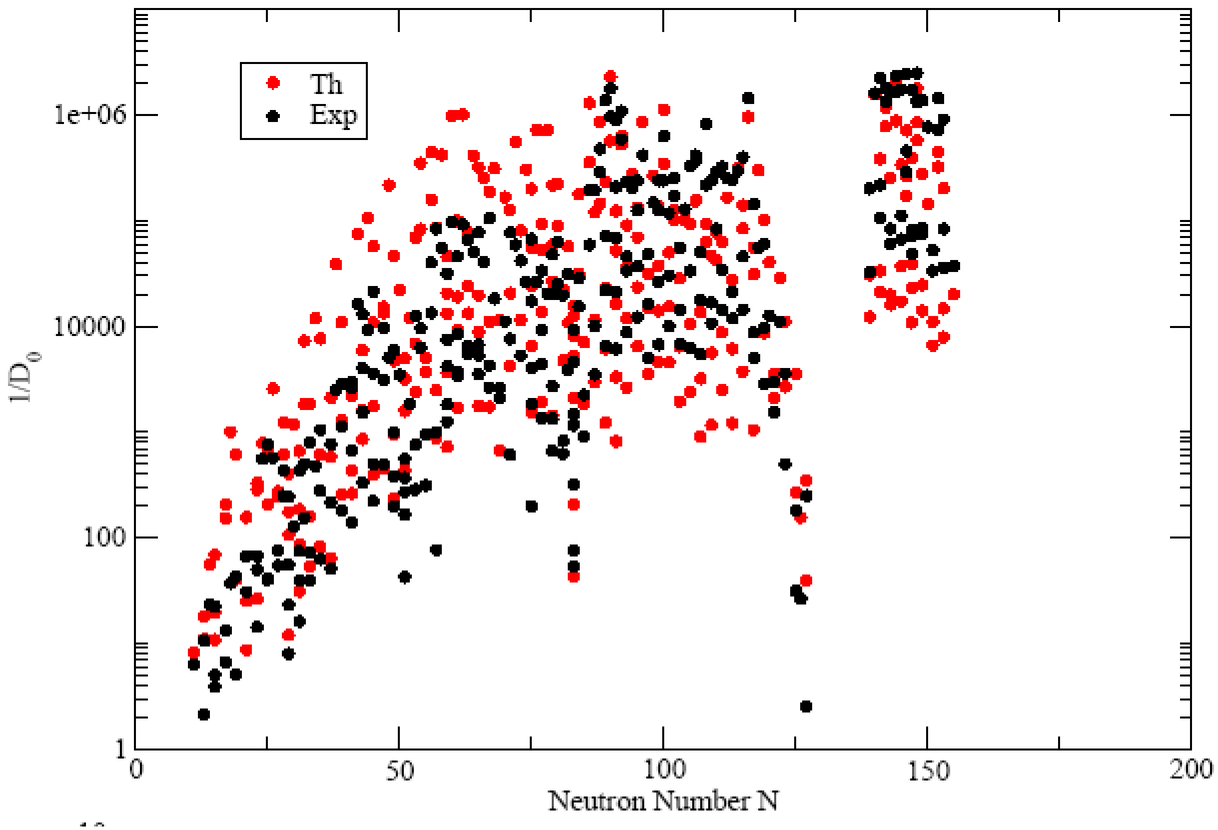
$$\rho_{tot}(E) = \sum_{I,\pi} \rho(E, I, \pi)$$

Level density of fixed parity:

$$\rho_{\pi}(E) = \sum_{I} \rho(E, I, \pi)$$

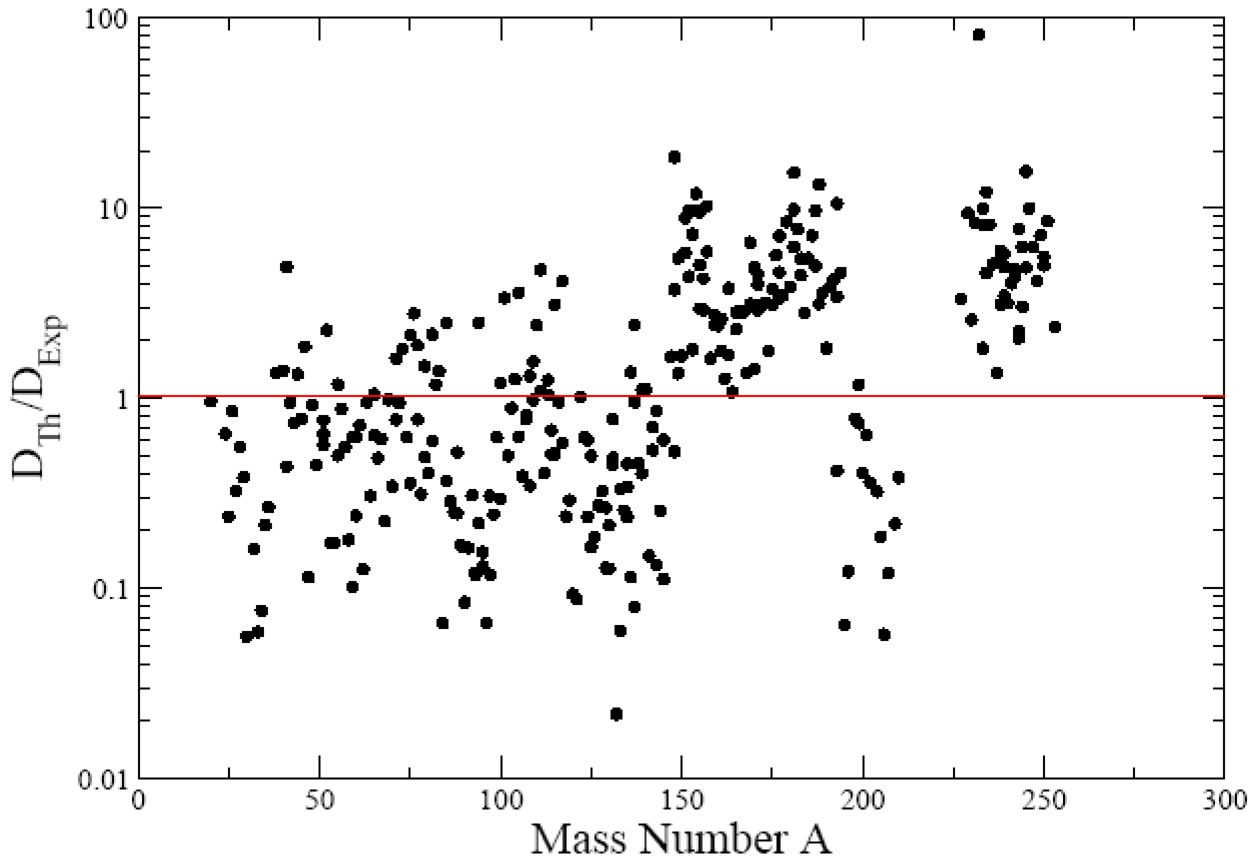
III.a Comparison to data – neutron resonance spacings

296 nuclei RIPL-2 database



Comparison to data – neutron resonance spacings

296 nuclei RIPL-2 database



$$f_{rms} = \exp\left[\frac{1}{N_e}\sum_{i=1}^{N_e}\ln^2\frac{D^i_{th}}{D^i_{exp}}\right]^{1/2}$$

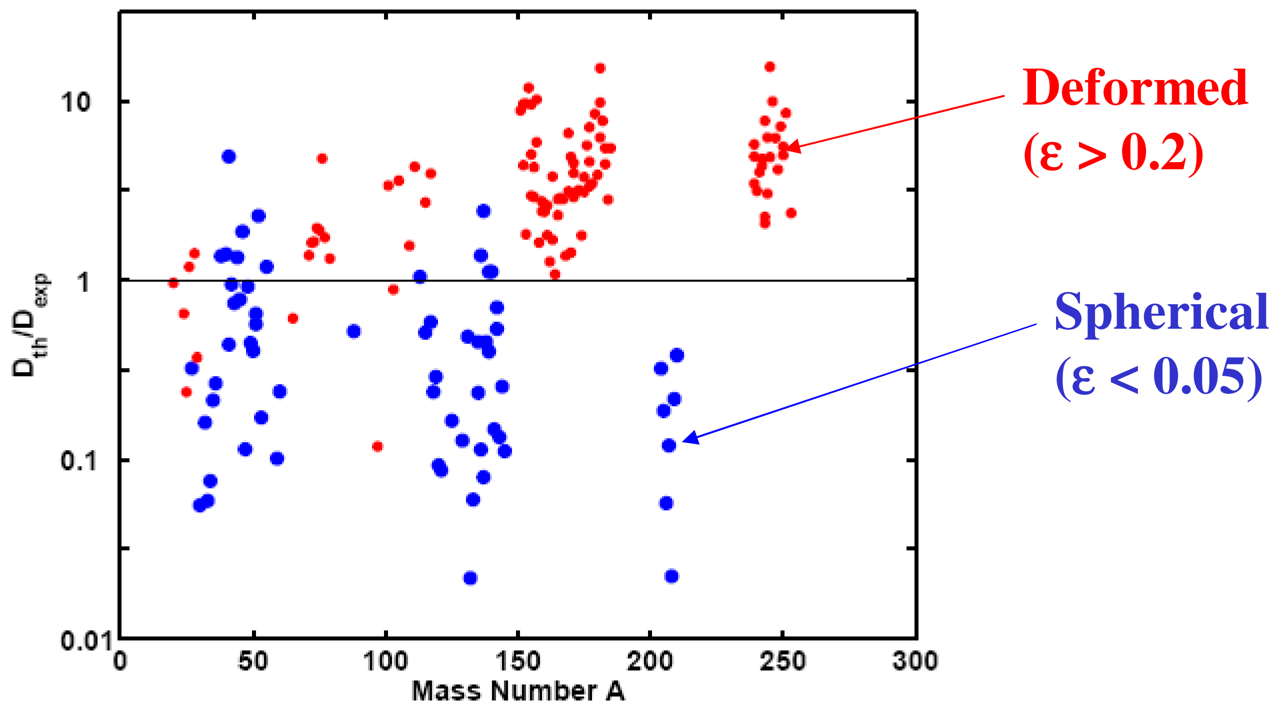
f_{rms}=4.18

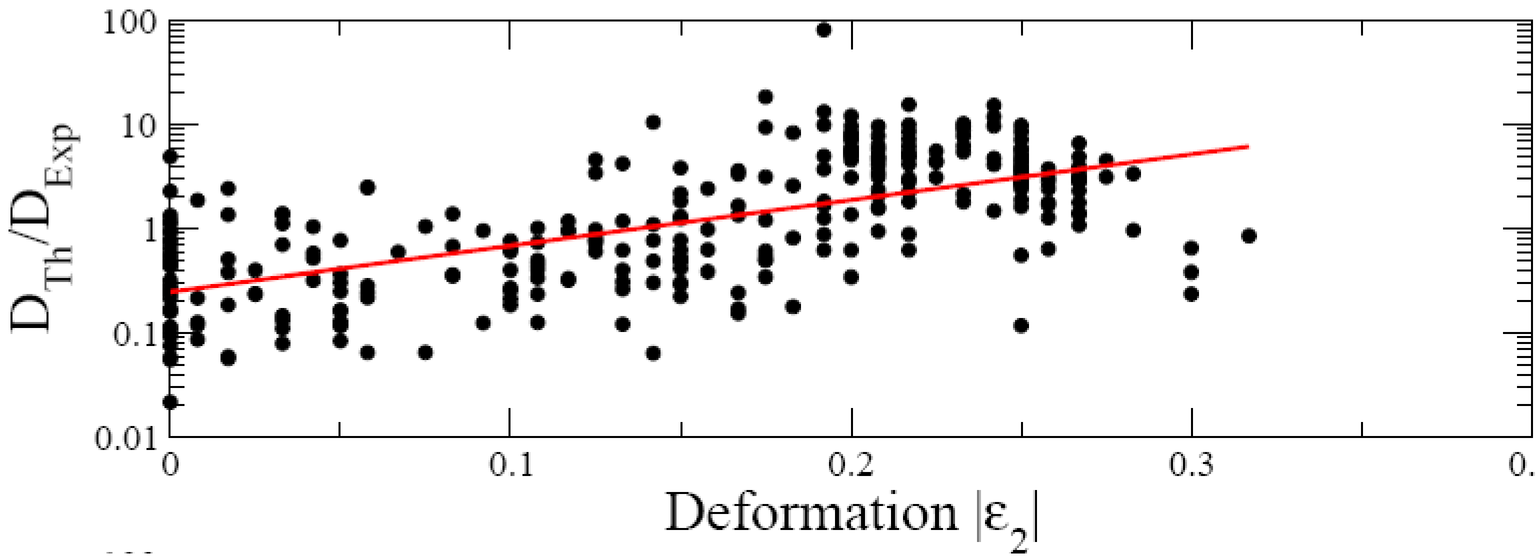
$$m = \exp\left[\frac{1}{N_e}\sum_{i=1}^{N_e}\ln\frac{D^i_{th}}{D^i_{exp}}\right]$$

m=1.09

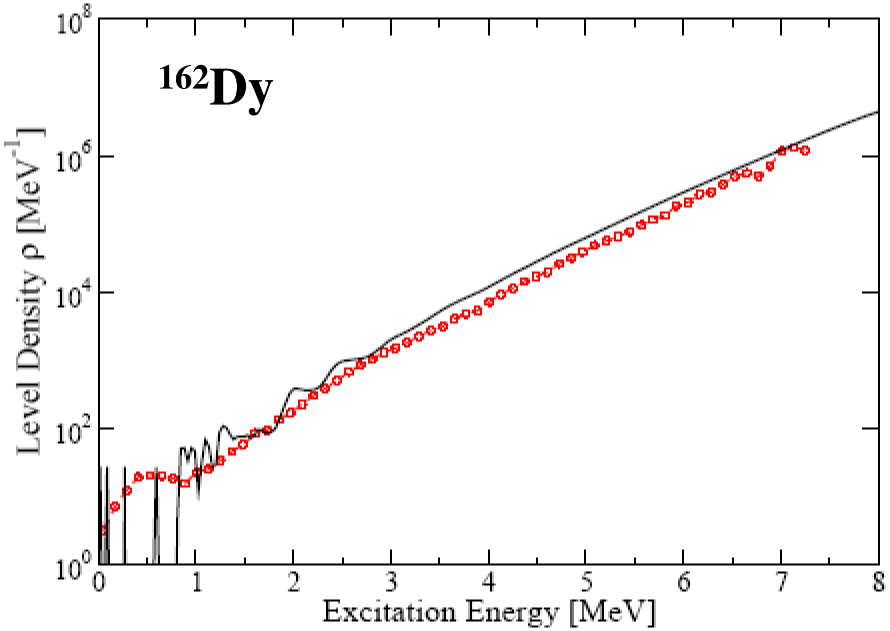
No free parameters!

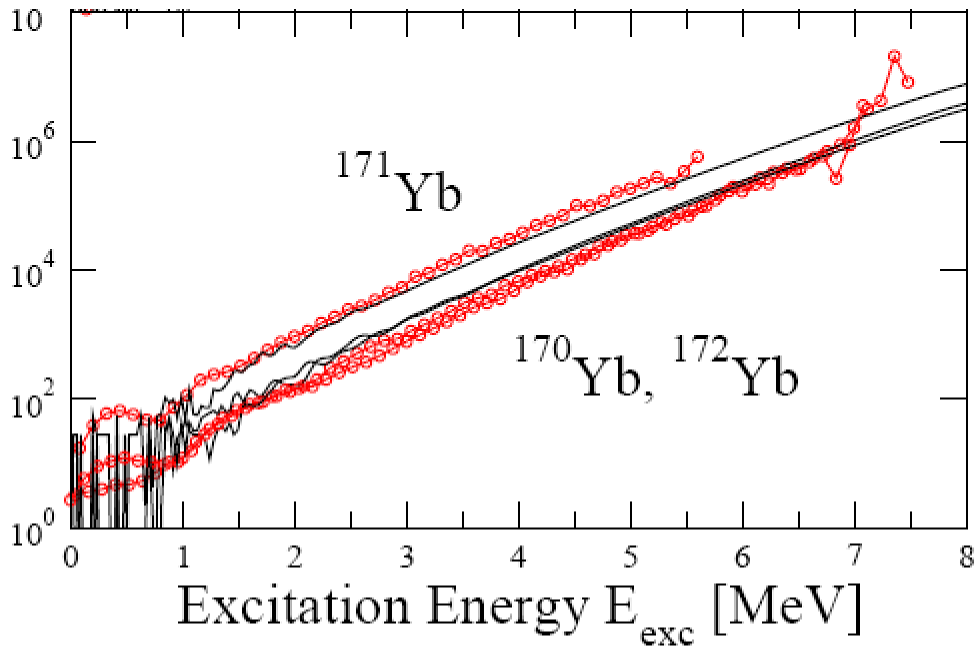
Systematics of the error in the model?





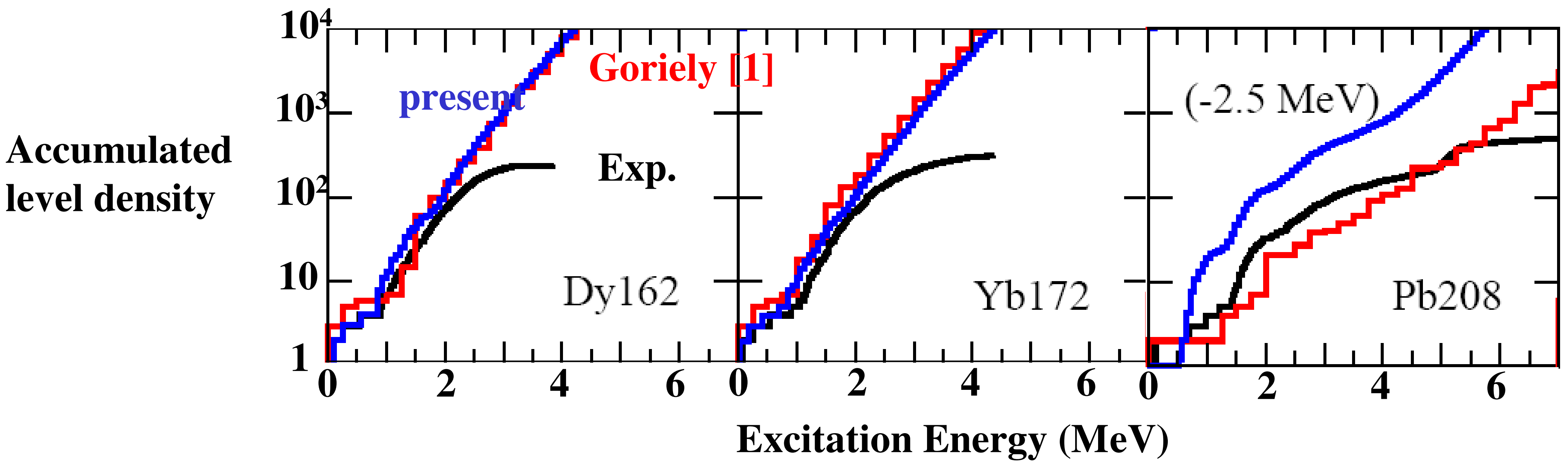
III.b Comparison to exp. data - Oslo data [1,2]





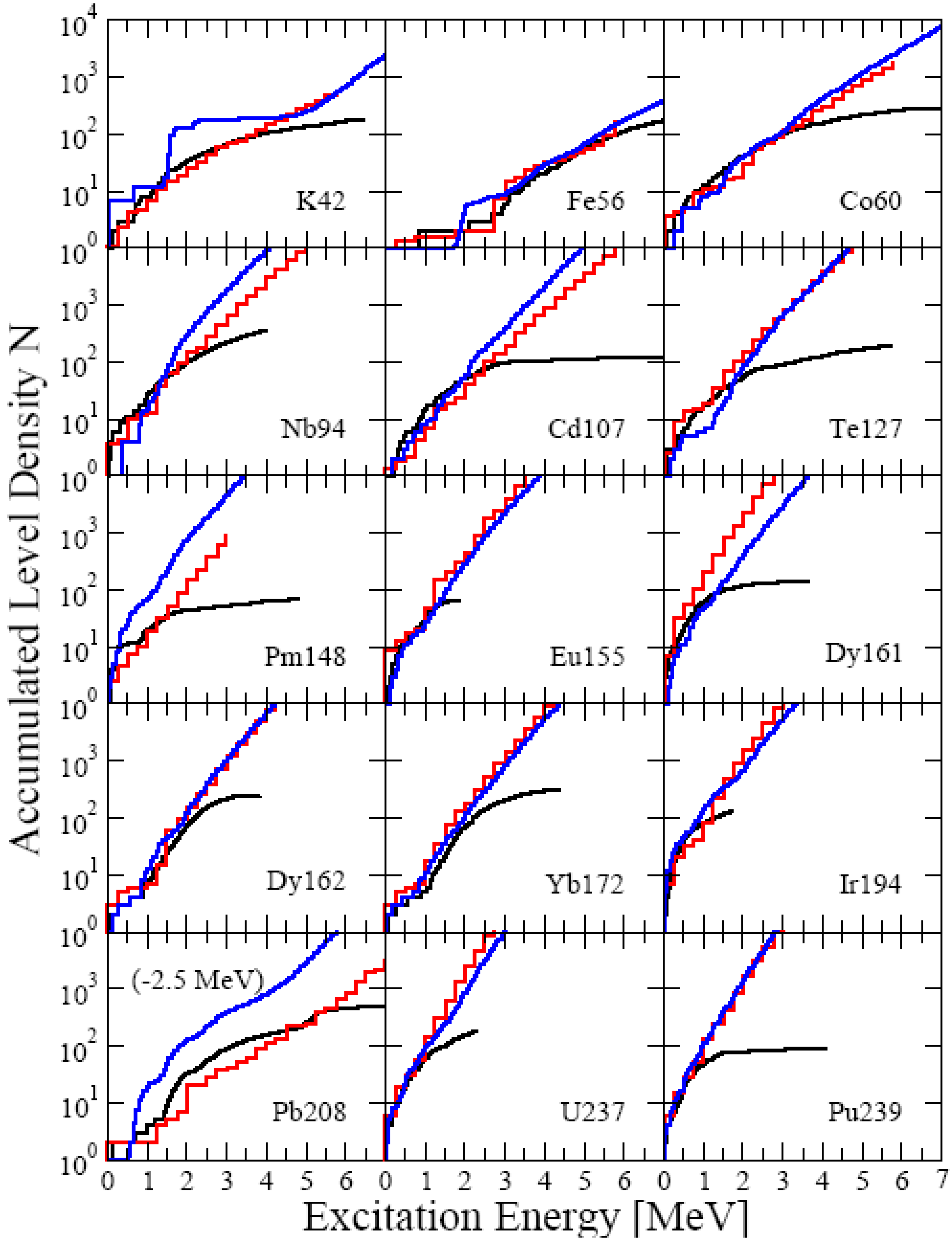
[1] M. Guttormsen et al, Phys. Rev. C68 (2003) 064306.
[2] A. Schiller et al, Phys. Rev. C63 (2001) 021306(R).

III.c Comparison to data – low-energy discrete states



[1] S. Goriely, S. Hilaire and A.J. Koning, Phys. Rev. C78 (2008) 064307:

Comparison to data – low-energy discrete states



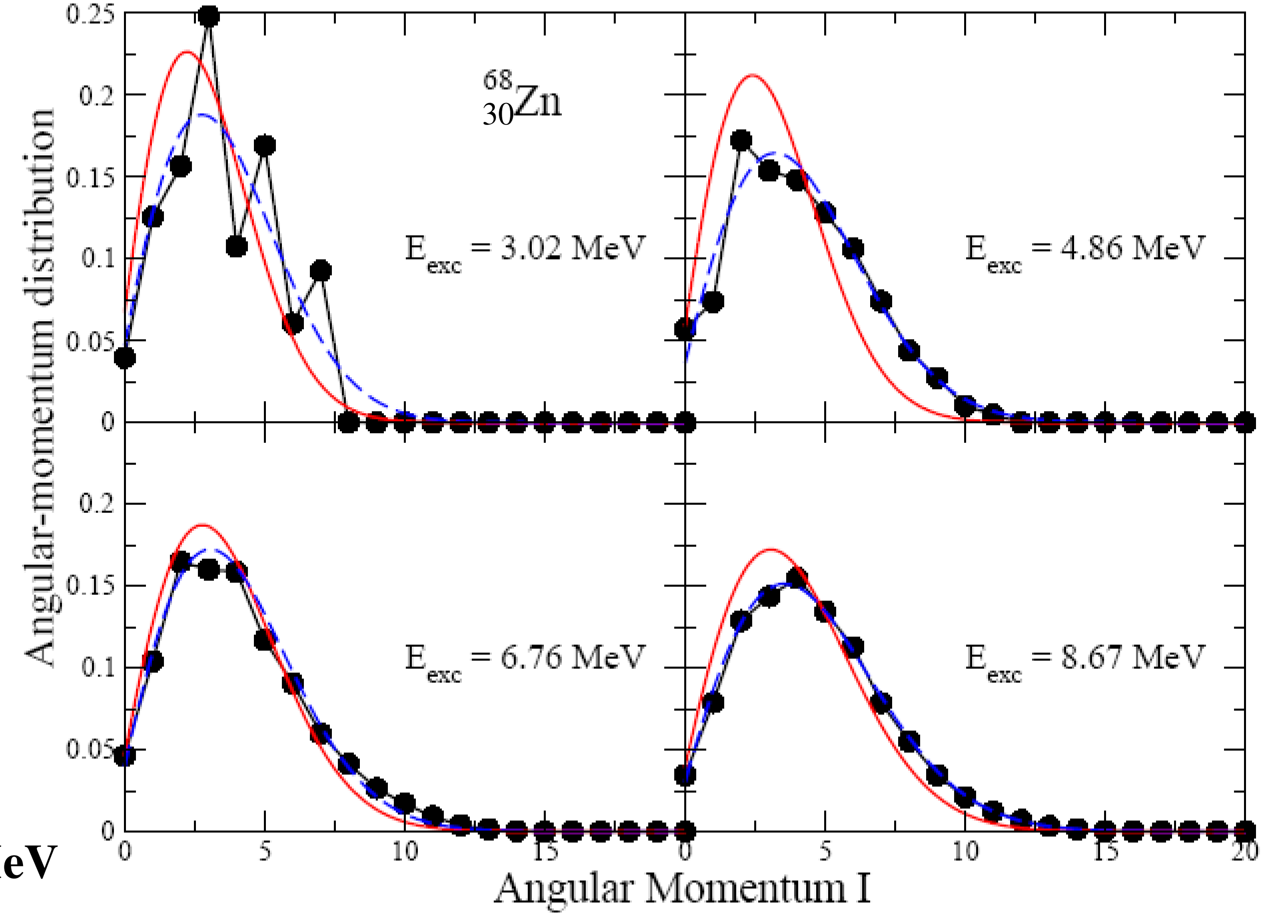
Nucl	D_{Th}/D_{Exp}	Nucl	D_{Th}/D_{Exp}	Nucl	D_{Th}/D_{Exp}
^{42}K	0.94	^{56}Fe	0.55	^{60}Co	0.62
^{94}Nb	2.50	^{107}Cd	0.77	^{127}Te	0.26
^{148}Pm	1.72	^{155}Eu	2.95	^{161}Dy	2.61
^{162}Dy	1.27	^{172}Yb	3.10	^{194}Ir	4.58
^{208}Pb	0.02	^{237}U	5.18	^{239}Pu	3.46

Spin and parity functions in microscopic level density model
- compared to Fermi gas functions

III.d Angular momentum distribution

In Fermi gas model: $$F(E_{exc}, I) = \frac{I+0.5}{\sigma^2} \exp\left(-\frac{(I+0.5)^2}{2\sigma^2}\right)$$

with spin cut-off factor: $$\sigma^2 = \frac{J_{rigid}}{\hbar^2}\sqrt{\frac{E_{exc}}{a}}$$



Large structure effects but approaches FG curve at high excitation energies

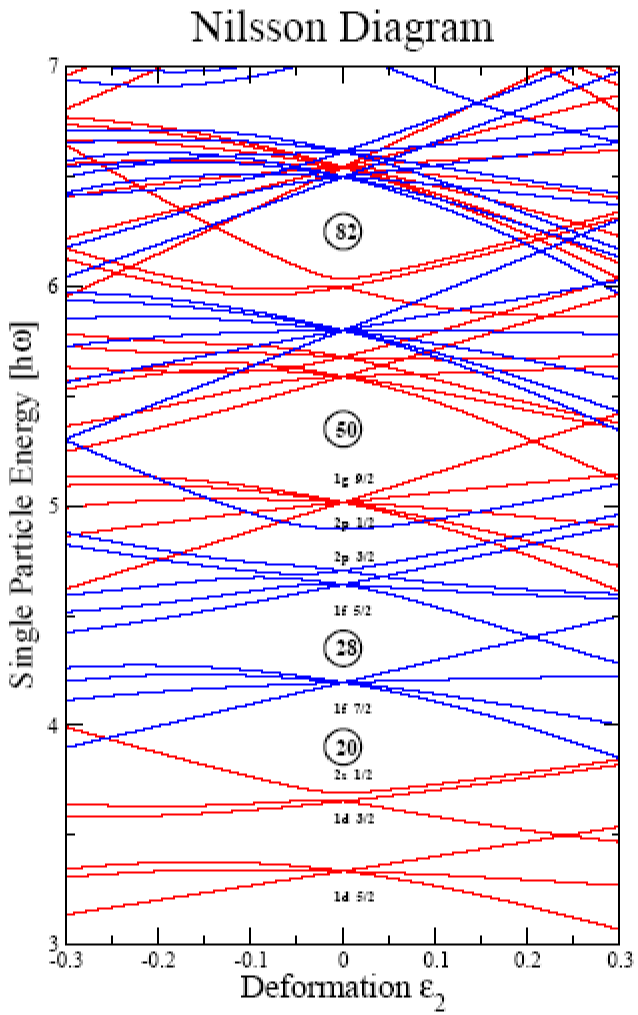
S_n=6.48 MeV

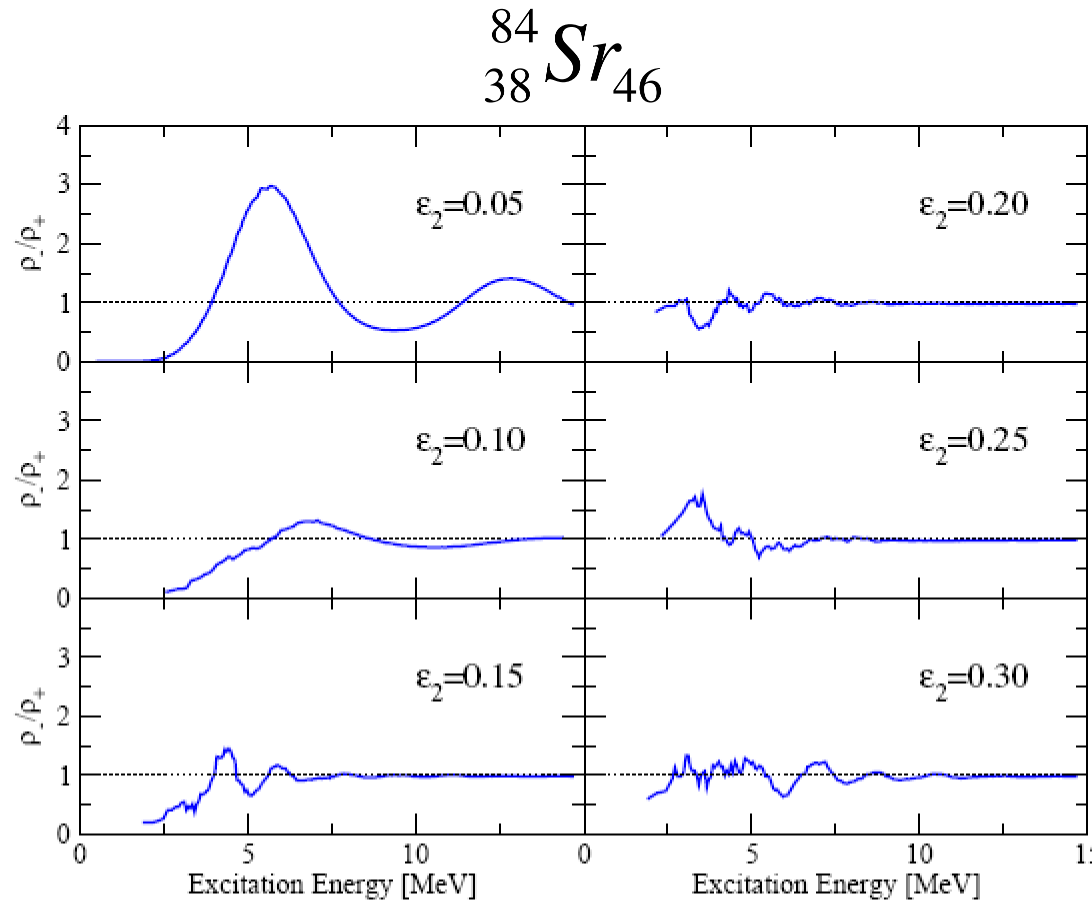
III.e Parity enhancement

Fermi gas model:
Equal level density of positive and negative parity

Microscopic model:
Shell structure may give an enhancement of one parity

Role of deformation

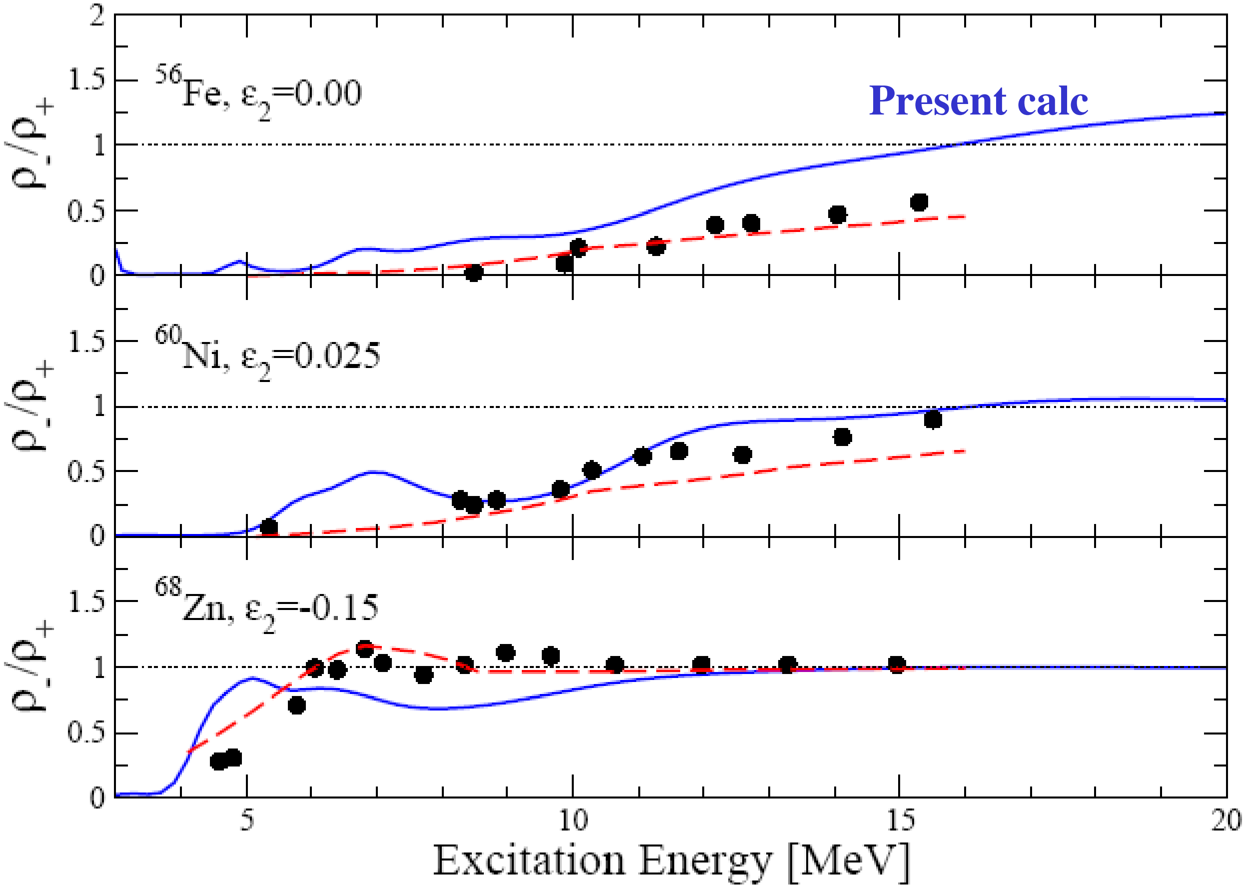




Parity enhancement stronger for spherical shape!

Compared to other models

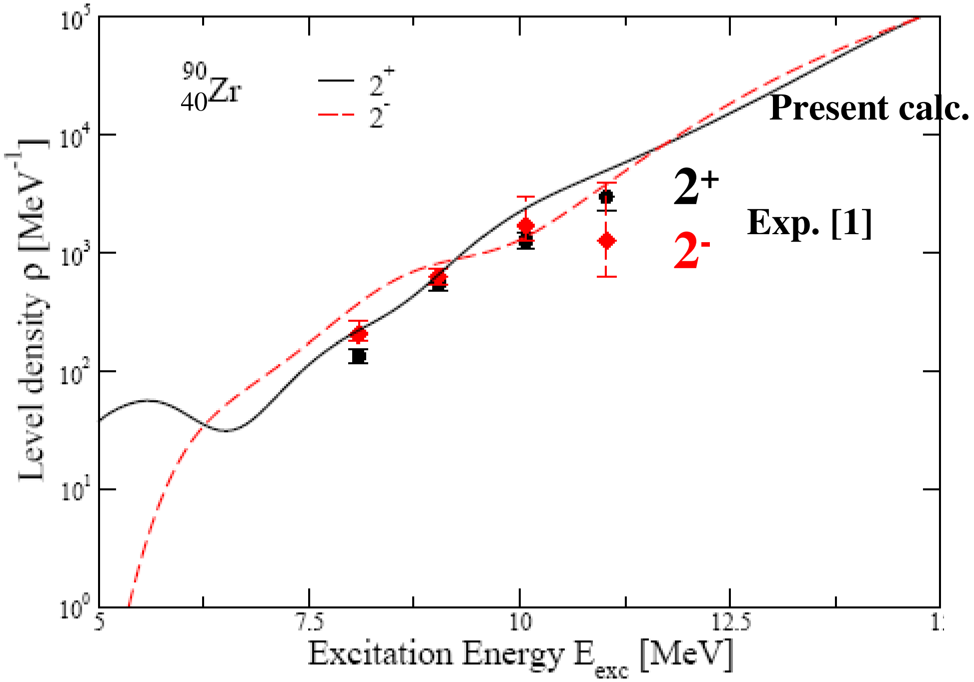
Parity enhancement in Monte Carlo calc (based on Shell Model) [1]



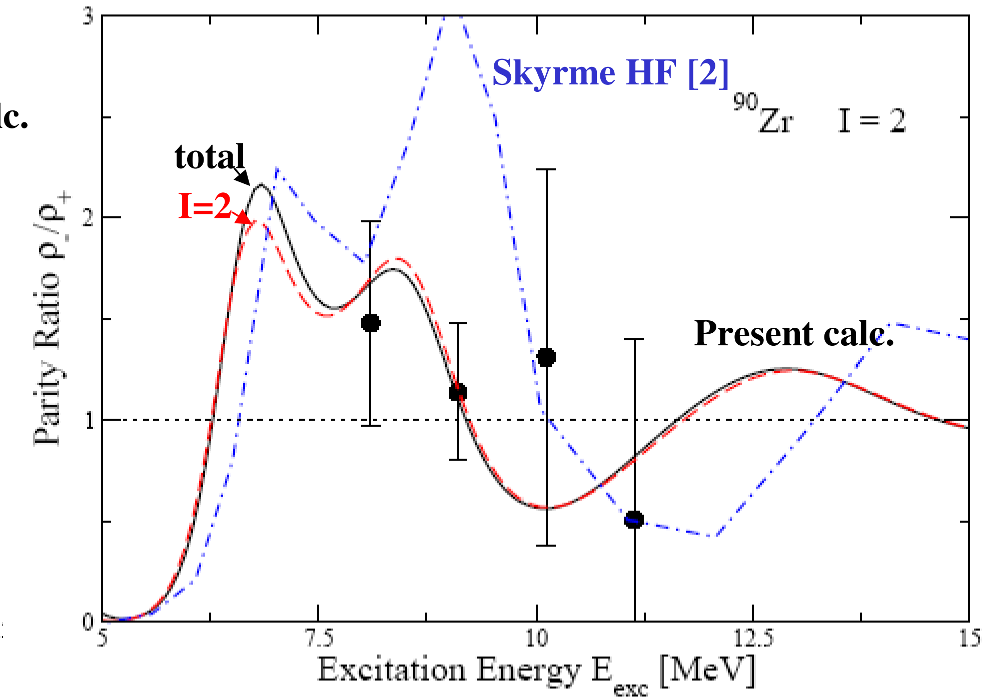
[1] Y. Alhassid, GF Bertsch, S Liu and H Nakada, PRL 84, 4313 (2000)

Measured parity enhancement in ^{90}Zr

Individual 2+ and 2- level dens.



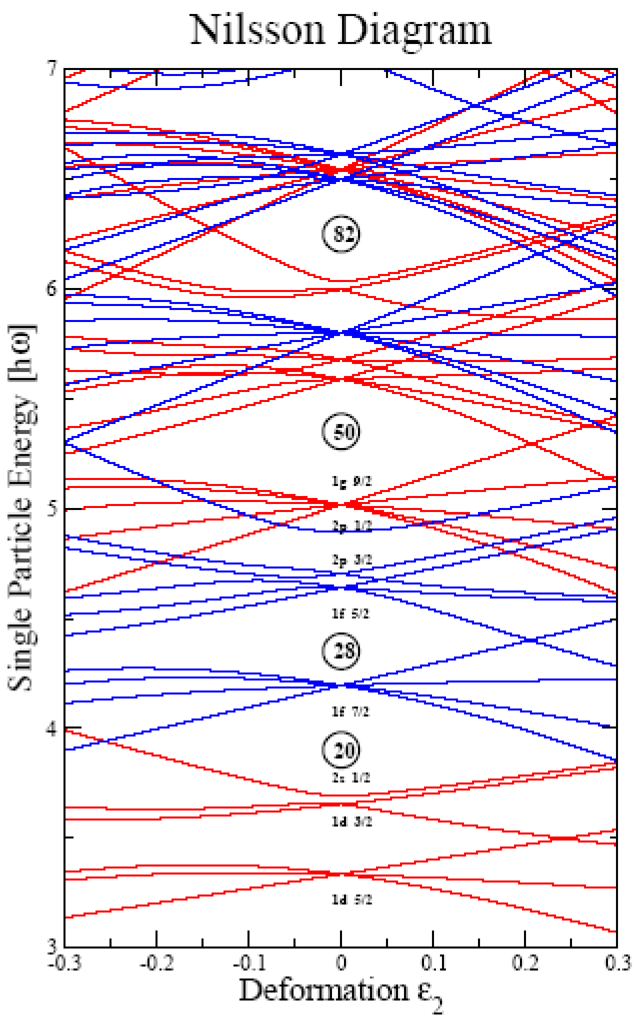
Parity ratio

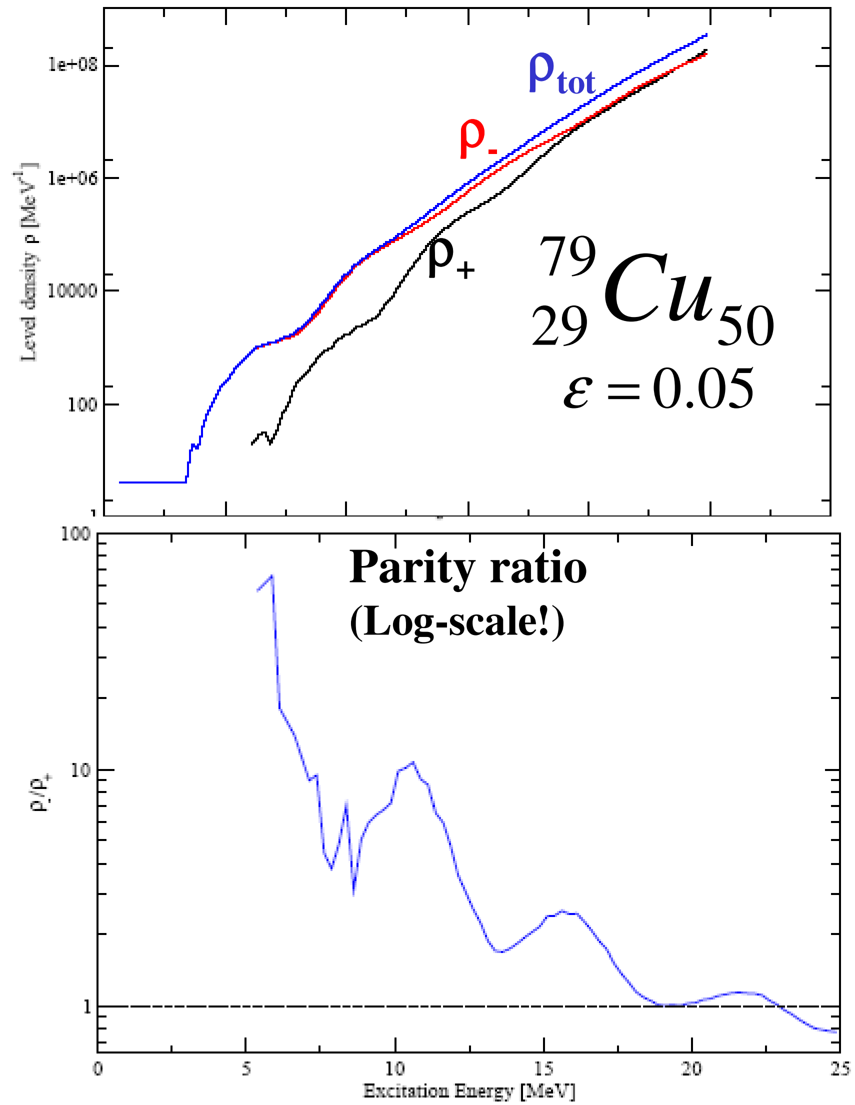


[1] High-res E2 (p-scatt.) and M2 (el. scatt.) giant res.
Y. Kalmykov et al Phys Rev Lett 99 (2007) 202502 (Richter exp.)

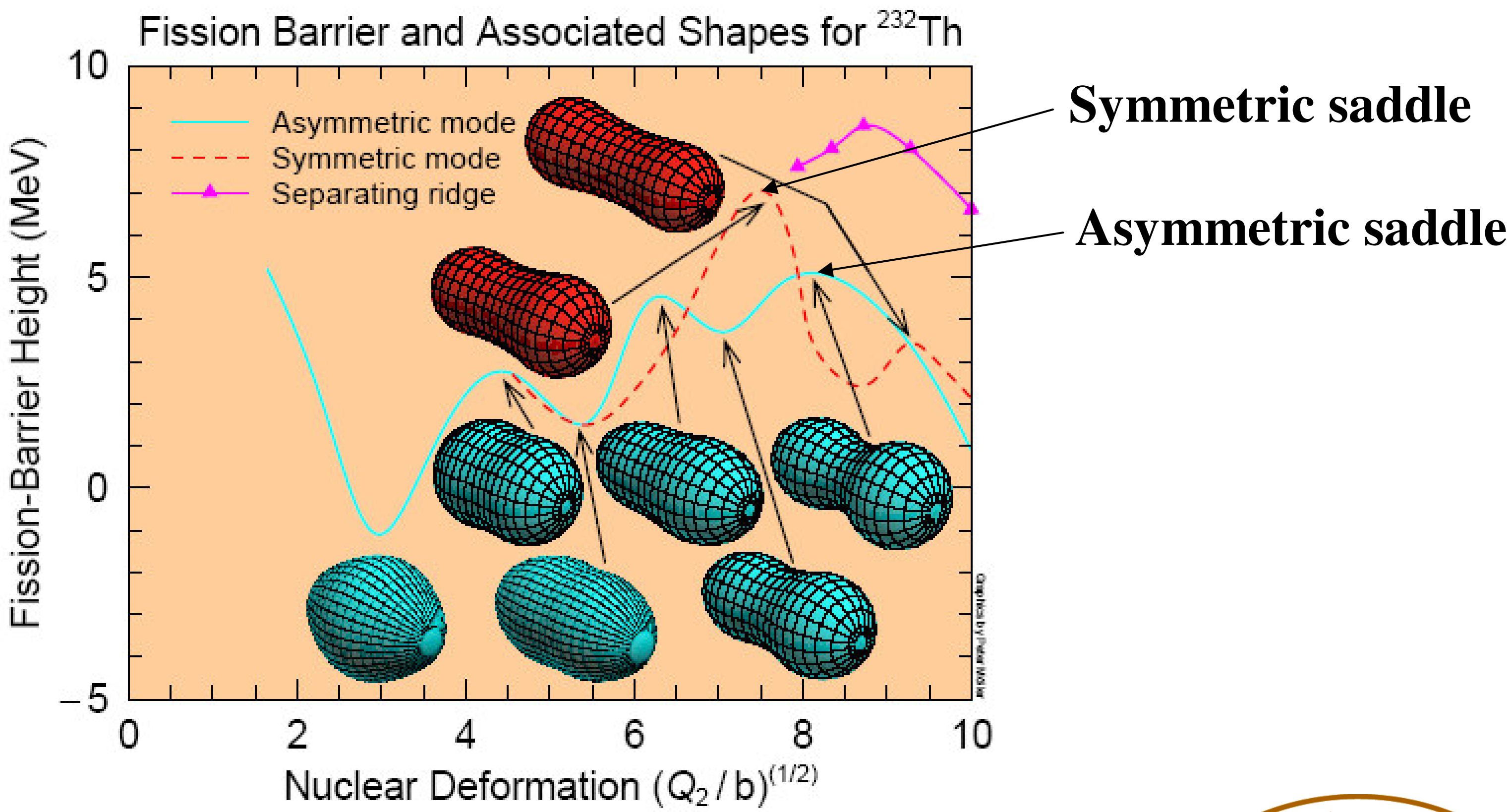
[2] Skyrme-Hartree-Fock calc.
S. Hilaire and G. Goriely, Nucl. Phys. A779 (2006) 63

Extreme enhancement for negative-parity states



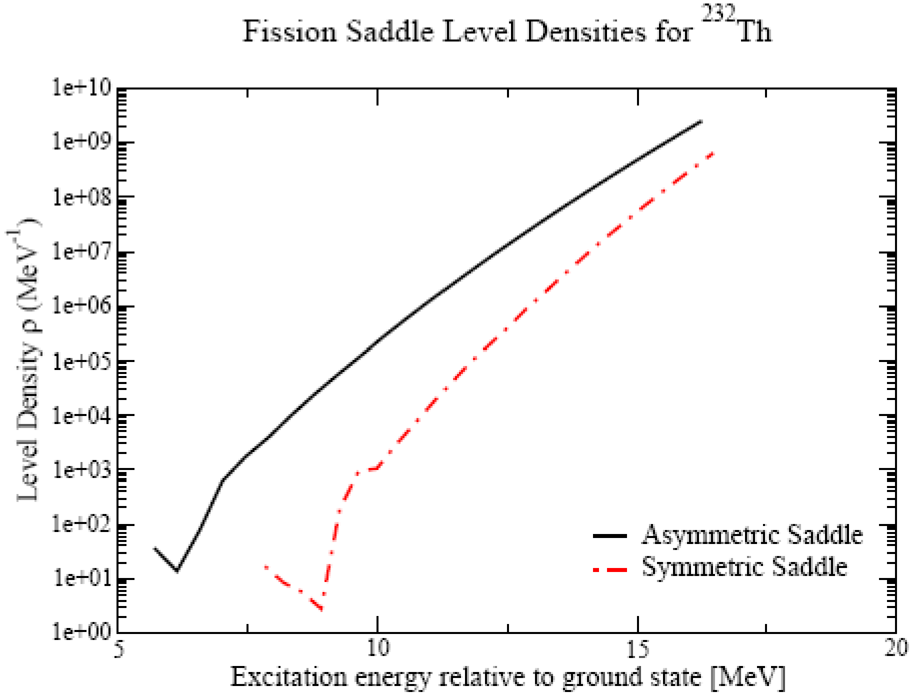


III.f Fission dynamics



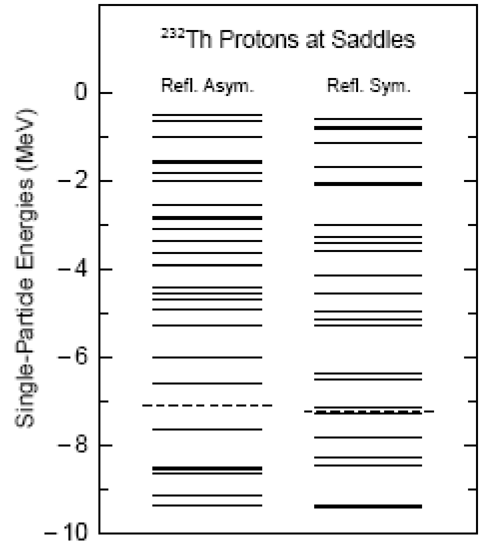
P. Möller et al, to appear in PRC (2009)

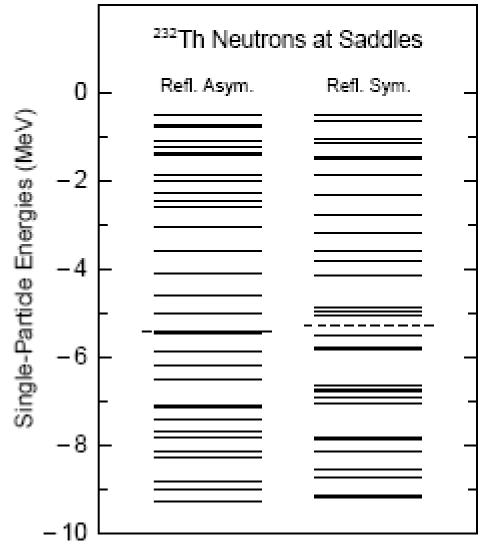
Asymetric vs symmetric shape of outer saddle



At higher excitation energy: Level density larger at symmetric fission, that will dominate.

Larger slope, for symmetric saddle, i.e. larger s.p. level density around Fermi surface:





How to improve?

- Better treatment of ground state correlations
- Improved mean field
- Improved pairing field/treatment
- Account for deformation changes vs excitation energy
- Level density of drip-line nuclei

SUMMARY

I. Microscopic model (micro canonical) for level densities including:
- well tested mean field (Möller et al)
- pairing, rotational and vibrational enhancements
- residual interaction schematically included

II. Vibrational enhancement VERY small

III. Fair agreement with data with NO parameters

IV. Pairing remains at high excitation energies

V. Detailed data on parity asymmetry – can be very large!

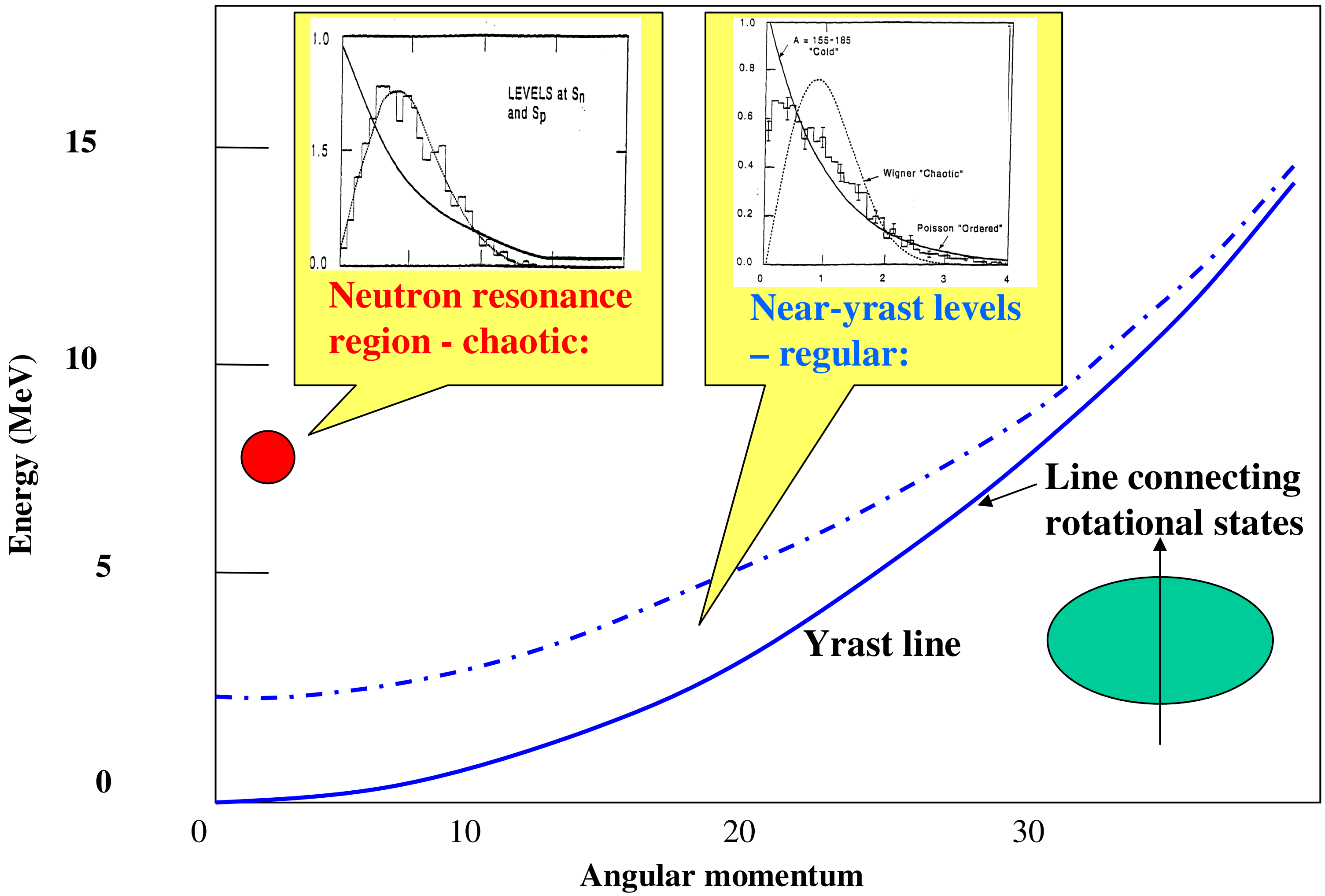
VI. Structure of level density important for fission dynamics: symmetric-asymmetric fission

VII. Level densities important test for structure models

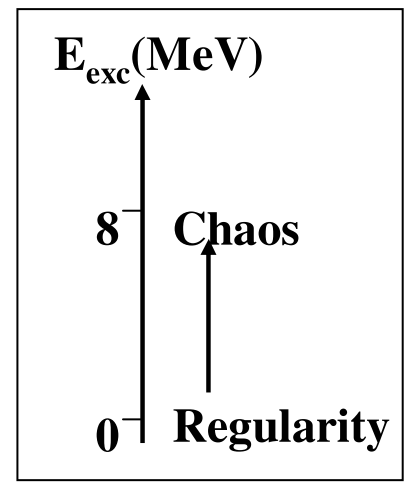
Onset of Chaos: Experimental knowledge

Nuclear Data Ensemble

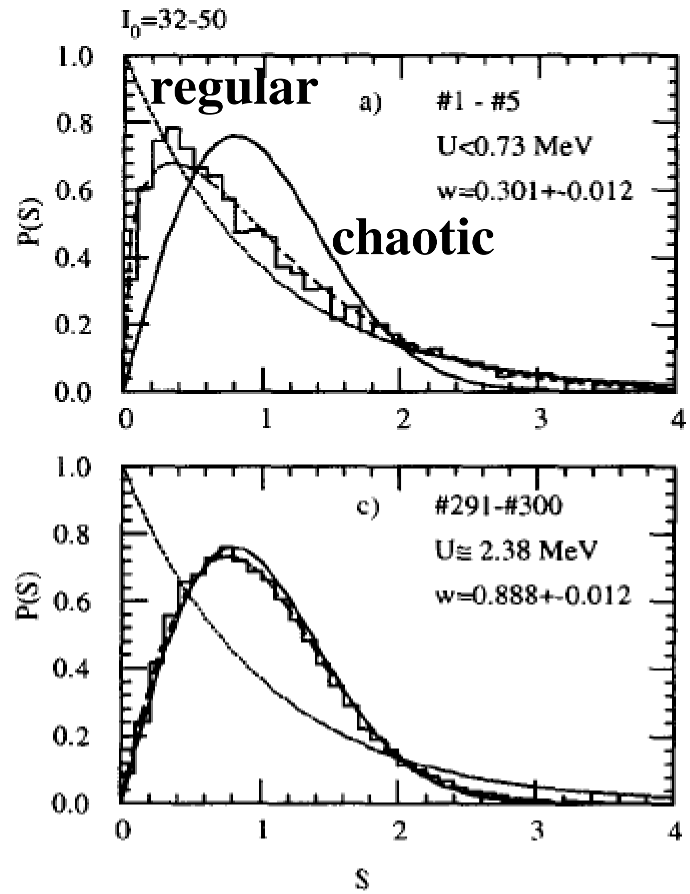
J.D. Garrett et al, Phys. Lett. B392 (1997) 24



Transition from regularity to chaos with increasing excitation energy:

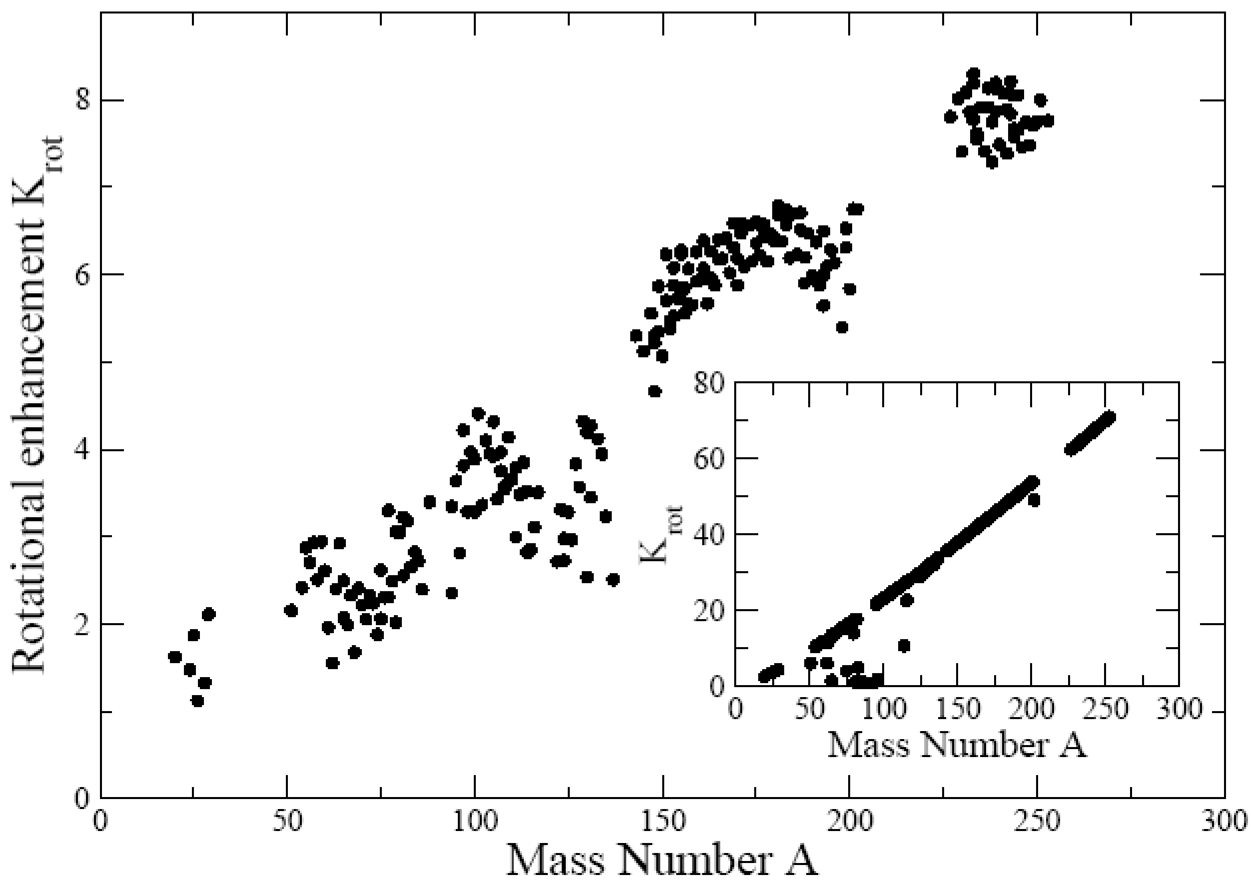


Onset of Chaos: Theoretical knowledge



Chaos sets in at 2-3 MeV above yrast in deformed highly rotating nuclei. [1]

[1] M. Matsuo et al, Nucl Phys A620, 296 (1997)


Rotational enhancement K_{rot}
Mass Number A
0
2
4
6
8
0
50
100
150
200
250
300
K_{rot}
Mass Number A
0
20
40
60
80
0
50
100
150
200
250
300